The beautiful colors from the surface of this soap bubble can be nicely explained by the wave theory of light. A soap bubble is a very thin spherical film filled with air. Light reflected from the outer and inner surfaces of this thin film of soapy water interferes constructively to produce the bright colors. Which color we see at any point depends on the thickness of the soapy water film at that point and also on the viewing angle. Near the top of the bubble, we see a small black area surrounded by a silver or white area. The bubble's thickness is smallest at that black spot, perhaps only about 30 nm thick, and is fully transparent (we see the black background).

We cover fundamental aspects of the wave nature of light, including two-slit interference and interference in thin films.

CHAPTER 24

The Wave Nature of Light

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

When a thin layer of oil lies on top of water or wet pavement, you can often see swirls of color. We also see swirls of color on the soap bubble shown above. What causes these colors?

(a) Additives in the oil or soap reflect various colors.
(b) Chemicals in the oil or soap absorb various colors.
(c) Dispersion due to differences in index of refraction in the oil or soap.
(d) The interactions of the light with a thin boundary layer where the oil (or soap) and the water have mixed irregularly.
(e) Light waves reflected from the top and bottom surfaces of the thin oil or soap film can add up constructively for particular wavelengths.

Light carries energy. Evidence for this can come from focusing the Sun's rays with a magnifying glass on a piece of paper and burning a hole in it. But how does light travel, and in what form is this energy carried? In our discussion of waves in Chapter 11, we noted that energy can be carried from place to place in basically two ways: by particles or by waves. In the first case, material objects or particles can carry energy, such as an avalanche of rocks or rushing water. In the second case, water waves and sound waves, for example, can carry energy over long distances even though the oscillating particles of the medium do not travel these distances. In view of this, what can we say about the nature of light: does light travel as a stream of particles away from its source, or does light travel in the form of waves that spread outward from the source?

Historically, this question has turned out to be a difficult one. For one thing, light does not reveal itself in any obvious way as being made up of tiny particles; nor do we see tiny light waves passing by as we do water waves. The evidence seemed to favor first one side and then the other until about 1830, when most physicists had accepted the wave theory. By the end of the nineteenth century, light was considered to be an *electromagnetic wave* (Chapter 22). In the early twentieth century, light was shown to have a particle nature as well, as we shall discuss in Chapter 27. We now speak of the wave–particle duality of light. The wave theory of light remains valid and has proved very successful. In this Chapter we investigate the evidence for the wave theory and how it has been used to explain a wide range of phenomena.

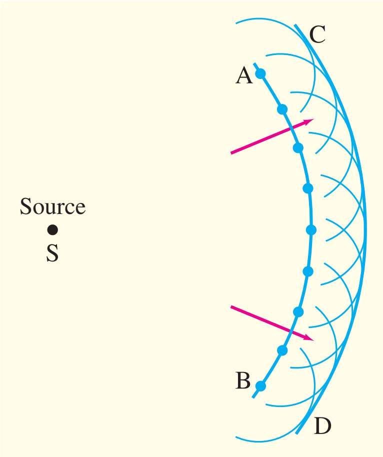


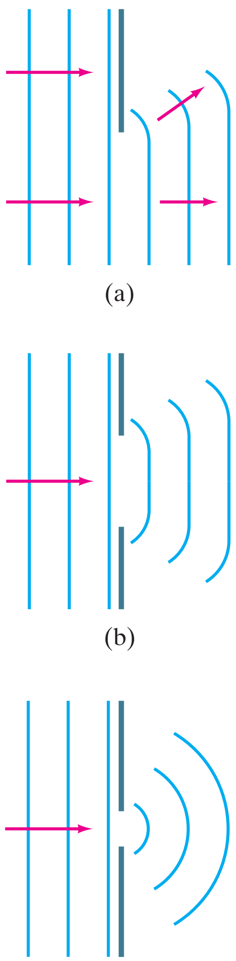
FIGURE 24–1 Huygens' principle, used to determine wave front CD when wave front AB is given.

24–1 Waves vs. Particles; Huygens' Principle and Diffraction

The Dutch scientist Christian Huygens (1629–1695), a contemporary of Newton, proposed a wave theory of light that had much merit. Still useful today is a technique Huygens developed for predicting the future position of a wave front when an earlier position is known. By a **wave front**, we mean all the points along a two- or three-dimensional wave that form a wave crest—what we simply call a "wave" as seen on the ocean. Wave fronts are perpendicular to rays as discussed in Chapter 11 (Fig. 11–35). **Huygens' principle** can be stated as follows: *Every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope of all the wavelets*—that is, *the tangent to all of them*.

As an example of the use of Huygens' principle, consider the wave front AB in Fig. 24–1, which is traveling away from a source S. We assume the medium is *isotropic*—that is, the speed v of the waves is the same in all directions. To find the wave front a short time t after it is at AB, tiny circles are drawn at points along AB with radius $r = vt$. The centers of these tiny circles are shown as blue dots on the original wave front AB, and the circles represent Huygens' (imaginary) wavelets. The tangent to all these wavelets, the curved line CD, is the new position of the wave front after a time t.

FIGURE 24–2 Huygens' principle is consistent with diffraction (a) around the edge of an obstacle, (b) through a large hole, (c) through a small hole whose size is on the order of the wavelength of the wave.



Huygens' principle is particularly useful for analyzing what happens when waves run into an obstacle and the wave fronts are partially interrupted. Huygens' principle predicts that waves bend in behind an obstacle, as shown in Fig. 24–2. This is just what water waves do, as we saw in Chapter 11 (Figs. 11–45 and 11–46). The bending of waves behind obstacles into the "shadow region" is known as **diffraction**. Since diffraction occurs for waves, but not for particles, it can serve as one means for distinguishing the nature of light.

Note, as shown in Fig. 24–2, that diffraction is most prominent when the size of the opening is on the order of the wavelength of the wave. If the opening is much larger than the wavelength, diffraction may go unnoticed.

Does light exhibit diffraction? In the mid-seventeenth century, the Jesuit priest Francesco Grimaldi (1618–1663) had observed that when sunlight entered a darkened room through a tiny hole in a screen, the spot on the opposite wall was larger than would be expected from geometric rays. He also observed that the border of the image was not clear but was surrounded by colored fringes. Grimaldi attributed this to the diffraction of light.

The wave model of light nicely accounts for diffraction. But the ray model (Chapter 23) cannot account for diffraction, and it is important to be aware of such limitations to the ray model. Geometric optics using rays is successful in a wide range of situations only because normal openings and obstacles are much larger than the wavelength of the light, and so relatively little diffraction or bending occurs.

*24–2 Huygens' Principle and the Law of Refraction

The laws of reflection and refraction were well known in Newton's time. The law of reflection could not distinguish between the two theories we just discussed: waves versus particles. When waves reflect from an obstacle, the angle of incidence equals the angle of reflection (Fig. 11–36). The same is true of particles—think of a tennis ball without spin striking a flat surface.

The law of refraction is another matter. Consider a ray of light entering a medium where it is bent toward the normal, as when traveling from air into water. As shown in Fig. 24–3, this bending can be constructed using Huygens' principle if we assume the speed of light is less in the second medium $(v_2 < v_1)$. In time t, point B on wave front AB (perpendicular to the incoming ray) travels a distance $v_1 t$ to reach point D. Point A on the wave front, traveling in the second medium, goes a distance $v_2 t$ to reach point C, and $v_2 t < v_1 t$. Huygens' principle is applied to points A and B to obtain the curved wavelets shown at C and D. The wave front is tangent to these two wavelets, so the new wave front is the line CD. Hence the rays, which are perpendicular to the wave fronts, bend toward the normal if $v_2 < v_1$, as drawn. (This is basically the same discussion as we used around Fig. 11–42.)

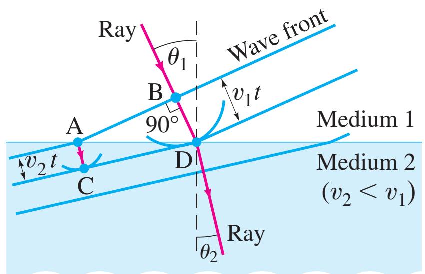


FIGURE 24–3 Refraction explained, using Huygens' principle. Wave fronts are perpendicular to the rays.

Newton favored a particle theory of light which predicted the opposite result, that the speed of light would be greater in the second medium $(v_2 > v_1)$. Thus the wave theory predicts that the speed of light in water, for example, is less than in air; and Newton's particle theory predicts the reverse. An experiment to actually measure the speed of light in water was performed in 1850 by the French physicist Jean Foucault, and it confirmed the wave-theory prediction. By then, however, the wave theory was already fully accepted, as we shall see in the next Section.

Snell's law of refraction follows directly from Huygens' principle, given that the speed of light v in any medium is related to the speed in a vacuum, c, and the index of refraction, n, by Eq. 23–4: that is, $v = c/n$. From the Huygens' construction of Fig. 24–3, angle ADC is equal to θ_2 and angle BAD is equal to θ_1. Then for the two triangles that have the common side AD, we have

$$\sin\theta_1 = \frac{v_1 t}{\mathrm{AD}}, \qquad \sin\theta_2 = \frac{v_2 t}{\mathrm{AD}}.$$

We divide these two equations and obtain

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}.$$

Then, by Eq. 23–4, $v_1 = c/n_1$ and $v_2 = c/n_2$, so we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2,$$

which is Snell's law of refraction, Eq. 23–5. (The law of reflection can be derived from Huygens' principle in a similar way.)

When a light wave travels from one medium to another, its frequency does not change, but its wavelength does. This can be seen from Fig. 24–3, where each of the blue lines representing a wave front corresponds to a crest (peak) of the wave. Then

CAUTION
Frequency is fixed, wavelength can change

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2 t}{v_1 t} = \frac{v_2}{v_1} = \frac{n_1}{n_2},$$

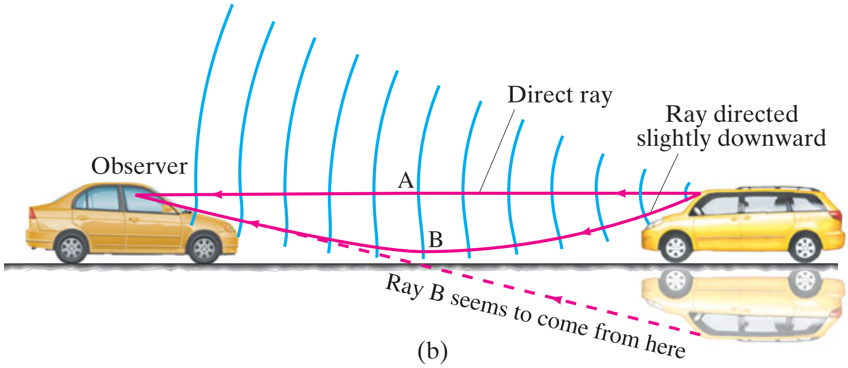
where, in the last step, we used Eq. 23–4, $v = c/n$. If medium 1 is a vacuum (or air), so $n_1 = 1$, $v_1 = c$, and we call λ_1 simply λ, then the wavelength in another medium of index of refraction n $(= n_2)$ will be

$$\lambda_n = \frac{\lambda}{n}. \qquad \textbf{(24–1)}$$

This result is consistent with the frequency f being unchanged no matter what medium the wave is traveling in, since $c = f\lambda$.

EXERCISE A A light beam in air with wavelength = 500 nm, frequency = 6.0×10^{14} Hz, and speed = 3.0×10^8 m/s goes into glass which has an index of refraction = 1.5. What are the wavelength, frequency, and speed of the light in the glass?

(a)



(b)

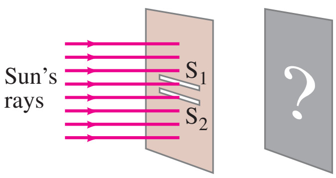
FIGURE 24–4 (a) A highway mirage. (b) Drawing (greatly exaggerated) showing wave fronts and rays to explain highway mirages. Note how sections of the wave fronts near the ground are farther apart and so are moving faster.

PHYSICS APPLIED
Highway mirages

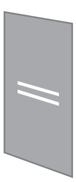
Wave fronts can be used to explain how mirages are produced by refraction of light. For example, on a hot day motorists sometimes see a mirage of water on the highway ahead of them, with distant vehicles seemingly reflected in it (Fig. 24–4a). On a hot day, there can be a layer of very hot air next to the roadway (made hot by the Sun beating down on the road). Hot air is less dense than cooler air, so the index of refraction is slightly lower in the hot air. In Fig. 24–4b, we see a diagram of light coming from one point on a distant car (on the right) heading left toward the observer. Wave fronts and two rays (perpendicular to the wave fronts) are shown. Ray A heads directly at the observer and follows a straight-line path, and represents the normal view of the distant car. Ray B is a ray initially directed slightly downward but, instead of hitting the road, it bends slightly as it moves through layers of air of different index of refraction. The wave fronts, shown in blue in Fig. 24–4b, move slightly faster in the layers of (less dense) air nearer the ground. Thus ray B is bent as shown, and seems to the observer to be coming from below (dashed line) as if reflected off the road. Hence the mirage.

24–3 Interference—Young's Double-Slit Experiment

FIGURE 24–5 (a) Young's double-slit experiment. (b) If light consists of particles, we would expect to see two bright lines on the screen behind the slits. (c) In fact, many lines are observed. The slits and their separation need to be very thin.



(a) Viewing screen



(b) Viewing screen (particle theory prediction)

(c) Viewing screen (actual)

In 1801, the Englishman Thomas Young (1773–1829) obtained convincing evidence for the wave nature of light and was even able to measure wavelengths for visible light. Figure 24–5a shows a schematic diagram of Young's famous double-slit experiment. To have light from a single source, Young used the sunlight passing through a very narrow slit in a window covering. This beam of parallel rays falls on a screen containing two closely spaced slits, S_1 and S_2. (The slits and their separation are very narrow, not much larger than the wavelength of the light.) If light consists of tiny particles, we would expect to see two bright lines on a screen placed behind the slits as in (b). But instead, a series of bright lines are seen as in (c). Young was able to explain this result as a **wave-interference** phenomenon.

To understand why, consider **plane waves**[†] of light of a single wavelength—called **monochromatic**, meaning "one color"—falling on the two slits as shown in Fig. 24–6. Because of diffraction, the waves leaving the two small slits spread out as shown. This is equivalent to the interference pattern produced when two rocks are thrown into a lake (Fig. 11–38), or when sound from two loudspeakers interferes (Fig. 12–16). Recall Section 11–11 on wave interference.

[†]See pages 312 and 628.

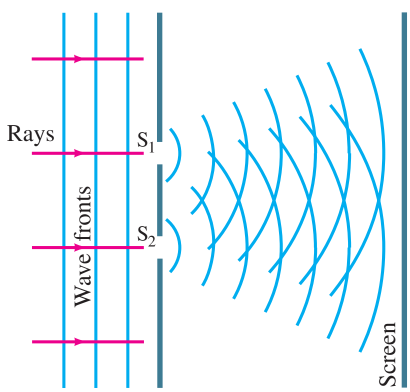


FIGURE 24–6 Plane waves (parallel flat wave fronts) fall on two slits. If light is a wave, light passing through one of two slits should interfere with light passing through the other slit.

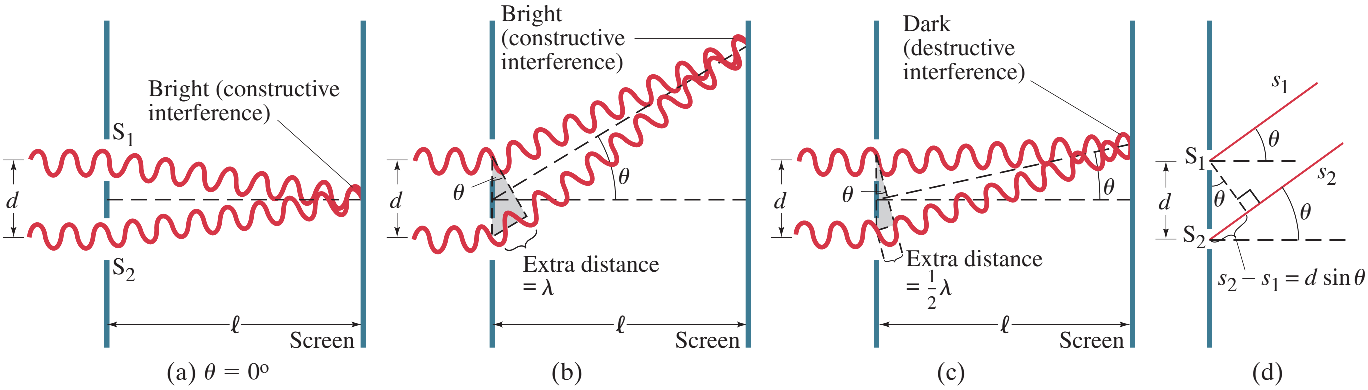
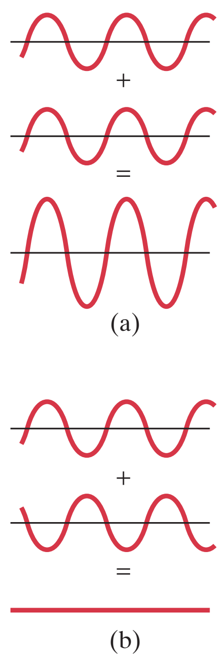


FIGURE 24–7 How the wave theory explains the pattern of lines seen in the double-slit experiment. (a) At the center of the screen, waves from each slit travel the same distance and are in phase. [Assume $\ell \gg d$.] (b) At this angle θ, the lower wave travels an extra distance of one whole wavelength, and the waves are in phase; note from the shaded triangle that the path difference equals $d \sin\theta$. (c) For this angle θ, the lower wave travels an extra distance equal to one-half wavelength, so the two waves arrive at the screen fully out of phase. (d) A more detailed diagram showing the geometry for parts (b) and (c).

To see how an interference pattern is produced on the screen, we make use of Fig. 24–7. Waves of wavelength λ are shown entering the slits S_1 and S_2, which are a distance d apart. The waves spread out in all directions after passing through the slits (Fig. 24–6), but they are shown in Figs. 24–7a, b, and c only for three different angles θ. In Fig. 24–7a, the waves reaching the center of the screen are shown ($\theta = 0°$). Waves from the two slits travel the same distance, so they are **in phase**: a crest of one wave arrives at the same time as a crest of the other wave. Hence the amplitudes of the two waves add to form a larger amplitude as shown in Fig. 24–8a. This is **constructive interference**, and there is a bright line at the center of the screen. Constructive interference also occurs when the paths of the two rays differ by one wavelength (or any whole number of wavelengths), as shown in Fig. 24–7b; also here there will be a bright line on the screen. But if one ray travels an extra distance of one-half wavelength (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, and so on), the two waves are exactly **out of phase** (Section 11–11) when they reach the screen: the crests of one wave arrive at the same time as the troughs of the other wave, and so they add to produce zero amplitude (Fig. 24–8b). This is **destructive interference**, and the screen is dark, Fig. 24–7c. Thus, there will be a series of bright and dark lines (or **fringes**) on the viewing screen.

FIGURE 24–8 Two traveling waves are shown undergoing (a) constructive interference, (b) destructive interference. (See also Section 11–11.)



To determine exactly where the bright lines fall, first note that Fig. 24–7 is somewhat exaggerated; in real situations, the distance d between the slits is very small compared to the distance ℓ to the screen. The rays from each slit for each case will therefore be essentially parallel, and θ is the angle they make with the horizontal as shown in Fig. 24–7d. From the shaded right triangles shown in Figs. 24–7b and c, we can see that the extra distance traveled by the lower ray is $d \sin\theta$ (seen more clearly in Fig. 24–7d). Constructive interference will occur, and a bright fringe will appear on the screen, when the *path difference*, $d \sin\theta$, equals a whole number of wavelengths:

$$d \sin\theta = m\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix}\text{constructive}\\ \text{interference}\\ \text{(bright)}\end{bmatrix} \qquad \textbf{(24–2a)}$$

The value of m is called the **order** of the interference fringe. The first order ($m = 1$), for example, is the first fringe on each side of the central fringe (which is at $\theta = 0$, $m = 0$). Destructive interference occurs when the path difference $d \sin\theta$ is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on:

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix}\text{destructive}\\ \text{interference}\\ \text{(dark)}\end{bmatrix} \qquad \textbf{(24–2b)}$$

The bright fringes are peaks or maxima of light intensity, the dark fringes are minima.

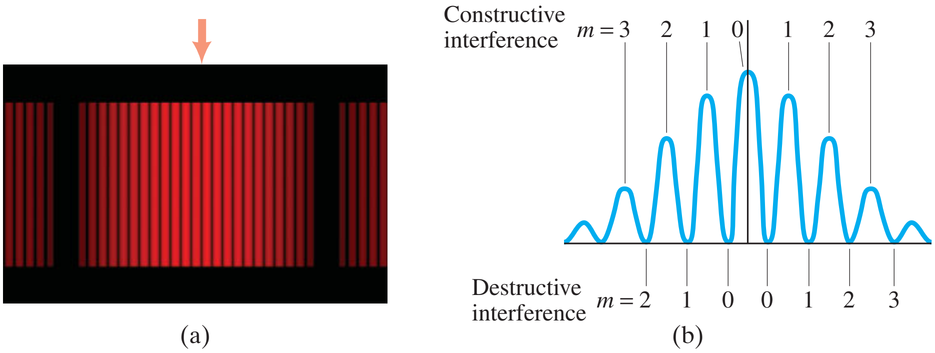


FIGURE 24–9 (a) Interference fringes produced by a double-slit experiment and detected by photographic film placed on the viewing screen. The arrow marks the central fringe. (b) Graph of the intensity of light in the interference pattern. Also shown are values of m for Eq. 24–2a (constructive interference) and Eq. 24–2b (destructive interference).

The intensity of the bright fringes is greatest for the central fringe ($m = 0$) and decreases for higher orders, as shown in Fig. 24–9. How much the intensity decreases with increasing order depends on the width of the two slits.

CAUTION
Use the approximation $\theta \approx \tan\theta$ or $\theta \approx \sin\theta$ only if θ is small and in radians

EXAMPLE 24–1 **Line spacing for double-slit interference.** A screen containing two slits 0.100 mm apart is 1.20 m from the viewing screen. Light of wavelength $\lambda = 500$ nm falls on the slits from a distant source. Approximately how far apart will adjacent bright interference fringes be on the screen?

APPROACH The angular position of bright (constructive interference) fringes is found using Eq. 24–2a. The distance between the first two fringes (say) can be found using right triangles as shown in Fig. 24–10.

SOLUTION Given $d = 0.100\text{ mm} = 1.00 \times 10^{-4}\text{ m}$, $\lambda = 500 \times 10^{-9}\text{ m}$, and $\ell = 1.20\text{ m}$, the first-order fringe ($m = 1$) occurs at an angle θ given by

$$\sin\theta_1 = \frac{m\lambda}{d} = \frac{(1)(500 \times 10^{-9}\text{ m})}{1.00 \times 10^{-4}\text{ m}} = 5.00 \times 10^{-3}.$$

This is a very small angle, so we can take $\sin\theta \approx \theta$, with θ in radians. The first-order fringe will occur a distance x_1 above the center of the screen (see Fig. 24–10), given by $x_1/\ell = \tan\theta_1 \approx \theta_1$, so

$$x_1 \approx \ell\theta_1 = (1.20\text{ m})(5.00 \times 10^{-3}) = 6.00\text{ mm}.$$

The second-order fringe ($m = 2$) will occur at

$$x_2 \approx \ell\theta_2 = \ell\frac{2\lambda}{d} = 12.0\text{ mm}$$

above the center, and so on. Thus the lower-order fringes are 6.00 mm apart.

NOTE The spacing between fringes is essentially uniform until the approximation $\sin\theta \approx \theta$ is no longer valid.

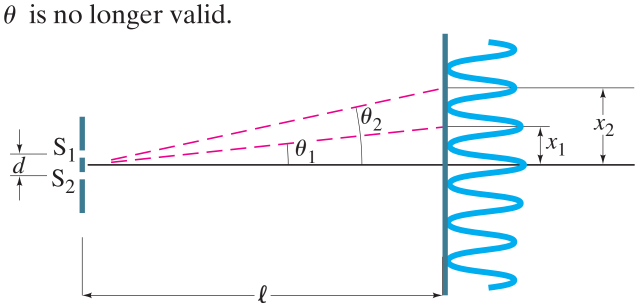


FIGURE 24–10 Examples 24–1 and 24–2. For small angles θ (give θ in radians), the interference fringes occur at distance $x = \theta\ell$ above the center fringe ($m = 0$); θ_1 and x_1 are for the first-order fringe ($m = 1$); θ_2 and x_2 are for $m = 2$.

CONCEPTUAL EXAMPLE 24–2 **Changing the wavelength.** (*a*) What happens to the interference pattern shown in Fig. 24–10, Example 24–1, if the incident light (500 nm) is replaced by light of wavelength 700 nm? (*b*) What happens instead if the wavelength stays at 500 nm but the slits are moved farther apart?

RESPONSE (*a*) When λ increases in Eq. 24–2a but d stays the same, the angle θ for bright fringes increases and the interference pattern spreads out. (*b*) Increasing the slit spacing d reduces θ for each order, so the lines are closer together.

From Eqs. 24–2 we can see that, except for the zeroth-order fringe at the center, the position of the fringes depends on wavelength. When white light falls on the two slits, as Young found in his experiments, the central fringe is white, but the first (and higher) order fringes contain a spectrum of colors like a rainbow.

Using Eq. 24–2a, we can see that θ is smallest for violet light and largest for red (Fig. 24–11). By measuring the position of these fringes, Young was the first to determine the wavelengths of visible light. In doing so, he showed that what distinguishes different colors physically is their wavelength (or frequency), an idea put forward earlier by Grimaldi in 1665.

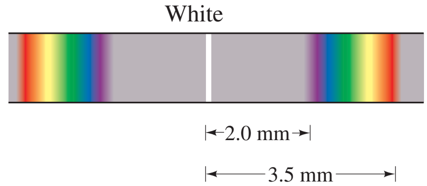


FIGURE 24–11 First-order fringes for a double slit are a full spectrum, like a rainbow. Also Example 24–3.

EXAMPLE 24–3 Wavelengths from double-slit interference. White light passes through two slits 0.50 mm apart, and an interference pattern is observed on a screen 2.5 m away. The first-order fringe resembles a rainbow with violet and red light at opposite ends. The violet light is about 2.0 mm and the red 3.5 mm from the center of the central white fringe (Fig. 24–11). Estimate the wavelengths for the violet and red light.

APPROACH We find the angles for violet and red light from the distances given and the diagram of Fig. 24–10. Then we use Eq. 24–2a to obtain the wavelengths. Because 3.5 mm is much less than 2.5 m, we can use the small-angle approximation.

SOLUTION We use Eq. 24–2a ($d\sin\theta = m\lambda$) with $m = 1$, $d = 5.0\times10^{-4}$ m, and $\sin\theta \approx \tan\theta \approx \theta$. Also $\theta \approx x/\ell$ (Fig. 24–10), so for violet light, $x = 2.0$ mm, and

$$\lambda = \frac{d\sin\theta}{m} \approx \frac{d\theta}{m} \approx \frac{d}{m}\frac{x}{\ell} = \left(\frac{5.0\times10^{-4}\,\text{m}}{1}\right)\left(\frac{2.0\times10^{-3}\,\text{m}}{2.5\,\text{m}}\right) = 4.0\times10^{-7}\,\text{m},$$

or 400 nm. For red light, $x = 3.5$ mm, so

$$\lambda \approx \frac{d}{m}\frac{x}{\ell} = \left(\frac{5.0\times10^{-4}\,\text{m}}{1}\right)\left(\frac{3.5\times10^{-3}\,\text{m}}{2.5\,\text{m}}\right) = 7.0\times10^{-7}\,\text{m} = 700\,\text{nm}.$$

EXERCISE B For the setup in Example 24–3, how far from the central white fringe is the first-order fringe for green light $\lambda = 500$ nm?

Coherence

The two slits in Figs. 24–6 and 24–7 act as if they were two sources of radiation. They are called **coherent sources** because the waves leaving them have the same wavelength and frequency, and bear the same phase relationship to each other at all times. This happens because the waves come from a single source to the left of the two slits. An interference pattern is observed only when the sources are coherent. If two tiny lightbulbs replaced the two slits, an interference pattern would not be seen. The light emitted by one lightbulb would have a random phase with respect to the second bulb, and the screen would be more or less uniformly illuminated. Two such sources, whose output waves have phases that bear no fixed relationship to each other over time, are called **incoherent sources**.

24–4 The Visible Spectrum and Dispersion

Two of the most important properties of light are readily describable in terms of the wave theory of light: intensity (or brightness) and color. The **intensity** of light is the energy it carries per unit area per unit time, and is related to the square of the amplitude of the wave, as for any wave (see Section 11–9, or Eqs. 22–7 and 22–8). The **color** of light is related to the frequency f or wavelength λ of the light. (Recall $\lambda f = c = 3.0\times10^8$ m/s, Eq. 22–4.) Visible light—that to which our eyes are sensitive—consists of frequencies from 4×10^{14} Hz to 7.5×10^{14} Hz, corresponding to wavelengths in air of about 400 nm to 750 nm.† This is the **visible spectrum**, and within it lie the different colors from violet to red, as shown in Fig. 24–12.

†Sometimes the angstrom (Å) unit is used when referring to light: 1 Å = 1×10^{-10} m. Visible light has wavelengths in air of 4000 Å to 7500 Å.

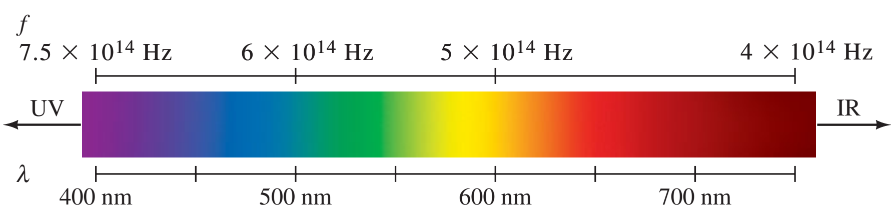


FIGURE 24–12 The spectrum of visible light, showing the range of frequencies and wavelengths in air for the various colors. Many colors, such as brown, do not appear in the spectrum; they are made from a mixture of wavelengths.

FIGURE 24–13 White light passing through a prism is spread out into its constituent colors.

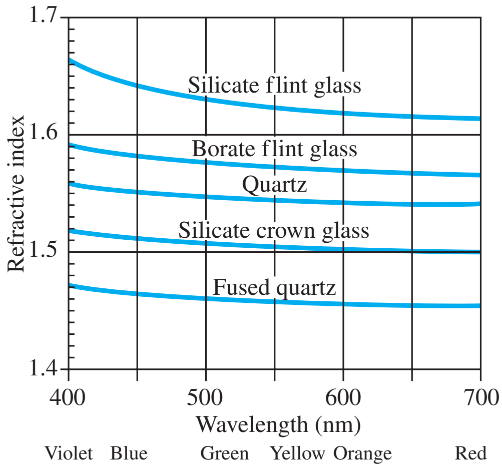
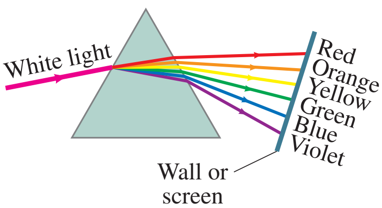


FIGURE 24–14 Index of refraction as a function of wavelength for various transparent solids.

FIGURE 24–15 White light dispersed by a prism into the visible spectrum.



PHYSICS APPLIED
Rainbows

Light with wavelength (in air) shorter than 400 nm (= violet) is called **ultraviolet** (UV), and light with wavelength longer than 750 nm (= red) is called **infrared** (IR).† Although human eyes are not sensitive to UV or IR, some types of photographic film and other detectors do respond to them.

A prism can separate white light into a rainbow of colors, as shown in Fig. 24–13. This happens when the index of refraction of a material depends on the wavelength, as shown for several materials in Fig. 24–14. White light is a mixture of all visible wavelengths, and when incident on a prism, as in Fig. 24–15, the different wavelengths are bent to varying degrees. Because the index of refraction is greater for the shorter wavelengths, violet light is bent the most and red the least, as shown in Fig. 24–15. This spreading of white light into the full spectrum is called **dispersion**.

Rainbows are a spectacular example of dispersion—by drops of water. You can see rainbows when you look at falling water droplets with the Sun behind you. Figure 24–16 shows how red and violet rays are bent by spherical water droplets and are reflected off the back surface of the droplet. Red is bent the least and so reaches the observer's eyes from droplets higher in the sky, as shown in Fig. 24–16a. Thus the top of the rainbow is red.

FIGURE 24–16 (a) Ray diagram showing how a rainbow (b) is formed.

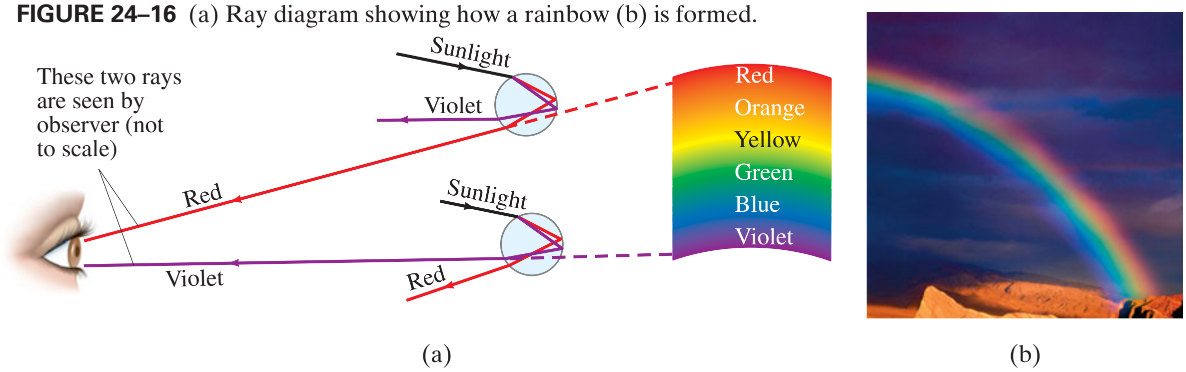


FIGURE 24–17 Diamond.

Diamonds achieve their brilliance (Fig. 24–17) from a combination of dispersion and total internal reflection. Because diamonds have a very high index of refraction of about 2.4, the critical angle for total internal reflection is only 25°. The light dispersed into a spectrum inside the diamond therefore strikes many of the internal surfaces of the diamond before it strikes one at less than 25° and emerges. After many such reflections, the light has traveled far enough that the colors have become sufficiently separated to be seen individually and brilliantly by the eye after leaving the diamond.

The visible spectrum, Fig. 24–12, does not show all the colors seen in nature. For example, there is no brown in Fig. 24–12. Many of the colors we see are a mixture of wavelengths. For practical purposes, most natural colors can be reproduced using three primary colors. They are red, green, and blue for direct source viewing such as TV and computer monitors. For inks used in printing, the primary colors are cyan (the blue color of the margin notes in this book), yellow, and magenta (the pinkish red color we use for light rays in ray diagrams).

†The complete electromagnetic spectrum is illustrated in Fig. 22–8.

CONCEPTUAL EXAMPLE 24–4 **Observed color of light under water.** We said that color depends on wavelength. For example, light of wavelength $\lambda_0 = 650$ nm in air, we see red. If we observe the same object when under water, it still looks red. But the wavelength in water λ_w is (Eq. 24–1) $\lambda_w = \lambda_0/n_w = 650\text{ nm}/1.33 = 489$ nm. Light with wavelength 489 nm in air would appear blue in air. Can you explain why the light appears red rather than blue when observed under water?

RESPONSE Today we have little doubt that it is our brains that express colors, based on the wavelengths of light that strike the receptor cells within the retina (at the rear of the eyeball, as diagrammed in the next Chapter, Fig. 25–9). For objects under water, the water does nothing to change the frequency, but does change the wavelength to λ_0/n_w. When that light enters the eye, the frequency is still unchanged, but the speed is changed to c/n_{eye} where n_{eye} is the index of refraction of the fluid that fills the interior of the eye and is in contact with the retina. The wavelength of light that reaches the retina is $\lambda_{eye} = \lambda_0/n_{eye}$, and is the same whether the light enters from the air or from water.

24–5 Diffraction by a Single Slit or Disk

Young's double-slit experiment put the wave theory of light on a firm footing. But full acceptance came only with studies on diffraction (Section 24–1) more than a decade later, in the 1810s and 1820s.

We have already discussed diffraction briefly with regard to water waves (Section 11–14) as well as for light (Section 24–1). We have seen that diffraction refers to the spreading or bending of waves around edges. Let's look in more detail.

In 1819 Augustin Fresnel (1788–1827) presented to the French Academy a wave theory of light that predicted and explained interference and diffraction effects. Almost immediately Siméon Poisson (1781–1840) pointed out a counter-intuitive inference: according to Fresnel's wave theory, if light from a point source were to fall on a solid disk, part of the incident light would be diffracted around the edges and would constructively interfere at the center of the shadow (Fig. 24–18). That prediction seemed very unlikely. But when the experiment was actually carried out by Francois Arago, the bright spot was seen at the very center of the shadow (Fig. 24–19a). This was strong evidence for the wave theory.

Figure 24–19a is a photograph of the shadow cast by a coin using a coherent point source of light, a laser in this case. The bright spot is clearly present at the center. Note also the bright and dark fringes beyond the shadow. These resemble the interference fringes of a double slit. Indeed, they are due to interference of waves diffracted around the outer edge of the disk, and the group of fringes is referred to as a **diffraction pattern**. A diffraction pattern exists around any sharp-edged object illuminated by a point source, as shown in Figs. 24–19b and c. We are not always aware of diffraction because most sources of light in everyday life are not points, so light from different parts of the source washes out the pattern.

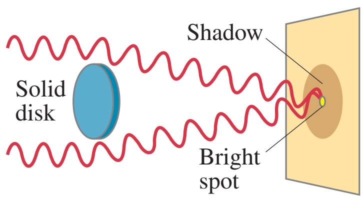
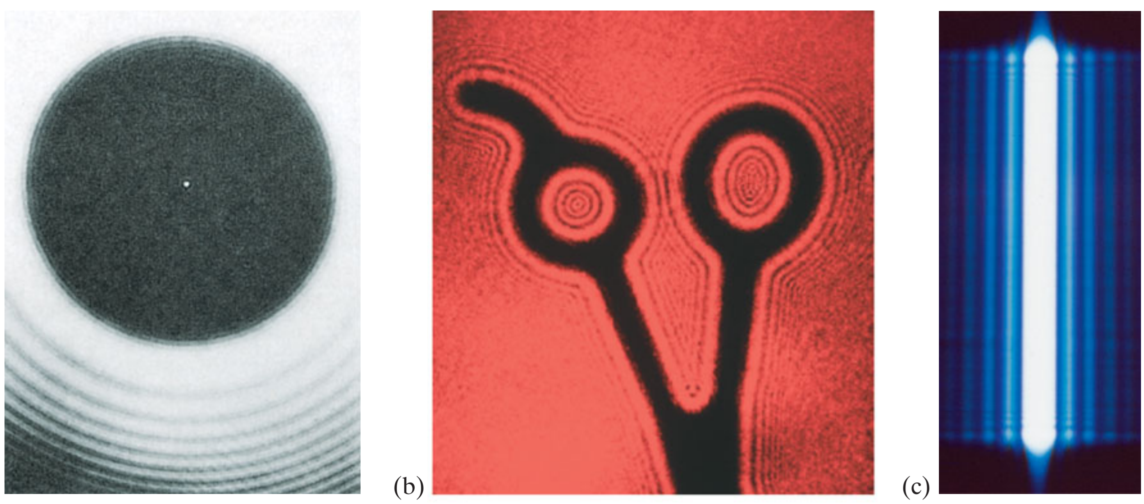


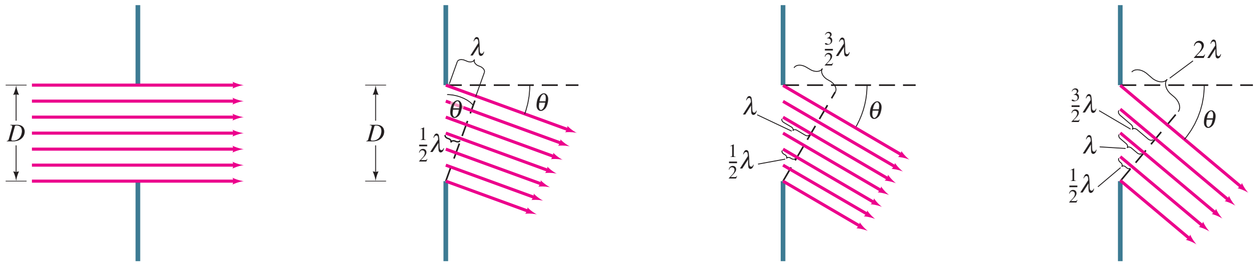
FIGURE 24–18 If light is a wave, a bright spot will appear at the center of the shadow of a solid disk illuminated by a point source of monochromatic light.

FIGURE 24–19 Diffraction pattern of (a) a circular disk (a coin), (b) scissors, (c) a single slit, each illuminated by a (nearly) point source of coherent monochromatic light.



(a) (b) (c)

To see how a diffraction pattern arises, we analyze the important case of monochromatic light passing through a narrow slit (as for Fig. 24–19c). We assume that parallel rays (plane waves) of light pass straight through a slit of width D to a viewing screen very far away.† As we know from studying water waves and from Huygens' principle, waves passing through a slit spread out in all directions. We will now examine how the waves passing through different parts of the slit interfere with each other.



(a) $\theta = 0$ Bright (b) $\sin\theta = \frac{\lambda}{D}$ Dark (c) $\sin\theta = \frac{3\lambda}{2D}$ Bright (d) $\sin\theta = \frac{2\lambda}{D}$ Dark

FIGURE 24–20 Analysis of diffraction pattern formed by light passing through a narrow slit of width D.

Parallel rays of monochromatic light pass through the narrow slit as shown in Fig. 24–20a. The slit width D is on the order of the wavelength λ of the light, but the slit's length (into and out of page) may be large compared to λ. The light falls on a screen which is assumed to be very far away, so the rays heading toward any point are very nearly parallel before they meet at the screen. First we consider rays that pass straight through as in Fig. 24–20a. They are all in phase, so there will be a central bright spot on the screen (see Fig. 24–19c). In Fig. 24–20b, we consider rays moving at an angle θ such that the ray from the top of the slit travels exactly one wavelength farther than the ray from the bottom edge to reach the screen. The ray passing through the very center of the slit will travel one-half wavelength farther than the ray at the bottom of the slit. These two rays will be exactly out of phase with one another and so will destructively interfere when they overlap at the screen. Similarly, a ray slightly above the bottom one will cancel a ray that is the same distance above the central one. Indeed, each ray passing through the lower half of the slit will cancel with a corresponding ray passing through the upper half. Thus, all the rays destructively interfere in pairs, and so the light intensity will be zero on the viewing screen at this angle. The angle θ at which this takes place can be seen from Fig. 24–20b to occur when $\lambda = D\sin\theta$, so

$$\sin\theta = \frac{\lambda}{D}. \qquad \text{[first minimum]} \quad \textbf{(24–3a)}$$

The light intensity is a maximum at $\theta = 0°$ and decreases to a minimum (intensity = zero) at the angle θ given by Eq. 24–3a.

Now consider a larger angle θ such that the top ray travels $\frac{3}{2}\lambda$ farther than the bottom ray, as in Fig. 24–20c. In this case, the rays from the bottom third of the slit will cancel in pairs with those in the middle third because they will be $\lambda/2$ out of phase. However, light from the top third of the slit will still reach the screen, so there will be a bright spot (or fringe) centered near $\sin\theta \approx 3\lambda/2D$, but it will not be nearly as bright as the central spot at $\theta = 0°$. For an even larger angle θ such that the top ray travels 2λ farther than the bottom ray, Fig. 24–20d, rays from the bottom quarter of the slit will cancel with those in the quarter just above it because the path lengths differ by $\lambda/2$. And the rays through the quarter of the slit just above center will cancel with those through the top quarter. At this angle there will again be a minimum of zero intensity in the diffraction pattern.

†If the viewing screen is not far away, lenses can be used to make the rays parallel.

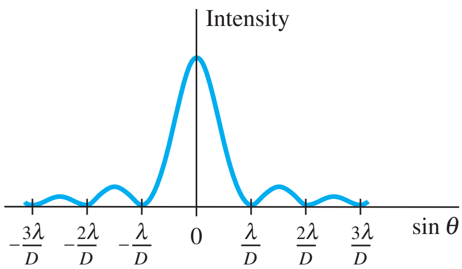


FIGURE 24–21 Intensity in the diffraction pattern of a single slit as a function of $\sin\theta$. Note that the central maximum is not only much higher than the maxima to each side, but it is also twice as wide ($2\lambda/D$ wide) as any of the others (each only λ/D wide).

A plot of the intensity as a function of angle is shown in Fig. 24–21. This corresponds well with the photo of Fig. 24–19c. Notice that minima (zero intensity) occur on both sides of center at

$$D\sin\theta = m\lambda, \qquad m = \pm1, \pm2, \pm3, \cdots, \qquad \text{[minima]} \quad \textbf{(24–3b)}$$

but *not* at $m = 0$ where there is the strongest maximum. Between the minima, smaller intensity maxima occur at approximately (not exactly) $m \approx \frac{3}{2}, \frac{5}{2}, \cdots$.

Note that the *minima* for a diffraction pattern, Eq. 24–3b, satisfy a criterion that looks very similar to that for the *maxima* (bright spots or fringes) for double-slit interference, Eq. 24–2a. Also note that D is a single slit width, whereas d in Eqs. 24–2 is the distance between two slits.

CAUTION

Don't confuse Eqs. 24–2 for interference with Eqs. 24–3 for diffraction; note the differences

EXAMPLE 24–5 Single-slit diffraction maximum. Light of wavelength 750 nm passes through a slit 1.0×10^{-3} mm wide. How wide is the central maximum (*a*) in degrees, and (*b*) in centimeters, on a screen 20 cm away?

APPROACH The width of the central maximum goes from the first minimum on one side to the first minimum on the other side. We use Eq. 24–3a to find the angular position of the first single-slit diffraction minimum.

SOLUTION (*a*) The first minimum occurs at

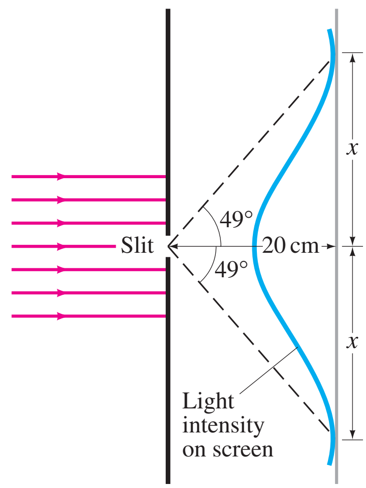
$$\sin\theta = \frac{\lambda}{D} = \frac{7.5 \times 10^{-7}\,\text{m}}{1.0 \times 10^{-6}\,\text{m}} = 0.75.$$

So $\theta = 49°$. This is the angle between the center and the first minimum, Fig. 24–22. The angle subtended by the whole central maximum, between the minima above and below the center, is twice this, or 98°.

(*b*) The width of the central maximum is $2x$, where $\tan\theta = x/20\,\text{cm}$. So $2x = 2(20\,\text{cm})(\tan 49°) = 46\,\text{cm}$.

NOTE A large width of the screen will be illuminated, but it will not normally be very bright since the amount of light that passes through such a small slit will be small and it is spread over a large area. Note also that we *cannot* use the small-angle approximation here ($\theta \approx \sin\theta \approx \tan\theta$) because θ is large.

FIGURE 24–22 Example 24–5.



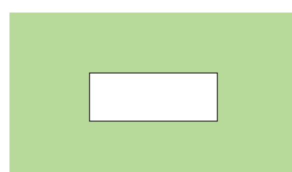
EXERCISE C In Example 24–5, red light ($\lambda = 750$ nm) was used. If instead yellow light ($\lambda = 550$ nm) had been used, would the central maximum be wider or narrower?

CONCEPTUAL EXAMPLE 24–6 Diffraction spreads. Light shines through a small rectangular slit that is narrower in the vertical direction than the horizontal, Fig. 24–23. (*a*) Would you expect the diffraction pattern to be more spread out in the vertical direction or in the horizontal direction? (*b*) Should a rectangular loudspeaker horn at a stadium be tall and narrow, or wide and flat?

RESPONSE (*a*) From Eq. 24–3a we can see that if we make the slit width D smaller, the pattern spreads out more (θ will be larger in Eq. 24–3a). This is consistent with our study of waves in Chapter 11. The diffraction through the rectangular hole will be wider vertically, since the opening is smaller in that direction.

(*b*) For a stadium loudspeaker, the sound pattern desired is one spread out horizontally, so the horn should be tall and narrow (rotate Fig. 24–23 by 90°).

FIGURE 24–23 Example 24–6.



24–6 Diffraction Grating

A large number of equally spaced parallel slits is called a **diffraction grating**, although the term “interference grating” might be as appropriate. Gratings can be made by precision machining of very fine parallel lines on a glass plate. The untouched spaces between the lines serve as the slits. Photographic transparencies of an original grating serve as inexpensive gratings. Gratings containing 10,000 lines or slits per centimeter are common, and are very useful for precise measurements of wavelengths. A diffraction grating containing slits is called a **transmission grating**. Another type of diffraction grating is the **reflection grating**, made by ruling fine lines on a metallic or glass surface from which light is reflected and analyzed. The analysis is basically the same as for a transmission grating, which we now discuss.

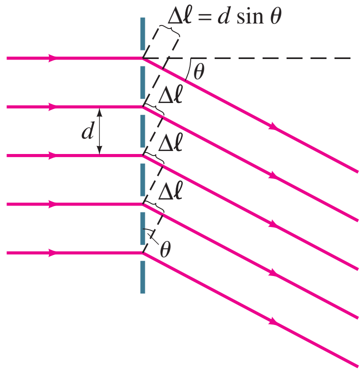


FIGURE 24–24 Diffraction grating.

The analysis of a diffraction grating is much like that of Young’s double-slit experiment. We assume parallel rays of light are incident on the grating as shown in Fig. 24–24. We also assume that the slits are narrow enough so that diffraction by each of them spreads light over a very wide angle on a distant screen beyond the grating, and interference can occur with light from all the other slits. Light rays that pass through each slit without deviation ($\theta = 0°$) interfere constructively to produce a bright maximum at the center of the screen. Constructive interference also occurs at an angle θ such that rays from adjacent slits travel an extra distance of $\Delta\ell = m\lambda$, where m is an integer. If d is the distance *between* slits, then we see from Fig. 24–24 that $\Delta\ell = d\sin\theta$, and

$$\sin\theta = \frac{m\lambda}{d}, \qquad m = 0,\ 1,\ 2,\ \cdots \qquad \begin{bmatrix}\text{diffraction grating}\\ \text{principal maxima}\end{bmatrix} \qquad \textbf{(24–4)}$$

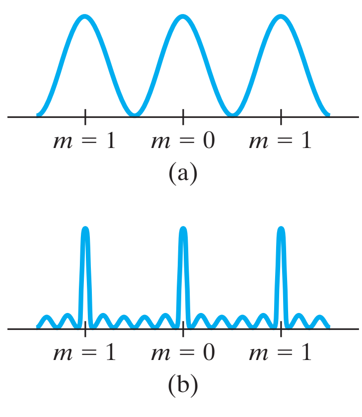
CAUTION

Diffraction grating is analyzed using interference formulas, not diffraction formulas

is the criterion to have a brightness maximum. This is the same equation as for the double-slit situation, and again m is called the **order** of the pattern.

There is an important difference between a double-slit and a multiple-slit pattern. The bright maxima are much *sharper* and *narrower* for a grating. Why? Suppose the angle θ in Fig. 24–24 is increased just slightly beyond θ required for a maximum. For only two slits, the two waves will be only slightly out of phase, so nearly full constructive interference occurs. This means the maxima are wide (see Fig. 24–9). For a grating, the waves from two adjacent slits will also not be significantly out of phase. But waves from one slit and those from a second one a few hundred slits away may be exactly out of phase; all or nearly all the light can cancel in pairs in this way. For example, suppose the angle θ is very slightly different from its first-order maximum, so that the extra path length for a pair of adjacent slits is not exactly λ but rather 1.0010λ. The wave through one slit and another one 500 slits below will have a path difference of $1\lambda + (500)(0.0010\lambda) = 1.5000\lambda$, or $1\frac{1}{2}$ wavelengths, so the two will be out of phase and cancel. A pair of slits, one below each of these, will also cancel. That is, the light from slit 1 cancels with light from slit 501; light from slit 2 cancels with light from slit 502, and so on. Thus even for a tiny angle[†] corresponding to an extra path length of $\frac{1}{1000}\lambda$, there is much destructive interference, and so the maxima of a diffraction grating are very narrow. The more slits there are in a grating, the sharper will be the peaks (see Fig. 24–25). Because a grating produces much sharper maxima than two slits alone, and also much brighter maxima because there are many more slits, a grating is a far more precise device for measuring wavelengths.

FIGURE 24–25 Intensity as a function of viewing angle θ (or position on the screen) for (a) two slits, (b) six slits. For a diffraction grating, the number of slits is very large ($\approx 10^4$) and the peaks are narrower still.



Suppose the light striking a diffraction grating is not monochromatic, but consists of two or more distinct wavelengths. Then for all orders other than $m = 0$, each wavelength will produce a maximum at a different angle (Eq. 24–4), forming a line on the screen as shown in Fig. 24–26a.

[†]Depending on the total number of slits, there may or may not be complete cancellation for such an angle, so there will be very tiny peaks between the main maxima (see Fig. 24–25b), but they are usually much too small to be seen.

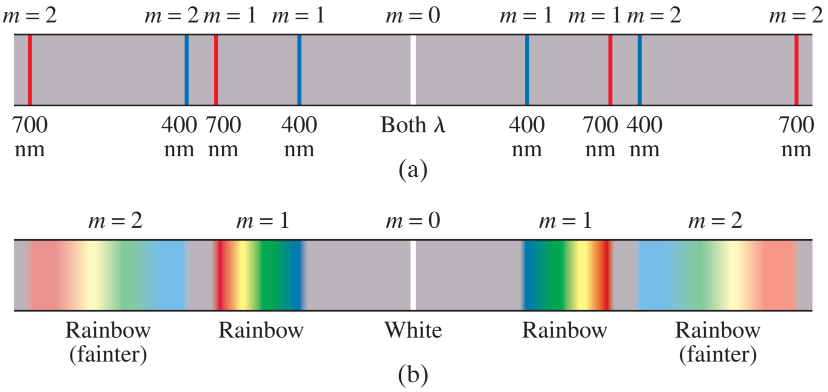


FIGURE 24–26 Spectra produced by a grating: (a) two wavelengths, 400 nm and 700 nm; (b) white light. The second order will normally be dimmer than the first order. (Higher orders are not shown.) If the grating spacing is small enough, the second and higher orders will be missing.

If white light strikes a grating, the central $(m = 0)$ maximum will be a sharp white line. But for all other orders, there will be a distinct spectrum of colors spread out over a certain angular width, Fig. 24–26b. Because a diffraction grating spreads out light into its component wavelengths, the resulting pattern is called a **spectrum**.

EXAMPLE 24–7 **Diffraction grating: line positions.** Determine the angular positions of the first- and second-order lines (maxima) for light of wavelength 400 nm and 700 nm incident on a grating containing 10,000 slits per centimeter.

APPROACH First we find the distance d between grating slits: if the grating has N slits in 1 m, then the distance between slits is $d = 1/N$ meters. Then we use Eq. 24–4, $\sin\theta = m\lambda/d$, to get the angles for the two wavelengths for $m = 1$ and 2.

SOLUTION The grating contains 1.00×10^4 slits/cm $= 1.00 \times 10^6$ slits/m, which means the distance between slits is $d = (1/1.00 \times 10^6)\,\text{m} = 1.00 \times 10^{-6}\,\text{m} = 1.00\,\mu\text{m}$. In first order $(m = 1)$, the angles are

$$\sin\theta_{400} = \frac{m\lambda}{d} = \frac{(1)(4.00 \times 10^{-7}\,\text{m})}{1.00 \times 10^{-6}\,\text{m}} = 0.400$$

$$\sin\theta_{700} = \frac{(1)(7.00 \times 10^{-7}\,\text{m})}{1.00 \times 10^{-6}\,\text{m}} = 0.700$$

so $\theta_{400} = 23.6°$ and $\theta_{700} = 44.4°$. In second order,

$$\sin\theta_{400} = \frac{2\lambda}{d} = \frac{(2)(4.00 \times 10^{-7}\,\text{m})}{1.00 \times 10^{-6}\,\text{m}} = 0.800$$

$$\sin\theta_{700} = \frac{(2)(7.00 \times 10^{-7}\,\text{m})}{1.00 \times 10^{-6}\,\text{m}} = 1.40$$

so $\theta_{400} = 53.1°$. But the second order does not exist for $\lambda = 700\,\text{nm}$ because $\sin\theta$ cannot exceed 1. No higher orders will appear.

EXAMPLE 24–8 **Spectra overlap.** White light containing wavelengths from 400 nm to 750 nm strikes a grating containing 4000 slits/cm. Show that the blue at $\lambda = 450\,\text{nm}$ of the third-order spectrum overlaps the red at 700 nm of the second order.

APPROACH We use $\sin\theta = m\lambda/d$ to calculate the angular positions of the $m = 3$ blue maximum and the $m = 2$ red one.

SOLUTION The grating spacing is $d = (1/4000)\,\text{cm} = 2.50 \times 10^{-6}\,\text{m}$. The blue of the third order occurs at an angle θ given by

$$\sin\theta = \frac{m\lambda}{d} = \frac{(3)(4.50 \times 10^{-7}\,\text{m})}{(2.50 \times 10^{-6}\,\text{m})} = 0.540.$$

Red in second order occurs at

$$\sin\theta = \frac{(2)(7.00 \times 10^{-7}\,\text{m})}{(2.50 \times 10^{-6}\,\text{m})} = 0.560,$$

which is a greater angle; so the second order overlaps into the beginning of the third-order spectrum.

EXERCISE D You are shown the spectra produced by red light shining through two different gratings. The maxima in spectrum A are farther apart than those in spectrum B. Which grating has more slits/cm?

24–7 The Spectrometer and Spectroscopy

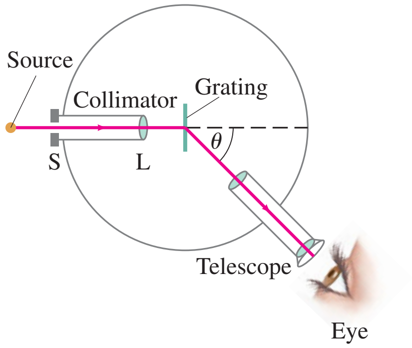


FIGURE 24–27 Spectrometer or spectroscope.

A **spectrometer** or **spectroscope**, Fig. 24–27, is a device to measure wavelengths accurately using a diffraction grating (or a prism) to separate different wavelengths of light. Light from a source passes through a narrow slit S in the "collimator." The slit is at the focal point of the lens L, so parallel light falls on the grating. The movable telescope can bring the rays to a focus. Nothing will be seen in the viewing telescope unless it is positioned at an angle θ that corresponds to a diffraction peak (first order is usually used) of a wavelength emitted by the source. The angle θ can be measured to very high accuracy, so the wavelength can be determined to high accuracy using Eq. 24–4:

$$\lambda = \frac{d}{m}\sin\theta,$$

where m is an integer representing the order, and d is the distance between grating slits. The bright line you see in a spectrometer corresponding to a discrete particular wavelength is actually an image of the slit S. A narrower slit results in dimmer light, but we can measure the angular position more precisely. If the light contains a continuous range of wavelengths, then a continuous spectrum is seen in the spectroscope.

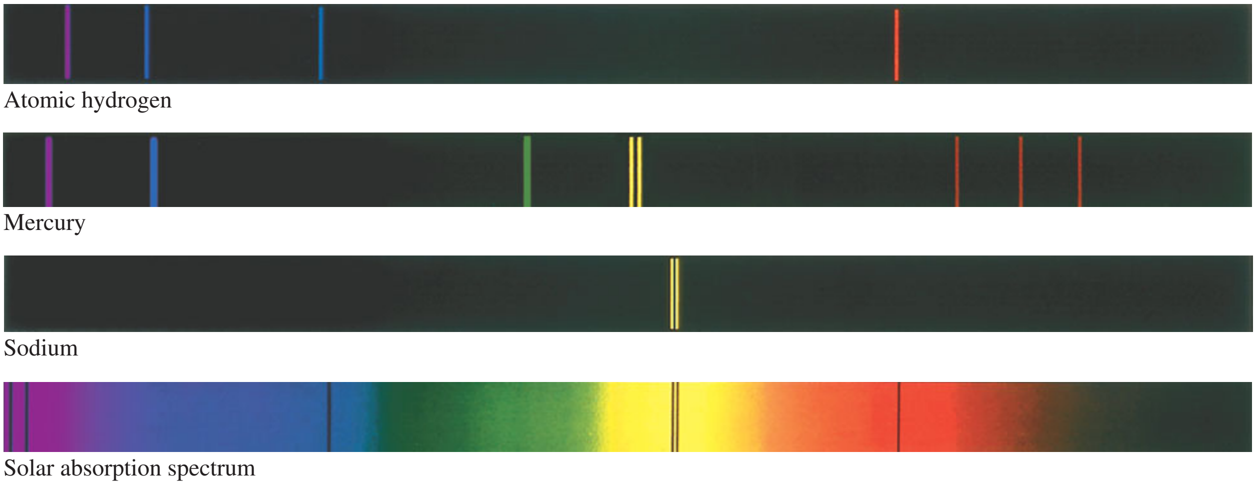
The spectrometer in Fig. 24–27 uses a transmission grating. Others may use a reflection grating, or sometimes a prism. A prism works because of dispersion (Section 24–4), bending light of different wavelengths into different angles. A prism is not a linear device and must be calibrated because λ is not $\propto \sin\theta$; see Fig. 24–14.

An important use of a spectrometer is for the identification of atoms or molecules. When a gas is heated or an electric current is passed through it, the gas emits a characteristic **line spectrum**. That is, only certain discrete wavelengths of light are emitted, and these are different for different elements and compounds.† Figure 24–28 shows the line spectra for a number of elements in the gas state. Line spectra occur only for gases at high temperatures and low pressure and density. The light from heated solids, such as a lightbulb filament, and even from a dense gaseous object such as the Sun, produces a **continuous spectrum** including a wide range of wavelengths.

Figure 24–28 also shows the Sun's "continuous spectrum," which contains a number of *dark* lines (only the most prominent are shown), called **absorption lines**. Atoms and molecules can absorb light at the same wavelengths at which they emit light.

†Why atoms and molecules emit line spectra was a great mystery for many years and played a central role in the development of modern quantum theory, as we shall see in Chapter 27.

FIGURE 24–28 Line spectra for the gases indicated, and the spectrum from the Sun showing absorption lines.



The Sun's absorption lines are due to absorption by atoms and molecules in the cooler outer atmosphere of the Sun, as well as by atoms and molecules in the Earth's atmosphere. A careful analysis of all the Sun's thousands of absorption lines reveals that at least two-thirds of all elements are present in the Sun's atmosphere. The presence of elements in the atmosphere of nearby planets, in interstellar space, and in stars, is also determined by spectroscopy.

Spectroscopy is useful for determining the presence of certain types of molecules in laboratory specimens where chemical analysis would be difficult. For example, biological DNA and different types of protein absorb light in particular regions of the spectrum (such as in the UV). The material to be examined, which is often in solution, is placed in a monochromatic light beam whose wavelength is selected by the placement angle of a diffraction grating or prism. The amount of absorption, as compared to a standard solution without the specimen, can reveal not only the presence of a particular type of molecule, but also its concentration.

PHYSICS APPLIED
Chemical and biochemical analysis by spectroscopy

Light emission and absorption also occur outside the visible part of the spectrum, such as in the UV and IR regions. Glass absorbs light in these regions, so reflection gratings and mirrors (in place of lenses) are used. Special types of film or sensors are used for detection.

EXAMPLE 24–9 **Hydrogen spectrum.** Light emitted by hot hydrogen gas is observed with a spectroscope using a diffraction grating having 1.00×10^4 slits/cm. The spectral lines nearest to the center (0°) are a violet line at 24.2°, a blue line at 25.7°, a blue-green line at 29.1°, and a red line at 41.0° from the center. What are the wavelengths of these spectral lines of hydrogen?

APPROACH We get the wavelengths from the angles by using $\lambda = (d/m)\sin\theta$ where d is the spacing between slits, and m is the order of the spectrum (Eq. 24–4).

SOLUTION Since these are the closest lines to $\theta = 0°$, this is the first-order spectrum ($m = 1$). The slit spacing is $d = 1/(1.00 \times 10^4\,\text{cm}^{-1}) = 1.00 \times 10^{-6}\,\text{m}$. The wavelength of the violet line is

$$\lambda = \left(\frac{d}{m}\right)\sin\theta = \left(\frac{1.00 \times 10^{-6}\,\text{m}}{1}\right)\sin 24.2° = 4.10 \times 10^{-7}\,\text{m} = 410\,\text{nm}.$$

The other wavelengths are:

$$\begin{aligned} &\text{blue:} & \lambda &= (1.00 \times 10^{-6}\,\text{m})\sin 25.7° = 434\,\text{nm},\\ &\text{blue-green:} & \lambda &= (1.00 \times 10^{-6}\,\text{m})\sin 29.1° = 486\,\text{nm},\\ &\text{red:} & \lambda &= (1.00 \times 10^{-6}\,\text{m})\sin 41.0° = 656\,\text{nm}. \end{aligned}$$

NOTE In an unknown mixture of gases, these four spectral lines need to be seen to identify that the mixture contains hydrogen.

24–8 Interference in Thin Films

Interference of light gives rise to many everyday phenomena such as the bright colors reflected from soap bubbles and from thin oil or gasoline films on water, Fig. 24–29. In these and other cases, the colors are a result of constructive interference between light reflected from the two surfaces of the thin film. The effect is observed only if the thickness of the film is on the order of the wavelength of the light. If the film thickness is greater than a few wavelengths, the effect gets washed out.

(a) (b) (c)

FIGURE 24–29 Thin-film interference patterns seen in (a) a soap bubble, (b) a thin film of soapy water, and (c) a thin layer of oil on wet pavement.

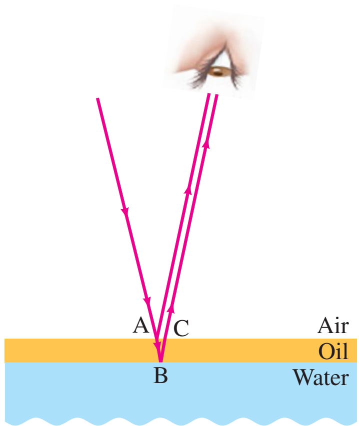


FIGURE 24–30 Light reflected from the upper and lower surfaces of a thin film of oil lying on water.

To see how this **thin-film interference** happens, consider a smooth surface of water on top of which is a thin uniform layer of another substance, say an oil whose index of refraction is less than that of water (we'll see why we assume this shortly); see Fig. 24–30. Assume for now that the incident light is of a single wavelength. Part of the incident light is reflected at A on the top surface, and part of the light transmitted is reflected at B on the lower surface. The part reflected at the lower surface must travel the extra distance ABC. If this *path difference* ABC equals one or a whole number of wavelengths in the film (λ_n), the two waves will reach the eye in phase and interfere constructively. Hence the region AC on the surface film will appear bright. But if ABC equals $\frac{1}{2}\lambda_n, \frac{3}{2}\lambda_n$, and so on, the two waves will be exactly out of phase and destructive interference occurs: the area AC on the film will show no reflection—it will be dark (transparent to the dark material below). The wavelength λ_n is *the wavelength in the film:* $\lambda_n = \lambda/n$, where n is the index of refraction in the film and λ is the wavelength in vacuum. See Eq. 24–1.

When white light falls on such a film, the path difference ABC will equal λ_n (or $m\lambda_n$, with m = an integer) for only one wavelength at a given viewing angle. The color corresponding to λ (λ in air) will be seen as very bright. For light viewed at a slightly different angle, the path difference ABC will be longer or shorter and a different color will undergo constructive interference. Thus, for an extended (nonpoint) source emitting white light, a series of bright colors will be seen next to one another. Variations in thickness of the film will also alter the path difference ABC and therefore affect the color of light that is most strongly reflected.

EXERCISE E Return to the Chapter-Opening Question, page 679, and answer it again now. Try to explain why you may have answered differently the first time.

When a curved glass surface is placed in contact with a flat glass surface, Fig. 24–31, a series of concentric rings is seen when illuminated from above by either white light (as shown) or by monochromatic light. These are called **Newton's rings**† and they are due to interference between waves reflected by the top and bottom surfaces of the very thin *air gap* between the two pieces of glass. Because this gap (which is equivalent to a thin film) increases in width from the central contact point out to the edges, the extra path length for the lower ray (equal to BCD) varies. Where it equals 0, $\frac{1}{2}\lambda$, λ, $\frac{3}{2}\lambda$, 2λ, and so on, it corresponds to constructive and destructive interference; and this gives rise to the series of bright colored circles seen in Fig. 24–31b. The color you see at a given radius corresponds to constructive interference; at that radius, other colors partially or fully destructively interfere. (If monochromatic light is used, the rings are alternately bright and dark.)

The point of contact of the two glass surfaces (A in Fig. 24–31a) is not bright in Fig. 24–31b. Since the path difference is zero here, our previous analysis would suggest that the waves reflected from each surface are in phase—so this central area ought to be bright. But it is dark, which tells us the two waves must be completely

†Although Newton gave an elaborate description of them, they had been first observed and described by his contemporary, Robert Hooke.

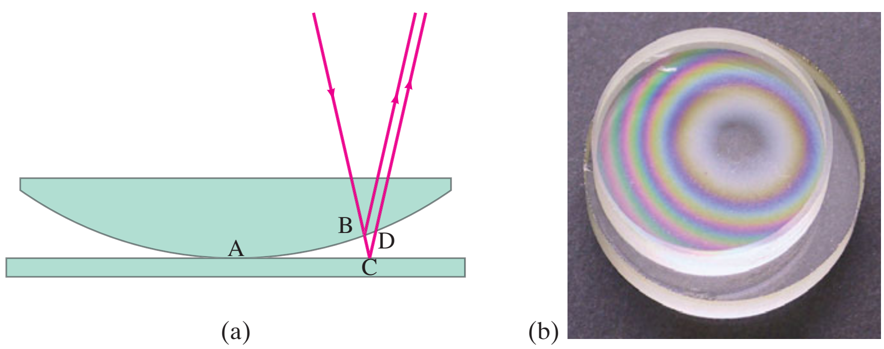


FIGURE 24–31 Newton's rings. (a) Light rays reflected from upper and lower surfaces of the thin air gap can interfere. (b) Photograph of interference patterns using white light.

out of phase. This can happen only if one of the waves, upon reflection, flips over—a crest becomes a trough—see Fig. 24–32. We say that the reflected wave has undergone a **phase shift** of 180°, or of half a wave cycle ($\frac{1}{2}\lambda$). Indeed, this and other experiments reveal that, at normal incidence,

> **a beam of light, reflected by a material with index of refraction greater than that of the material in which it is traveling, changes phase by 180° or $\frac{1}{2}$ cycle;**

see Fig. 24–32. This phase shift acts just like a path difference of $\frac{1}{2}\lambda$. If the index of refraction of the reflecting material is less than that of the material in which the light is traveling, no phase shift occurs.[†]

Thus the wave reflected at the curved surface above the air gap in Fig. 24–31a undergoes no change in phase. But the wave reflected at the lower surface, where the beam in air strikes the glass, undergoes a $\frac{1}{2}$-cycle phase shift, equivalent to a $\frac{1}{2}\lambda$ path difference. Thus the two waves reflected near the point of contact A of the two glass surfaces (where the air gap approaches zero thickness) will be a half cycle (or 180°) out of phase, and a dark spot occurs. Bright colored rings will occur when the path difference is $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on, because the phase shift at one surface effectively adds a path difference of $\frac{1}{2}\lambda$ $\left(= \frac{1}{2}\text{cycle}\right)$. (If monochromatic light is used, the bright Newton's rings will be separated by dark bands which occur when the path difference BCD in Fig. 24–31a is equal to an integral number of wavelengths.)

Returning for a moment to Fig. 24–30, the light reflecting at both interfaces, air–oil and oil–water, *each* underwent a phase shift of 180° equivalent to a path difference of $\frac{1}{2}\lambda$, since we assumed $n_{\text{water}} > n_{\text{oil}} > n_{\text{air}}$. Because the two phase shifts were equal, they didn't affect our analysis.

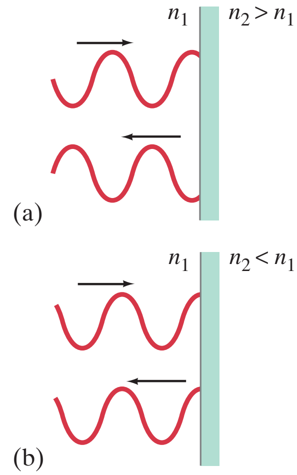


FIGURE 24–32 (a) Reflected ray changes phase by 180° or $\frac{1}{2}$ cycle if $n_2 > n_1$, but (b) does not if $n_2 < n_1$.

EXAMPLE 24–10 **Thin film of air, wedge-shaped.** A very fine wire 7.35×10^{-3} mm in diameter is placed between two flat glass plates as in Fig. 24–33a. Light whose wavelength in air is 600 nm falls (and is viewed) perpendicular to the plates and a series of bright and dark bands is seen, Fig. 24–33b. How many light and dark bands will there be in this case? Will the area next to the wire be bright or dark?

APPROACH We need to consider two effects: (1) path differences for rays reflecting from the two close surfaces (thin wedge of air between the two glass plates), and (2) the $\frac{1}{2}$-cycle phase shift at the lower surface (point E in Fig. 24–33a), where rays in air can enter glass (or be reflected). Because there is a phase shift only at the lower surface, there will be a dark band (no reflection) when the path difference is 0, λ, 2λ, 3λ, and so on. Since the light rays are perpendicular to the plates, the extra path length (DEF) equals $2t$, where t is the thickness of the air gap at any point.

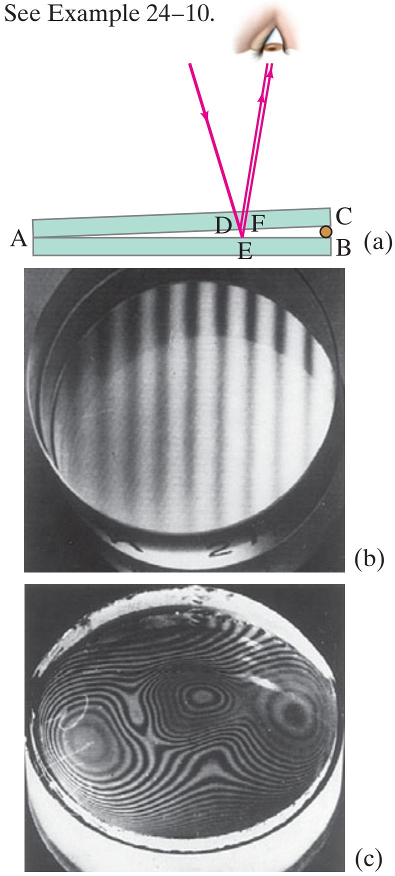
SOLUTION Dark bands will occur where

$$2t = m\lambda, \qquad m = 0,\ 1,\ 2,\ \cdots.$$

Bright bands occur when $2t = \left(m + \frac{1}{2}\right)\lambda$, where m is an integer. At the position of the wire, $t = 7.35 \times 10^{-6}$ m. At this point there will be $2t/\lambda = (2)\left(7.35 \times 10^{-6}\,\text{m}\right)/\left(6.00 \times 10^{-7}\,\text{m}\right) = 24.5$ wavelengths. This is a "half integer," so the area next to the wire will be bright. There will be a total of 25 dark lines along the plates, corresponding to path lengths DEF of 0λ, 1λ, 2λ, 3λ, $\cdots$, 24λ, including the one at the point of contact A $(m = 0)$. Between them, there will be 24 bright lines plus the one at the end, or 25.

NOTE The bright and dark bands will be straight only if the glass plates are extremely flat. If they are not, the pattern is uneven, as in Fig. 24–33c. Thus we see a very precise way of testing a glass surface for flatness. Spherical lens surfaces can be tested for precision by placing the lens on a flat glass surface and observing Newton's rings (Fig. 24–31b) for perfect circularity.

FIGURE 24–33 (a) Light rays reflected from the upper and lower surfaces of a thin wedge of air (between two glass plates) interfere to produce bright and dark bands. (b) Pattern observed when glass plates are optically flat; (c) pattern when plates are not so flat. See Example 24–10.



PHYSICS APPLIED
Testing glass for flatness

[†]This result corresponds to the reflection of a wave traveling along a cord when it reaches the end. As we saw in Fig. 11–33, if the end is tied down, the wave changes phase and the pulse flips over, but if the end is free, no phase shift occurs.

When white light (rather than monochromatic light) is incident on the thin wedge of air in Fig. 24–31a or 24–33a, a colorful series of fringes is seen because constructive interference occurs for different wavelengths in the reflected light at different thicknesses along the wedge.

A soap bubble (Fig. 24–29a and Chapter-Opening Photo) is a thin spherical shell (or film) with air inside. The variations in thickness of a soap bubble film give rise to bright colors reflected from the soap bubble. (There is air on both sides of the bubble film.) Similar variations in film thickness produce the bright colors seen reflecting from a thin layer of oil or gasoline on a puddle or lake (Fig. 24–29c). Which wavelengths appear brightest also depends on the viewing angle.

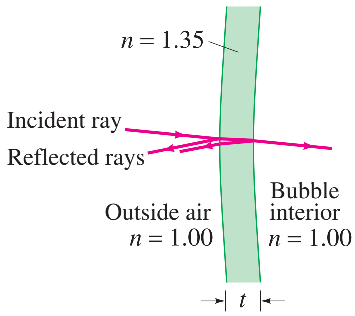


FIGURE 24–34 Soap bubble, Example 24–11. The incident and reflected rays are assumed to be perpendicular to the bubble's surface. They are shown at a slight angle so we can distinguish them.

CAUTION
A formula is not enough: you must also check for phase changes at surfaces

EXAMPLE 24–11 **Thickness of soap bubble skin.** A soap bubble appears green ($\lambda = 540\,\text{nm}$) at the point on its front surface nearest the viewer. What is the smallest thickness the soap bubble film could have? Assume $n = 1.35$.

APPROACH Assume the light is reflected perpendicularly from the point on a spherical surface nearest the viewer, Fig. 24–34. The light rays also reflect from the inner surface of the soap bubble film as shown. The path difference of these two reflected rays is $2t$, where t is the thickness of the soap film. Light reflected from the first (outer) surface undergoes a 180° phase change (index of refraction of soap is greater than that of air), whereas reflection at the second (inner) surface does not. To determine the thickness t for an interference maximum, we must use the wavelength of light in the soap ($n = 1.35$).

SOLUTION The 180° phase change at only one surface is equivalent to a $\frac{1}{2}\lambda$ path difference. Therefore, green light is bright when the minimum path difference equals $\frac{1}{2}\lambda_n$. Thus, $2t = \lambda_n/2$, so

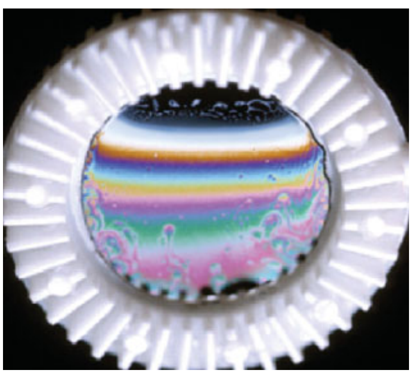
$$t = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{(540\,\text{nm})}{(4)(1.35)} = 100\,\text{nm}.$$

This is the smallest thickness.

NOTE At this small thickness, blue (450 nm) and red (600 nm) also would reflect fairly constructively, so the bubble would appear almost white. The green color is more likely to be seen at the *next* thickness that gives constructive interference, $2t = 3\lambda/2n$, because other colors would be more fully cancelled by destructive interference. Then t would be $t = 3\lambda/4n = 300\,\text{nm}$. Note that green is seen in air, so $\lambda = 540\,\text{nm}$ (not λ/n).

* Colors in a Thin Soap Film

FIGURE 24–29b (Repeated.)



The thin film of soapy water (in a plastic loop) shown in Fig. 24–29b (repeated here) has stood vertically for a long time. Gravity has pulled the soapy water downward, so the film increases in thickness going toward the bottom. The top section is so thin (perhaps 30 nm thick $\ll \lambda$) that light reflected from the front and back surfaces have almost zero path difference. Thus the 180° phase change at the front surface assures that the two reflected waves are 180° out of phase for all wavelengths of visible light. The white light incident on this thin film does not reflect at the top part of the film, so the top is transparent and we see the background which is black.

Below the black area at the top, there is a thin blue line, and then a white band. The film has thickened to perhaps 75 to 100 nm, so the shortest wavelength (blue) light begins to partially interfere constructively. But just below, where the thickness is slightly greater (100 nm), the path difference is reasonably close to $\lambda/2$ for much of the spectrum and we see white or silver.[†]

Immediately below the white band in this Figure we see a brown band, where $t \approx 200\,\text{nm}$, and many wavelengths (not all) are close to λ—and those colors destructively interfere, leaving only a few colors to partially interfere constructively, giving us murky brown.

[†] Why? Recall that red starts at 600 nm in air; so most colors in the spectrum lie between 450 nm and 600 nm in air; but in water the wavelengths are $n = 1.33$ times smaller, 340 nm to 450 nm, so a 100-nm thickness is a 200-nm path difference, not far from $\lambda/2$ for most colors.

Farther down in Fig. 24–29b, with increasing thickness t, a path difference $2t = 510\,\text{nm}$ corresponds nicely to $\frac{3}{2}\lambda$ for blue, but not for other colors, so we see blue ($\frac{3}{2}\lambda$ path difference plus $\frac{1}{2}\lambda$ phase change = constructive interference). Other colors experience constructive interference (at $\frac{3}{2}\lambda$ and then at $\frac{5}{2}\lambda$) at still greater thicknesses, so going down we see a series of separated colors something like a rainbow.

In the soap bubble of our Chapter-Opening Photo (page 679), similar things happen: at the top (where the film is thinnest) we see black and then silver-white, just as in the soap film shown in Fig. 24–29b.

Also examine the oil film on wet pavement shown in Fig. 24–29c (repeated here). The oil film is thickest at the center and thins out toward the edges. Notice the whitish outer ring where most colors constructively interfere, which would suggest a thickness on the order of 100 nm as discussed above for the white band in the soap film. Beyond the outer white band of the oil film, Fig. 24–29c, there is still some oil, but the film is so thin that reflected light from upper and lower surfaces destructively interfere and you can see right through this very thin oil film.

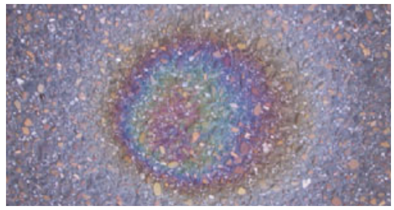


FIGURE 24–29c (Repeated.)

Lens Coatings

An important application of thin-film interference is in the coating of glass to make it "nonreflecting," particularly for lenses. A glass surface reflects about 4% of the light incident upon it. Good-quality cameras, microscopes, and other optical devices may contain six to ten thin lenses. Reflection from all these surfaces can reduce the light level considerably, and multiple reflections produce a background haze that reduces the quality of the image. By reducing reflection, transmission and sharpness are increased.

PHYSICS APPLIED
Lens coatings

A very thin coating on the lens surfaces can reduce reflections considerably. The thickness of the coating is chosen so that light (at least for one wavelength) reflecting from the front and rear surfaces of the film destructively interferes. Destructive interference can occur nearly completely for one particular wavelength depending on the thickness of the coating. Nearby wavelengths will at least partially destructively interfere, but a single coating cannot eliminate reflections for all wavelengths. Nonetheless, a single coating can reduce total reflection from 4% to 1% of the incident light. Often the coating is designed to eliminate the center of the reflected spectrum (around 550 nm). The extremes of the spectrum—red and violet—will not be reduced as much. Since a mixture of red and violet produces purple, the light seen reflected from such coated lenses is purple (Fig. 24–35). Lenses containing two or three separate coatings can more effectively reduce a wider range of reflecting wavelengths.

FIGURE 24–35 A coated lens. Note color of light reflected from the front lens surface.

PROBLEM SOLVING
Interference

1. **Interference effects** depend on the simultaneous arrival of two or more waves at the same point in space.
2. **Constructive interference** occurs when waves with the same wavelength arrive in phase with each other: a crest of one wave arrives at the same time as a crest of the other wave(s). The amplitudes of the waves then add to form a larger amplitude. Constructive interference also occurs when the path difference is exactly one full wavelength or any integer multiple of a full wavelength: $1\lambda, 2\lambda, 3\lambda, \cdots$.
3. **Destructive interference** occurs when a crest of one wave arrives at the same time as a trough of the other wave. The amplitudes add, but they are of opposite sign, so the total amplitude is reduced to zero if the two amplitudes are equal. Destructive interference occurs whenever the phase difference is half a wave cycle, or the path difference is a half-integral number of wavelengths. Thus, the total amplitude will be zero if two identical waves arrive one-half wavelength out of phase, or $\left(m + \frac{1}{2}\right)\lambda$ out of phase, where m is an integer.
4. For thin-film interference, an extra half-wavelength **phase shift** occurs when light **reflects** from an optically more dense medium (going from a material of lesser toward greater index of refraction).

EXAMPLE 24–12 **Nonreflective coating.** What is the thickness of an optical coating of MgF_2 whose index of refraction is $n = 1.38$ and which is designed to eliminate reflected light at wavelengths (in air) around 550 nm when incident normally on glass for which $n = 1.50$?

APPROACH We explicitly follow the procedure outlined in the Problem Solving Strategy on page 697.

SOLUTION

1. **Interference effects.** Consider two rays reflected from the front and rear surfaces of the coating on the lens as shown in Fig. 24–36. The rays are drawn not quite perpendicular to the lens so we can see each of them. These two reflected rays will interfere with each other.
2. **Constructive interference.** We want to eliminate reflected light, so we do not consider constructive interference.
3. **Destructive interference.** To eliminate reflection, we want reflected rays 1 and 2 to be $\frac{1}{2}$ cycle out of phase with each other so that they destructively interfere. The phase difference is due to the path difference $2t$ traveled by ray 2, as well as any phase change in either ray due to reflection.
4. **Reflection phase shift.** Rays 1 and 2 *both* undergo a change of phase by $\frac{1}{2}$ cycle when they reflect from the coating's front and rear surfaces, respectively (at both surfaces the index of refraction increases). Thus there is no net change in phase due to the reflections. The net phase difference will be due to the extra path $2t$ taken by ray 2 in the coating, where $n = 1.38$. We want $2t$ to equal $\frac{1}{2}\lambda_n$ so that destructive interference occurs, where $\lambda_n = \lambda/n$ is the wavelength in the coating. With $2t = \lambda_n/2 = \lambda/2n$, then

$$t = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = \frac{(550\text{ nm})}{(4)(1.38)} = 99.6\text{ nm}.$$

NOTE We could have set $2t = \left(m + \frac{1}{2}\right)\lambda_n$, where m is an integer. The smallest thickness ($m = 0$) is usually chosen because destructive interference will occur over the widest angle.

NOTE Complete destructive interference occurs only for the given wavelength of visible light. Longer and shorter wavelengths will have only partial cancellation.

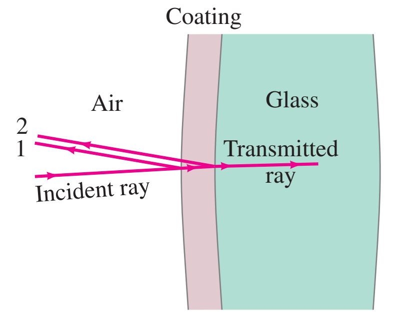
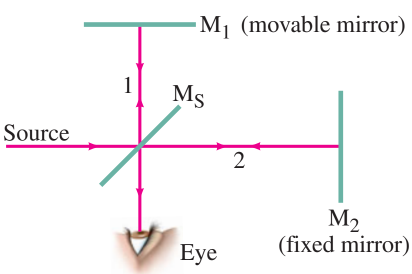


FIGURE 24–36 Lens coating, Example 24–12. Incident ray of light is partially reflected at the front surface of a lens coating (ray 1) and again partially reflected at the rear surface of the coating (ray 2), with most of the energy passing through as the transmitted ray into the glass.

*24–9 Michelson Interferometer

A useful instrument involving wave interference is the **Michelson interferometer** (Fig. 24–37),[†] invented by the American Albert A. Michelson (Section 22–4). Monochromatic light from a single point on an extended source is shown striking a half-silvered mirror M_S. This **beam splitter** mirror M_S has a thin layer of silver that reflects only half the light that hits it, so that half of the beam passes through to a fixed mirror M_2, where it is reflected back. The other half is reflected by M_S to a mirror M_1 that is movable (by a fine-thread screw), where it is also reflected back. Upon its return, part of beam 1 passes through M_S and reaches a sensor or the eye; and part of beam 2, on its return, is reflected by M_S into the eye. If the two path lengths are identical, the two coherent beams entering the eye constructively interfere and brightness will be seen. If the movable mirror is moved a distance $\lambda/4$, one beam will travel an extra distance equal to $\lambda/2$ (because it travels back and forth over the distance $\lambda/4$). In this case, the two beams will destructively interfere and darkness will be seen. As M_1 is moved farther, brightness will recur (when the path difference is λ), then darkness, and so on.

FIGURE 24–37 Michelson interferometer.



Very precise length measurements can be made with an interferometer. The motion of mirror M_1 by only $\frac{1}{4}\lambda$ produces a clear difference between brightness and darkness. For $\lambda = 400$ nm, this means a precision of 100 nm, or 10^{-4} mm! If mirror M_1 is tilted very slightly, the bright or dark spots are seen instead as a series of bright and dark lines or "fringes" that move as M_1 moves. By counting the number of fringes (or fractions thereof) that pass a reference line, extremely precise length measurements can be made.

[†]There are other types of interferometer, but Michelson's is the best known.

24–10 Polarization

An important and useful property of light is that it can be *polarized*. To see what this means, let us examine waves traveling on a rope. A rope can oscillate in a vertical plane, Fig. 24–38a, or in a horizontal plane, Fig. 24–38b. In either case, the wave is said to be **linearly polarized** or **plane-polarized**—*the oscillations are in a plane*.

If we now place an obstacle containing a vertical slit in the path of the wave, Fig. 24–39, a vertically polarized wave passes through the vertical slit, but a horizontally polarized wave will not. If a horizontal slit were used, the vertically polarized wave would be stopped. If both types of slit were used, both types of wave would be stopped by one slit or the other. Note that polarization can exist *only* for *transverse waves*, and not for longitudinal waves such as sound. The latter oscillate only along the direction of motion, and neither orientation of slit would stop them.

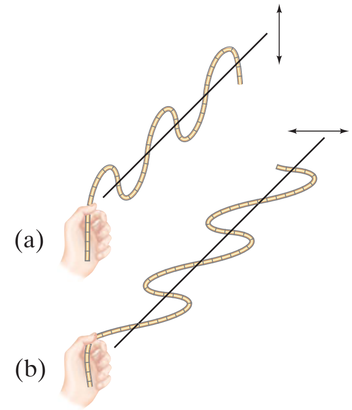


FIGURE 24–38 Transverse waves on a rope polarized (a) in a vertical plane and (b) in a horizontal plane.

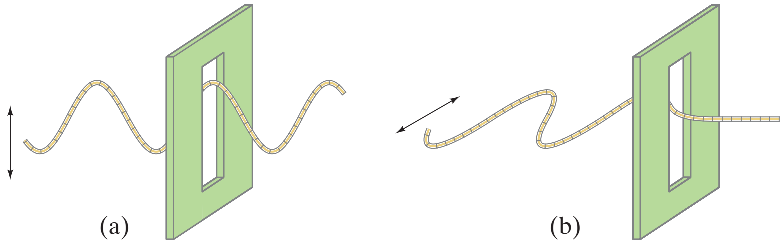


FIGURE 24–39 (a) A vertically polarized wave passes through a vertical slit, but (b) a horizontally polarized wave will not.

Maxwell's theory of light as electromagnetic (EM) waves predicted that light can be polarized since an EM wave is a transverse wave. The direction of polarization in a plane-polarized EM wave is taken as the direction of the electric field vector $\vec{\mathbf{E}}$.

Light is not necessarily polarized. It can also be **unpolarized**, which means that the source has oscillations in many planes at once, as shown in Fig. 24–40. Ordinary lightbulbs emit unpolarized light, as does the Sun.

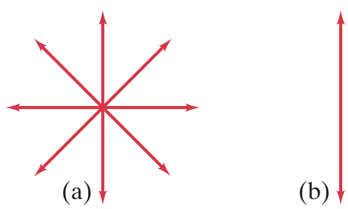


FIGURE 24–40 (a) Oscillation of the electric field vectors in unpolarized light. The light is traveling into or out of the page. (b) Electric field in linear polarized light.

Polaroids (Polarization by Absorption)

Plane-polarized light can be obtained from unpolarized light using certain crystals such as tourmaline. Or, more commonly, we use a **Polaroid sheet**. (Polaroid materials were invented in 1929 by Edwin Land.) A Polaroid sheet consists of long complex molecules arranged parallel to one another. Such a Polaroid acts like a series of parallel slits to allow one orientation of polarization to pass through nearly undiminished. This direction is called the *transmission axis* of the Polaroid. Polarization perpendicular to this direction is absorbed almost completely by the Polaroid.

Absorption by a Polaroid can be explained at the molecular level. An electric field $\vec{\mathbf{E}}$ that oscillates parallel to the long molecules can set electrons into motion along the molecules, thus doing work on them and transferring energy. Hence, if $\vec{\mathbf{E}}$ is parallel to the molecules, it gets absorbed. An electric field $\vec{\mathbf{E}}$ perpendicular to the long molecules does not have this possibility of doing work and transferring its energy, and so passes through freely. When we speak of the *transmission axis* of a Polaroid, we mean the direction for which $\vec{\mathbf{E}}$ is passed, so a Polaroid axis is *perpendicular* to the long molecules. [If we want to think of there being slits between the parallel molecules in the sense of Fig. 24–39, then Fig. 24–39 would apply for the $\vec{\mathbf{B}}$ field in the EM wave, not the $\vec{\mathbf{E}}$ field.]

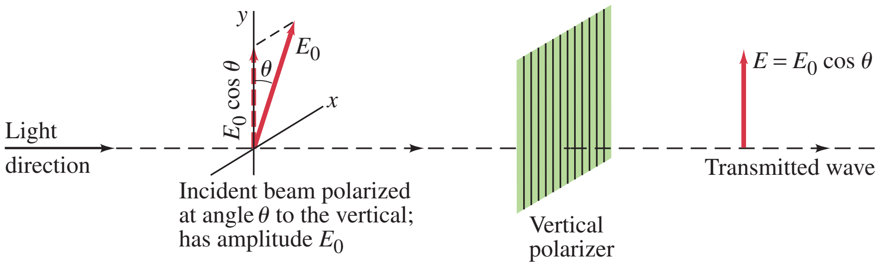


FIGURE 24–41 Vertical Polaroid transmits only the vertical component of a wave (electric field) incident upon it.

If a beam of plane-polarized light strikes a Polaroid whose transmission axis is at an angle θ to the incident polarization direction, the beam will emerge plane-polarized parallel to the Polaroid transmission axis, and the amplitude of E will be reduced to $E \cos\theta$, Fig. 24–41. Thus, a Polaroid passes only that component of polarization (the electric field vector, $\vec{\mathbf{E}}$) that is parallel to its transmission axis. Because the intensity of a light beam is proportional to the square of the amplitude (Sections 11–9 and 22–5), the intensity of a plane-polarized beam transmitted by a polarizer is proportional to $(E_0 \cos\theta)^2$, a relation called Malus' law,

$$I = I_0 \cos^2\theta, \quad \left[\begin{array}{c}\text{intensity of plane-polarized} \\ \text{wave passed by polarizer}\end{array}\right] \qquad \textbf{(24–5)}$$

where I_0 is the incoming intensity and θ is the angle between the polarizer transmission axis and the plane of polarization of the incoming wave.

A Polaroid can be used as a **polarizer** to *produce* plane-polarized light from unpolarized light, since only the component of light parallel to the axis is transmitted. A Polaroid can also be used as an **analyzer** to determine (1) if light is polarized and (2) the plane of polarization. A Polaroid acting as an analyzer will pass the same amount of light independent of the orientation of its axis if the light is unpolarized; try rotating one lens of a pair of Polaroid sunglasses while looking through it at a lightbulb. If the light is polarized, however, when you rotate the Polaroid the transmitted light will be a maximum when the plane of polarization is parallel to the Polaroid's transmission axis, and a minimum when perpendicular to it. If you do this while looking at the sky, preferably at right angles to the Sun's direction, you will see that skylight is polarized. (Direct sunlight is unpolarized, but don't look directly at the Sun, even through a polarizer, for damage to the eye may occur.) If the light transmitted by an analyzer Polaroid falls to zero at one orientation, then the light is 100% plane-polarized. If it merely reaches a minimum, the light is *partially polarized*.

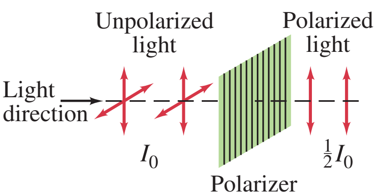


FIGURE 24–42 Unpolarized light has equal intensity vertical and horizontal components. After passing through a polarizer, one of these components is eliminated. The intensity of the light is reduced to half.

Unpolarized light consists of light with random directions of polarization. Each of these polarization directions can be resolved into components along two mutually perpendicular directions. On average, an unpolarized beam can be thought of as two plane-polarized beams of equal magnitude perpendicular to one another. When unpolarized light passes through a polarizer, one component is eliminated. So the intensity of the light passing through is reduced by half because half the light is eliminated: $I = \frac{1}{2} I_0$ (Fig. 24–42).

When two Polaroids are *crossed*—that is, their polarizing axes are perpendicular to one another—unpolarized light can be entirely stopped. As shown in Fig. 24–43, unpolarized light is made plane-polarized by the first Polaroid (the polarizer). The second Polaroid, the analyzer, then eliminates this component since its transmission axis is perpendicular to the first.

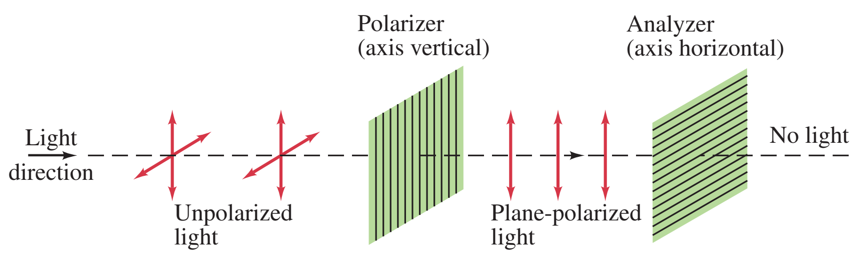


FIGURE 24–43 Crossed Polaroids completely eliminate light.

You can try this with Polaroid sunglasses (Fig. 24–44). Note that Polaroid sunglasses eliminate 50% of unpolarized light because of their polarizing property; they absorb even more because they are colored. Plane-polarized light in any direction is also stopped by crossed Polaroids.

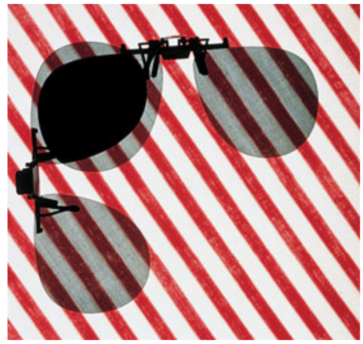


FIGURE 24–44 Crossed Polaroids. When the two polarized sunglass lenses overlap, with axes perpendicular, almost no light passes through.

EXAMPLE 24–13 **Two Polaroids at 60°.** Unpolarized light passes through two Polaroids; the axis of the first is vertical and that of the second is at 60° to the vertical. Describe the orientation and intensity of the transmitted light.

APPROACH Half of the unpolarized light is absorbed by the first Polaroid, and the remaining light emerges plane-polarized vertically. When that light passes through the second Polaroid, the intensity is further reduced according to Eq. 24–5, and the plane of polarization is then along the axis of the second Polaroid.

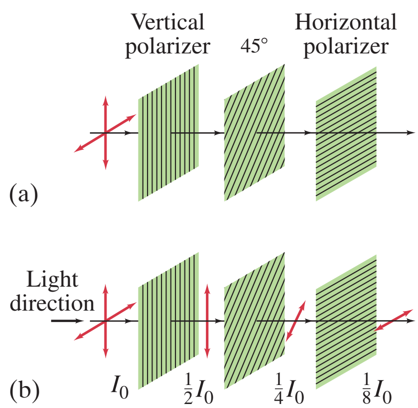
SOLUTION The first Polaroid eliminates half the light, so the intensity is reduced by half: $I_1 = \frac{1}{2}I_0$. The light reaching the second polarizer is vertically polarized and so is reduced in intensity (Eq. 24–5) to

$$I_2 = I_1(\cos 60°)^2 = \tfrac{1}{4}I_1.$$

Thus, $I_2 = \frac{1}{8}I_0$. The transmitted light has an intensity one-eighth that of the original and is plane-polarized at a 60° angle to the vertical.

CONCEPTUAL EXAMPLE 24–14 **Three Polaroids.** We saw in Fig. 24–43 that when unpolarized light falls on two crossed Polaroids (axes at 90°), no light passes through. What happens if a third Polaroid, with axis at 45° to each of the other two, is placed between them (Fig. 24–45a)?

FIGURE 24–45 Example 24–14.



RESPONSE We start just as in Example 24–13 and recall again that light emerging from each Polaroid is polarized parallel to that Polaroid's axis. Thus the angle in Eq. 24–5 is that between the transmission axes of each pair of Polaroids taken in turn. The first Polaroid changes the unpolarized light to plane-polarized and reduces the intensity from I_0 to $I_1 = \frac{1}{2}I_0$. The second polarizer further reduces the intensity by $(\cos 45°)^2$, Eq. 24–5:

$$I_2 = I_1(\cos 45°)^2 = \tfrac{1}{2}I_1 = \tfrac{1}{4}I_0.$$

The light leaving the second polarizer is plane-polarized at 45° (Fig. 24–45b) relative to the third polarizer, so the third one reduces the intensity to

$$I_3 = I_2(\cos 45°)^2 = \tfrac{1}{2}I_2,$$

or $I_3 = \frac{1}{8}I_0$. Thus $\frac{1}{8}$ of the original intensity gets transmitted.

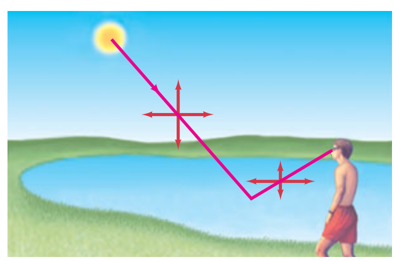
NOTE If we don't insert the 45° Polaroid, zero intensity results (Fig. 24–43).

EXERCISE F How much light would pass through if the 45° polarizer in Example 24–14 was placed not between the other two polarizers but (*a*) before the vertical (first) polarizer, or (*b*) after the horizontal polarizer?

Polarization by Reflection

Another means of producing polarized light from unpolarized light is by reflection. When light strikes a nonmetallic surface at any angle other than perpendicular, the reflected beam is polarized preferentially in the plane parallel to the surface, Fig. 24–46. In other words, the component with polarization in the plane perpendicular to the surface is preferentially transmitted or absorbed. You can check this by rotating Polaroid sunglasses while looking through them at a flat surface of a lake or road. Since most outdoor surfaces are horizontal, Polaroid sunglasses are made with their axes vertical to eliminate the more strongly reflected horizontal component, and thus reduce glare.

FIGURE 24–46 Light reflected from a nonmetallic surface, such as the smooth surface of water in a lake, is partially polarized parallel to the surface.



(a)

(b)

FIGURE 24–47 Photographs of a lake, (a) allowing all light into the camera lens, and (b) using a polarizer. The polarizer is adjusted to absorb most of the (polarized) light reflected from the water's surface, allowing the dimmer light from the bottom of the lake, and any fish lying there, to be seen more readily.

People who go fishing wear Polaroids to eliminate reflected glare from the surface of a lake or stream and thus see beneath the water more clearly (Fig. 24–47).

The amount of polarization in the reflected beam depends on the angle, varying from no polarization at normal incidence to 100% polarization at an angle known as the **polarizing angle** θ_p.[†] This angle is related to the index of refraction of the two materials on either side of the boundary by the equation

$$\tan\theta_p = \frac{n_2}{n_1}, \qquad \textbf{(24–6a)}$$

where n_1 is the index of refraction of the material in which the incident beam is traveling, and n_2 is that of the medium beyond the reflecting boundary. If the beam is traveling in air, $n_1 = 1$, and Eq. 24–6a becomes

$$\tan\theta_p = n. \qquad \textbf{(24–6b)}$$

The polarizing angle θ_p is also called **Brewster's angle**, and Eqs. 24–6 *Brewster's law*, after the Scottish physicist David Brewster (1781–1868), who worked it out experimentally in 1812. Equations 24–6 can be derived from the electromagnetic wave theory of light. It is interesting that at Brewster's angle, the reflected ray and the transmitted (refracted) ray make a 90° angle to each other; that is, $\theta_p + \theta_r = 90°$, where θ_r is the refraction angle (Fig. 24–48). This can be seen

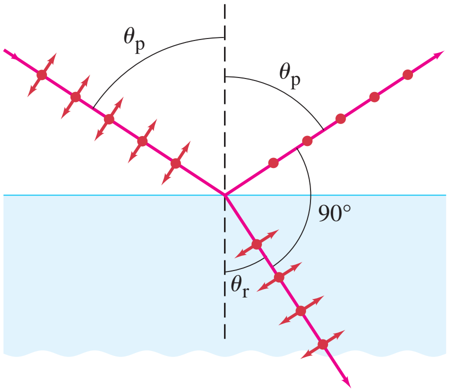


FIGURE 24–48 At θ_p the reflected light is plane-polarized parallel to the surface, and $\theta_p + \theta_r = 90°$, where θ_r is the refraction angle. (The large dots represent vibrations perpendicular to the page.)

by substituting Eq. 24–6a, $n_2 = n_1\tan\theta_p = n_1\sin\theta_p/\cos\theta_p$, into Snell's law, $n_1\sin\theta_p = n_2\sin\theta_r$, which gives $\cos\theta_p = \sin\theta_r$ which can only hold if $\theta_p = 90° - \theta_r$ (see Trigonometric identities inside back cover or Appendix page A-8).

EXAMPLE 24–15 Polarizing angle. (*a*) At what incident angle is sunlight reflected from a lake perfectly plane-polarized? (*b*) What is the refraction angle?

APPROACH The polarizing angle at the surface is Brewster's angle, Eq. 24–6b. We find the angle of refraction from Snell's law.

SOLUTION (*a*) We use Eq. 24–6b with $n = 1.33$, so $\tan\theta_p = 1.33$ giving $\theta_p = 53.1°$. (*b*) From Snell's law, $\sin\theta_r = \sin\theta_p/n = \sin 53.1°/1.33 = 0.601$ giving $\theta_r = 36.9°$.

NOTE $\theta_p + \theta_r = 53.1° + 36.9° = 90.0°$, as expected.

[†]Only a fraction of the incident light is reflected at the surface of a transparent medium. Although this reflected light is 100% polarized (if $\theta = \theta_p$), the remainder of the light, which is transmitted into the new medium, is only partially polarized.

*24–11 Liquid Crystal Displays (LCD)

A wonderful use of polarization is in a **liquid crystal display** (LCD). LCDs are used as the display in cell phones, other hand-held electronic devices, and flat-panel computer and television screens.

A liquid crystal display is made up of many tiny rectangles called **pixels**, or "picture elements." The picture you see depends on which pixels are dark or light and of what color, as suggested in Fig. 24–49 for a simple black and white picture.

Liquid crystals are organic materials that at room temperature exist in a phase that is neither fully solid nor fully liquid. They are sort of gooey, and their molecules display a randomness of position characteristic of liquids, as discussed in Section 13–1 and Fig. 13–2b. They also show some of the orderliness of a solid crystal (Fig. 13–2a), but only in one dimension.

The liquid crystals we find useful are made up of relatively rigid rod-like molecules that interact weakly with each other and tend to align parallel to each other, as shown in Fig. 24–50.

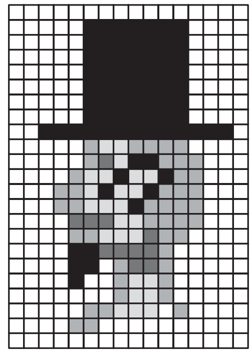


FIGURE 24–49 Example of an image made up of many small squares or *pixels* (picture elements).

FIGURE 24–50 Liquid crystal molecules tend to align in one dimension (parallel to each other) but have random positions (left-right, up-down).

In a simple LCD, each pixel (picture element) contains liquid crystal material sandwiched between two glass plates whose inner surfaces have been brushed to form nanometer-wide parallel scratches. The rod-like liquid crystal molecules in contact with the scratches tend to line up along the scratches. The two plates typically have their scratches at 90° to each other, and the weak electric forces between the rod-like molecules tend to keep them nearly aligned with their nearest neighbors, resulting in the twisted pattern shown in Fig. 24–51a.

The outer surfaces of the glass plates each have a thin film polarizer, they too oriented at 90° to each other. Unpolarized light incident from the left becomes plane-polarized, and the liquid crystal molecules keep this polarization aligned with their rod-like shape. That is, the plane of polarization of the light rotates with the molecules as the light passes through the liquid crystal. The light emerges with its plane of polarization rotated by 90°, and readily passes through the second polarizer, Fig. 24–51a. A tiny LCD pixel in this situation will appear bright.

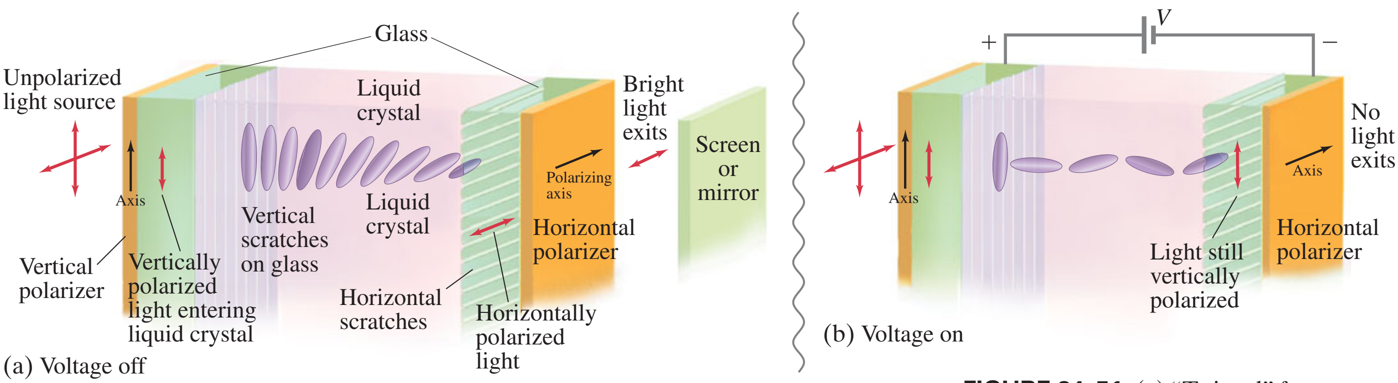


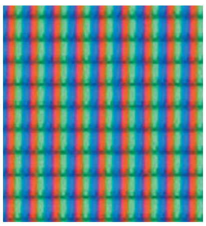
FIGURE 24–51 (a) "Twisted" form of liquid crystal. Light polarization plane is rotated 90°, and so is transmitted by the horizontal polarizer. Only one line of molecules is shown. (b) Molecules disoriented by electric field. The plane of polarization is not changed, so light does not pass through the horizontal polarizer. (The transparent electrodes are not shown.)

Now suppose a voltage is applied to transparent electrodes on each glass plate of the pixel. The rod-like molecules are polar (or can acquire an internal separation of charge due to the applied electric field). The applied voltage tends to align the molecules end-to-end, and they no longer follow the careful twisted pattern shown in Fig. 24–51a. Instead the applied electric field tends to align the molecules end-to-end, left to right (perpendicular to the glass plates), Fig. 24–51b, and then they don't affect the light polarization significantly. The entering plane-polarized light no longer has its plane of polarization rotated as it passes through the liquid crystal, and no light can exit through the second (horizontal) polarizer (Fig. 24–51b). With the voltage on, the pixel appears dark.†

†Some displays use an opposite system: the polarizers are parallel to each other (the scratches remain at 90° to maintain the twist). Then voltage *off* results in *black* (no light), and voltage *on* results in bright light.

FIGURE 24–52 Watch-face LCD display with altimeter. The black segments or pixels have a voltage applied to them. Note that the 8 uses all seven segments (pixels); other numbers use fewer.

FIGURE 24–53 Arrangement of subpixels on a TV or computer display (enlarged).



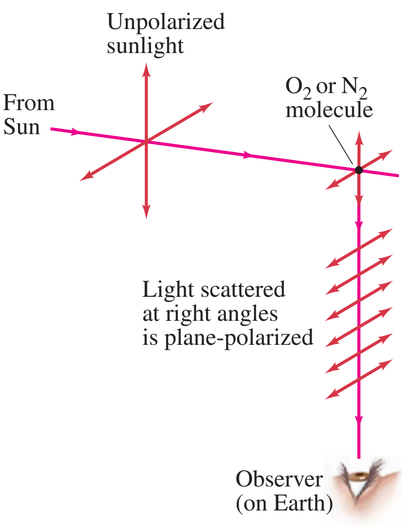
Simple display screens, such as for watches and calculators, use ambient light as the source (you can't see the display in the dark), with a mirror behind the LCD to reflect the light back. There are only a few pixels, corresponding to the elongated segments needed to form the numbers from 0 to 9 (and letters in some displays), as seen in Fig. 24–52. Any pixels to which a voltage is applied appear dark and form part of a number. With no voltage, pixels pass light through the polarizers to the mirror and back out, which forms a bright background. Displays with white numbers on a dark background have the voltages reversed.

Television, cell phone, and computer LCDs are more sophisticated. A color pixel consists of three cells, or subpixels, each covered with a red, green, or blue filter (Fig. 24–53). Varying brightnesses of these three primary colors can yield almost any natural color. A good-quality screen consists of millions of pixels. Behind this array of pixels is a light source, often thin fluorescent tubes the diameter of a straw, or light-emitting diodes (LEDs). The light passes through the liquid crystal subpixels, or not, depending on the voltage applied to each, as we discussed in detail in Section 17–11. See especially Figs. 17–31 and 17–33.

[To obtain a range of gray scale or range of color brightness, each subpixel cannot simply go on or off as in Fig. 24–51. Several techniques can be used depending on the construction of the LCD. If the voltage applied in Fig. 24–51b is small enough, the disorientation of the molecules may be small, allowing some rotation of the polarization vector and thus some light can pass through, the actual amount depending on the voltage. Alternatively, each subpixel can be pulsed—the length of time it is *on* affects the perceived brightness. The effect of stronger or weaker brightness can instead be provided by the number of nearby subpixels of the same color that are turned on or off; this third system lets the eye "average" over many pixels, but reduces the sharpness or resolution of the picture.]

*24–12 Scattering of Light by the Atmosphere

FIGURE 24–54 Unpolarized sunlight scattered by molecules of the air. An observer at right angles sees plane-polarized light, since the component of oscillation along the line of sight emits no light along that line.



PHYSICS APPLIED
Why the sky is blue
Why sunsets are red
Why clouds are white

Sunsets are red, the sky is blue, and skylight is polarized (at least partially). These phenomena can be explained on the basis of the *scattering* of light by the molecules of the atmosphere. In Fig. 24–54 we see unpolarized light from the Sun impinging on a molecule of the Earth's atmosphere. The electric field of the EM wave sets the electric charges within the molecule into oscillation, and the molecule absorbs some of the incident radiation. But the molecule quickly reemits this light since the charges are oscillating. As discussed in Section 22–2, oscillating electric charges produce EM waves. The intensity is strongest along the direction perpendicular to the oscillation, and drops to zero along the line of oscillation (Section 22–2). In Fig. 24–54 the motion of the charges is resolved into two components. An observer at right angles to the direction of the sunlight, as shown, will see plane-polarized light because no light is emitted along the line of the other component of the oscillation. (When viewing along the line of an oscillation, you don't see that oscillation, and hence see no waves made by it.) At other viewing angles, both components will be present; one will be stronger, however, so the light appears partially polarized. Thus, the process of scattering explains the polarization of skylight.

Scattering of light by the Earth's atmosphere depends on wavelength λ. For particles much smaller than the wavelength of light (such as molecules of air), the particles will be less of an obstruction to long wavelengths than to short ones. The scattering decreases, in fact, as $1/\lambda^4$. Blue and violet light are thus scattered much more than red and orange, which is why the sky looks blue. At sunset, the Sun's rays pass through a maximum length of atmosphere. Much of the blue has been taken out by scattering. The light that reaches us at this low angle where the Sun is near the horizon, and reflects off clouds and haze, is thus lacking in blue. That is why sunsets appear reddish.

The dependence of scattering on $1/\lambda^4$ is valid only if the scattering objects are much smaller than the wavelength of the light. This is valid for oxygen and nitrogen molecules whose diameters are about 0.2 nm. Clouds, however, contain water droplets or crystals that are much larger than λ. They scatter all frequencies of light nearly uniformly. Hence clouds appear white (or gray, if shadowed).

Summary

The wave theory of light is strongly supported by observations that light exhibits **interference** and **diffraction**. Wave theory also explains the refraction of light and the fact that light travels more slowly in transparent solids and liquids than it does in air.

[*An aid to predicting wave behavior is **Huygens' principle**, which states that every point on a wave front can be considered as a source of tiny wavelets that spread out in the forward direction at the speed of the wave itself. The new wave front is the envelope (the common tangent) of all the wavelets.]

The wavelength of light in a medium with index of refraction n is

$$\lambda_n = \frac{\lambda}{n}, \tag{24–1}$$

where λ is the wavelength in vacuum; the frequency is not changed.

Young's double-slit experiment demonstrated the interference of light. The observed bright spots of the interference pattern are explained as constructive interference between the beams coming through the two slits, where the beams differ in path length by an integral number of wavelengths. The dark areas in between are due to destructive interference when the path lengths differ by $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on. The angles θ at which **constructive interference** occurs are given by

$$\sin\theta = m\frac{\lambda}{d}, \tag{24–2a}$$

where λ is the wavelength of the light, d is the separation of the slits, and m is an integer $(0, 1, 2, \cdots)$. **Destructive interference** occurs at angles θ given by

$$\sin\theta = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}, \tag{24–2b}$$

where m is an integer $(0, 1, 2, \cdots)$.

Two sources of light are perfectly **coherent** if the waves leaving them are of the same single frequency and maintain the same phase relationship at all times. If the light waves from the two sources have a random phase with respect to each other over time (as for two lightbulbs), the two sources are **incoherent**.

The frequency or wavelength of light determines its color. The **visible spectrum** in air extends from about 400 nm (violet) to about 750 nm (red).

Glass prisms spread white light into its constituent colors because the index of refraction varies with wavelength, a phenomenon known as **dispersion**.

The formula $\sin\theta = m\lambda/d$ for constructive interference also holds for a **diffraction grating**, which consists of many parallel slits or lines, separated from each other by a distance d. The peaks of constructive interference are much brighter and sharper for a diffraction grating than for a two-slit apparatus.

A diffraction grating (or a prism) is used in a **spectrometer** to separate different colors and observe **line spectra**. For a given order m, θ depends on λ. Precise determination of wavelength can be done with a spectrometer by careful measurement of θ.

Diffraction refers to the fact that light, like other waves, bends around objects it passes, and spreads out after passing through narrow slits. This bending gives rise to a **diffraction pattern** due to interference between rays of light that travel different distances.

Light passing through a very narrow slit of width D (on the order of the wavelength λ) will produce a pattern with a bright central maximum of half-width θ given by

$$\sin\theta = \frac{\lambda}{D}, \tag{24–3a}$$

flanked by fainter lines to either side.

Light reflected from the front and rear surfaces of a thin film of transparent material can interfere constructively or destructively, depending on the path difference. A phase change of 180° or $\frac{1}{2}\lambda$ occurs when the light reflects at a surface where the index of refraction increases. Such **thin-film interference** has many practical applications, such as lens coatings and using Newton's rings to check uniformity of glass surfaces.

In **unpolarized light**, the electric field vectors oscillate in all transverse directions. If the electric vector oscillates only in one plane, the light is said to be **plane-polarized**. Light can also be partially polarized.

When an unpolarized light beam passes through a **Polaroid** sheet, the emerging beam is plane-polarized. When a light beam is polarized and passes through a Polaroid, the intensity varies as the Polaroid is rotated. Thus a Polaroid can act as a **polarizer** or as an **analyzer**.

The intensity I_0 of a plane-polarized light beam incident on a Polaroid is reduced to

$$I = I_0\cos^2\theta \tag{24–5}$$

where θ is the angle between the axis of the Polaroid and the initial plane of polarization.

Light can also be partially or fully **polarized by reflection**. If light traveling in air is reflected from a medium of index of refraction n, the reflected beam will be *completely* plane-polarized if the incident angle θ_p is given by

$$\tan\theta_p = n. \tag{24–6b}$$

The fact that light can be polarized shows that it must be a transverse wave.

Questions

1. Does Huygens' principle apply to sound waves? To water waves? Explain how Huygens' principle makes sense for water waves, where each point vibrates up and down.
2. Why is light sometimes described as rays and sometimes as waves?
3. We can hear sounds around corners but we cannot see around corners; yet both sound and light are waves. Explain the difference.
4. Two rays of light from the same source destructively interfere if their path lengths differ by how much?
5. Monochromatic red light is incident on a double slit, and the interference pattern is viewed on a screen some distance away. Explain how the fringe pattern would change if the red light source is replaced by a blue light source.
6. If Young's double-slit experiment were submerged in water, how would the fringe pattern be changed?

7. Why doesn't the light from the two headlights of a distant car produce an interference pattern?
8. Why are interference fringes noticeable only for a *thin* film like a soap bubble and not for a thick piece of glass?
9. Why are the fringes of Newton's rings (Fig. 24–31) closer together as you look farther from the center?
10. Some coated lenses appear greenish yellow when seen by reflected light. What reflected wavelengths do you suppose the coating is designed to eliminate completely?
11. A drop of oil on a pond appears bright at its edges, where its thickness is much less than the wavelengths of visible light. What can you say about the index of refraction of the oil compared to that of water?
12. Radio waves and visible light are both electromagnetic waves. Why can a radio receive a signal behind a hill when we cannot see the transmitting antenna?
13. Hold one hand close to your eye and focus on a distant light source through a narrow slit between two fingers. (Adjust your fingers to obtain the best pattern.) Describe the pattern that you see.
14. For diffraction by a single slit, what is the effect of increasing (*a*) the slit width, (*b*) the wavelength?
15. Describe the single-slit diffraction pattern produced when white light falls on a slit having a width of (*a*) 60 nm, (*b*) 60,000 nm.
16. What happens to the diffraction pattern of a single slit if the whole apparatus is immersed in (*a*) water, (*b*) a vacuum, instead of in air.
17. What is the difference in the interference patterns formed by two slits 10^{-4} cm apart as compared to a diffraction grating containing 10^4 slits/cm?
18. For a diffraction grating, what is the advantage of (*a*) many slits, (*b*) closely spaced slits?
19. White light strikes (*a*) a diffraction grating and (*b*) a prism. A rainbow appears on a wall just below the direction of the horizontal incident beam in each case. What is the color of the top of the rainbow in each case? Explain.
20. What does polarization tell us about the nature of light?
21. Explain the advantage of polarized sunglasses over plain tinted sunglasses.
22. How can you tell if a pair of sunglasses is polarizing or not?
*23. What would be the color of the sky if the Earth had no atmosphere?
*24. If the Earth's atmosphere were 50 times denser than it is, would sunlight still be white, or would it be some other color?

MisConceptual Questions

1. Light passing through a double-slit arrangement is viewed on a distant screen. The interference pattern observed on the screen would have the widest spaced fringes for the case of
(*a*) red light and a small slit spacing.
(*b*) blue light and a small slit spacing.
(*c*) red light and a large slit spacing.
(*d*) blue light and a large slit spacing.
2. Light from a green laser of wavelength 530 nm passes through two slits that are 400 nm apart. The resulting pattern formed on a screen in front of the slits is shown in Fig. 24–55. If point A is the same distance from both slits, how much closer is point B to one slit than to the other?
(*a*) 530 nm.
(*b*) 265 nm.
(*c*) 400 nm.
(*d*) 0 nm.
(*e*) It depends on the distance to the screen.

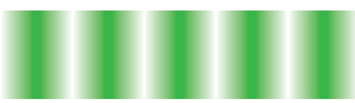


FIGURE 24–55 MisConceptual Question 2.

3. The colors in a rainbow are caused by
(*a*) the interaction of the light reflected from different raindrops.
(*b*) different amounts of absorption for light of different colors by the water in the raindrops.
(*c*) different amounts of refraction for light of different colors by the water in the raindrops.
(*d*) the downward motion of the raindrops.
4. A double-slit experiment yields an interference pattern due to the path length difference from light traveling through one slit versus the other. Why does a single slit show a diffraction pattern?
(*a*) There is a path length difference from waves originating at different parts of the slit.
(*b*) The wavelength of the light is shorter than the slit.
(*c*) The light passing through the slit interferes with light that does not pass through.
(*d*) The single slit must have something in the middle of it, causing it to act like a double slit.
5. If you hold two fingers very close together and look at a bright light, you see lines between the fingers. What is happening?
(*a*) You are holding your fingers too close to your eye to be able to focus on it.
(*b*) You are seeing a diffraction pattern.
(*c*) This is a quantum-mechanical tunneling effect.
(*d*) The brightness of the light is overwhelming your eye.
6. Light passes through a slit that is about 5×10^{-3} m high and 5×10^{-7} m wide. The central bright light visible on a distant screen will be
(*a*) about 5×10^{-3} m high and about 5×10^{-7} m wide.
(*b*) about 5×10^{-3} m high and wider than 5×10^{-7} m.
(*c*) about 5×10^{-3} m high and narrower than 5×10^{-7} m.
(*d*) taller than 5×10^{-3} m high and wider than 5×10^{-7} m.
(*e*) taller than 5×10^{-3} m high and about 5×10^{-7} m wide.

7. Blue light of wavelength λ passes through a single slit of width d and forms a diffraction pattern on a screen. If we replace the blue light by red light of wavelength 2λ, we can retain the original diffraction pattern if we change the slit width
(*a*) to $d/4$.
(*b*) to $d/2$.
(*c*) not at all.
(*d*) to $2d$.
(*e*) to $4d$.

8. Imagine holding a circular disk in a beam of monochromatic light (Fig. 24–56). If diffraction occurs at the edge of the disk, the center of the shadow is
(*a*) darker than the rest of the shadow.
(*b*) a bright spot.
(*c*) bright or dark, depending on the wavelength.
(*d*) bright or dark, depending on the distance to the screen.

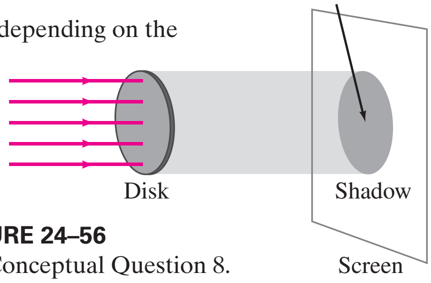


FIGURE 24–56 MisConceptual Question 8.

9. If someone is around a corner from you, what is the main reason you can hear him speaking but can't see him?
(*a*) Sound travels farther in air than light does.
(*b*) Sound can travel through walls, but light cannot.
(*c*) Sound waves have long enough wavelengths to bend around a corner; light wavelengths are too short to bend much.
(*d*) Sound waves reflect off walls, but light cannot.

10. When a CD is held at an angle, the reflected light contains many colors. What causes these colors?
(*a*) An anti-theft encoding intended to prevent copying of the CD.
(*b*) The different colors correspond to different data bits.
(*c*) Light reflected from the closely spaced grooves adds constructively for different wavelengths at different angles.
(*d*) It is part of the decorative label on the CD.

11. If a thin film has a thickness that is
(*a*) $\frac{1}{4}$ of a wavelength, constructive interference will always occur.
(*b*) $\frac{1}{4}$ of a wavelength, destructive interference will always occur.
(*c*) $\frac{1}{2}$ of a wavelength, constructive interference will always occur.
(*d*) $\frac{1}{2}$ of a wavelength, destructive interference will always occur.
(*e*) None of the above is always true.

12. If unpolarized light is incident from the left on three polarizers as shown in Fig. 24–57, in which case will some light get through?
(*a*) Case 1 only.
(*b*) Case 2 only.
(*c*) Case 3 only.
(*d*) Cases 1 and 3.
(*e*) All three cases.

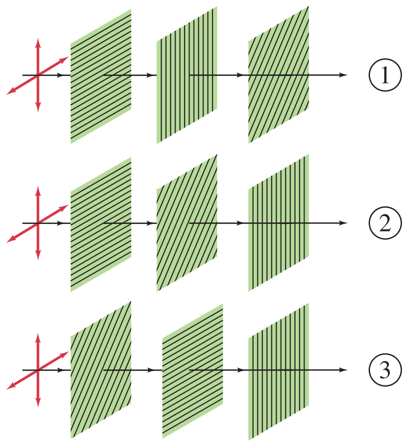


FIGURE 24–57 MisConceptual Question 12.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

24–3 Double-Slit Interference

1. (I) Monochromatic light falling on two slits 0.018 mm apart produces the fifth-order bright fringe at an 8.6° angle. What is the wavelength of the light used?

2. (I) The third-order bright fringe of 610-nm light is observed at an angle of 31° when the light falls on two narrow slits. How far apart are the slits?

3. (II) Monochromatic light falls on two very narrow slits 0.048 mm apart. Successive fringes on a screen 6.50 m away are 8.5 cm apart near the center of the pattern. Determine the wavelength and frequency of the light.

4. (II) If 720-nm and 660-nm light passes through two slits 0.62 mm apart, how far apart are the second-order fringes for these two wavelengths on a screen 1.0 m away?

5. (II) Water waves having parallel crests 4.5 cm apart pass through two openings 7.5 cm apart in a board. At a point 3.0 m beyond the board, at what angle relative to the "straight-through" direction would there be little or no wave action?

6. (II) A red laser from the physics lab is marked as producing 632.8-nm light. When light from this laser falls on two closely spaced slits, an interference pattern formed on a wall several meters away has bright red fringes spaced 5.00 mm apart near the center of the pattern. When the laser is replaced by a small laser pointer, the fringes are 5.14 mm apart. What is the wavelength of light produced by the laser pointer?

7. (II) Light of wavelength 680 nm falls on two slits and produces an interference pattern in which the third-order bright red fringe is 38 mm from the central fringe on a screen 2.8 m away. What is the separation of the two slits?

8. (II) Light of wavelength λ passes through a pair of slits separated by 0.17 mm, forming a double-slit interference pattern on a screen located a distance 37 cm away. Suppose that the image in Fig. 24–9a is an actual-size reproduction of this interference pattern. Use a ruler to measure a pertinent distance on this image; then utilize this measured value to determine λ (nm).

9. (II) A parallel beam of light from a He–Ne laser, with a wavelength 633 nm, falls on two very narrow slits 0.068 mm apart. How far apart are the fringes in the center of the pattern on a screen 3.3 m away?

10. (II) A physics professor wants to perform a lecture demonstration of Young's double-slit experiment for her class using the 633-nm light from a He–Ne laser. Because the lecture hall is very large, the interference pattern will be projected on a wall that is 5.0 m from the slits. For easy viewing by all students in the class, the professor wants the distance between the $m = 0$ and $m = 1$ maxima to be 35 cm. What slit separation is required in order to produce the desired interference pattern?

11. (II) Suppose a thin piece of glass is placed in front of the lower slit in Fig. 24–7 so that the two waves enter the slits 180° out of phase (Fig. 24–58). Draw in detail the interference pattern seen on the screen.

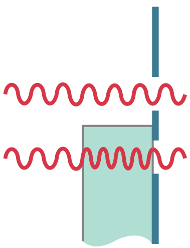


FIGURE 24–58
Problem 11.

12. (II) In a double-slit experiment it is found that blue light of wavelength 480 nm gives a second-order maximum at a certain location on the screen. What wavelength of visible light would have a minimum at the same location?

13. (II) Two narrow slits separated by 1.0 mm are illuminated by 544-nm light. Find the distance between adjacent bright fringes on a screen 4.0 m from the slits.

14. (II) Assume that light of a single color, rather than white light, passes through the two-slit setup described in Example 24–3. If the distance from the central fringe to a first-order fringe is measured to be 2.9 mm on the screen, determine the light's wavelength (in nm) and color (see Fig. 24–12).

15. (II) In a double-slit experiment, the third-order maximum for light of wavelength 480 nm is located 16 mm from the central bright spot on a screen 1.6 m from the slits. Light of wavelength 650 nm is then projected through the same slits. How far from the central bright spot will the second-order maximum of this light be located?

16. (II) Light of wavelength 470 nm in air shines on two slits 6.00×10^{-2} mm apart. The slits are immersed in water, as is a viewing screen 40.0 cm away. How far apart are the fringes on the screen?

17. (III) A very thin sheet of plastic ($n = 1.60$) covers one slit of a double-slit apparatus illuminated by 680-nm light. The center point on the screen, instead of being a maximum, is dark. What is the (minimum) thickness of the plastic?

24–4 Visible Spectrum; Dispersion

18. (I) By what percent is the speed of blue light (450 nm) less than the speed of red light (680 nm), in silicate flint glass (see Fig. 24–14)?

19. (II) A light beam strikes a piece of glass at a 65.00° incident angle. The beam contains two wavelengths, 450.0 nm and 700.0 nm, for which the index of refraction of the glass is 1.4831 and 1.4754, respectively. What is the angle between the two refracted beams?

20. (III) A parallel beam of light containing two wavelengths, $\lambda_1 = 455$ nm and $\lambda_2 = 642$ nm, enters the silicate flint glass of an equilateral prism as shown in Fig. 24–59. At what angles, θ_1 and θ_2, does each beam leave the prism (give angle with normal to the face)? See Fig. 24–14.

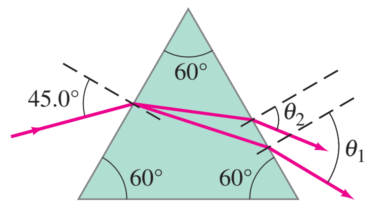


FIGURE 24–59
Problem 20.

24–5 Single-Slit Diffraction

21. (I) If 680-nm light falls on a slit 0.0425 mm wide, what is the angular width of the central diffraction peak?

22. (I) Monochromatic light falls on a slit that is 2.60×10^{-3} mm wide. If the angle between the first dark fringes on either side of the central maximum is 28.0° (dark fringe to dark fringe), what is the wavelength of the light used?

23. (II) When blue light of wavelength 440 nm falls on a single slit, the first dark bands on either side of center are separated by 51.0°. Determine the width of the slit.

24. (II) A single slit 1.0 mm wide is illuminated by 450-nm light. What is the width of the central maximum (in cm) in the diffraction pattern on a screen 6.0 m away?

25. (II) How wide is the central diffraction peak on a screen 2.30 m behind a 0.0348-mm-wide slit illuminated by 558-nm light?

26. (II) Consider microwaves which are incident perpendicular to a metal plate which has a 1.6-cm slit in it. Discuss the angles at which there are diffraction minima for wavelengths of (*a*) 0.50 cm, (*b*) 1.0 cm, and (*c*) 3.0 cm.

27. (II) (*a*) For a given wavelength λ, what is the minimum slit width for which there will be no diffraction minima? (*b*) What is the minimum slit width so that no visible light exhibits a diffraction minimum?

28. (II) Light of wavelength 620 nm falls on a slit that is 3.80×10^{-3} mm wide. Estimate how far the first bright diffraction fringe is from the strong central maximum if the screen is 10.0 m away.

29. (II) Monochromatic light of wavelength 633 nm falls on a slit. If the angle between the first two bright fringes on either side of the central maximum is 32°, estimate the slit width.

30. (II) Coherent light from a laser diode is emitted through a rectangular area 3.0 μm × 1.5 μm (horizontal-by-vertical). If the laser light has a wavelength of 780 nm, determine the angle between the first diffraction minima (*a*) above and below the central maximum, (*b*) to the left and right of the central maximum.

31. (III) If parallel light falls on a single slit of width D at a 28.0° angle to the normal, describe the diffraction pattern.

24–6 and 24–7 Diffraction Gratings

32. (I) At what angle will 510-nm light produce a second-order maximum when falling on a grating whose slits are 1.35×10^{-3} cm apart?

33. (I) A grating that has 3800 slits per cm produces a third-order fringe at a 22.0° angle. What wavelength of light is being used?

34. (I) A grating has 7400 slits/cm. How many spectral orders can be seen (400 to 700 nm) when it is illuminated by white light?

35. (II) Red laser light from a He–Ne laser ($\lambda = 632.8$ nm) creates a second-order fringe at 53.2° after passing through the grating. What is the wavelength λ of light that creates a first-order fringe at 20.6°?

36. (II) How many slits per centimeter does a grating have if the third order occurs at a 15.0° angle for 620-nm light?

37. (II) A source produces first-order lines when incident normally on a 9800-slit/cm diffraction grating at angles 28.8°, 36.7°, 38.6°, and 41.2°. What are the wavelengths?

38. (II) White light containing wavelengths from 410 nm to 750 nm falls on a grating with 7800 slits/cm. How wide is the first-order spectrum on a screen 3.40 m away?

39. (II) A diffraction grating has 6.5×10^5 slits/m. Find the angular spread in the second-order spectrum between red light of wavelength 7.0×10^{-7} m and blue light of wavelength 4.5×10^{-7} m.

40. (II) Two first-order spectrum lines are measured by a 9650-slit/cm spectroscope at angles, on each side of center, of +26°38′, +41°02′ and −26°18′, −40°27′. Calculate the wavelengths based on these data.

41. (II) What is the highest spectral order that can be seen if a grating with 6500 slits per cm is illuminated with 633-nm laser light? Assume normal incidence.

42. (II) The first-order line of 589-nm light falling on a diffraction grating is observed at a 14.5° angle. How far apart are the slits? At what angle will the third order be observed?

43. (II) Two (and only two) full spectral orders can be seen on either side of the central maximum when white light is sent through a diffraction grating. What is the maximum number of slits per cm for the grating?

24–8 Thin-Film Interference

44. (I) If a soap bubble is 120 nm thick, what wavelength is most strongly reflected at the center of the outer surface when illuminated normally by white light? Assume that $n = 1.32$.

45. (I) How far apart are the dark bands in Example 24–10 if the glass plates are each 21.5 cm long?

46. (II) (*a*) What is the smallest thickness of a soap film ($n = 1.33$) that would appear black if illuminated with 480-nm light? Assume there is air on both sides of the soap film. (*b*) What are two other possible thicknesses for the film to appear black? (*c*) If the thickness t was much less than λ, why would the film also appear black?

47. (II) A lens appears greenish yellow ($\lambda = 570$ nm is strongest) when white light reflects from it. What minimum thickness of coating ($n = 1.25$) do you think is used on such a glass lens ($n = 1.52$), and why?

48. (II) A thin film of oil ($n_o = 1.50$) with varying thickness floats on water ($n_w = 1.33$). When it is illuminated from above by white light, the reflected colors are as shown in Fig. 24–60. In air, the wavelength of yellow light is 580 nm. (*a*) Why are there no reflected colors at point A? (*b*) What is the oil's thickness t at point B?

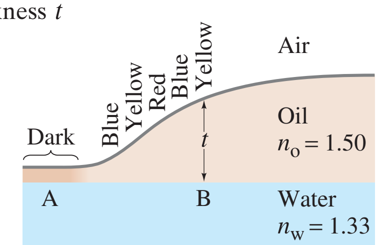


FIGURE 24–60 Problem 48.

49. (II) How many uncoated thin lenses in an optical instrument would reduce the amount of light passing through the instrument to 50% or less? (Assume the same transmission percent at each of the two surfaces—see page 697.)

50. (II) A total of 35 bright and 35 dark Newton's rings (not counting the dark spot at the center) are observed when 560-nm light falls normally on a planoconvex lens resting on a flat glass surface (Fig. 24–31). How much thicker is the lens at the center than the edges?

51. (II) If the wedge between the glass plates of Example 24–10 is filled with some transparent substance other than air—say, water—the pattern shifts because the wavelength of the light changes. In a material where the index of refraction is n, the wavelength is $\lambda_n = \lambda/n$, where λ is the wavelength in vacuum (Eq. 24–1). How many dark bands would there be if the wedge of Example 24–10 were filled with water?

52. (II) A fine metal foil separates one end of two pieces of optically flat glass, as in Fig. 24–33. When light of wavelength 670 nm is incident normally, 24 dark bands are observed (with one at each end). How thick is the foil?

53. (II) How thick (minimum) should the air layer be between two flat glass surfaces if the glass is to appear bright when 450-nm light is incident normally? What if the glass is to appear dark?

54. (III) A thin oil slick $(n_o = 1.50)$ floats on water $(n_w = 1.33)$. When a beam of white light strikes this film at normal incidence from air, the only enhanced reflected colors are red (650 nm) and violet (390 nm). From this information, deduce the (minimum) thickness t of the oil slick.

55. (III) A uniform thin film of alcohol $(n = 1.36)$ lies on a flat glass plate $(n = 1.56)$. When monochromatic light, whose wavelength can be changed, is incident normally, the reflected light is a minimum for $\lambda = 525$ nm and a maximum for $\lambda = 655$ nm. What is the minimum thickness of the film?

*24–9 Michelson Interferometer

***56.** (II) How far must the mirror M_1 in a Michelson interferometer be moved if 680 fringes of 589-nm light are to pass by a reference line?

***57.** (II) What is the wavelength of the light entering an interferometer if 362 bright fringes are counted when the movable mirror moves 0.125 mm?

***58.** (II) A micrometer is connected to the movable mirror of an interferometer. When the micrometer is tightened down on a thin metal foil, the net number of bright fringes that move, compared to closing the empty micrometer, is 296. What is the thickness of the foil? The wavelength of light used is 589 nm.

***59.** (III) One of the beams of an interferometer (Fig. 24–61) passes through a small evacuated glass container 1.155 cm deep. When a gas is allowed to slowly fill the container, a total of 158 dark fringes are counted to move past a reference line. The light used has a wavelength of 632.8 nm. Calculate the index of refraction of the gas at its final density, assuming that the interferometer is in vacuum.

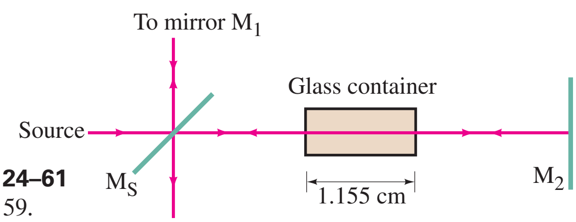


FIGURE 24–61 Problem 59.

24–10 Polarization

60. (I) Two polarizers are oriented at 72° to one another. Unpolarized light falls on them. What fraction of the light intensity is transmitted?

61. (I) What is Brewster's angle for an air–glass $(n = 1.56)$ surface?

62. (II) At what angle should the axes of two Polaroids be placed so as to reduce the intensity of the incident unpolarized light to (*a*) $\frac{1}{3}$, (*b*) $\frac{1}{10}$?

63. (II) Two polarizers are oriented at 42.0° to one another. Light polarized at a 21.0° angle to each polarizer passes through both. What is the transmitted intensity (%)?

64. (II) Three perfectly polarizing sheets are spaced 2 cm apart and in parallel planes. The transmission axis of the second sheet is 30° relative to the first one. The transmission axis of the third sheet is 90° relative to the *first* one. Unpolarized light impinges on the first polarizing sheet. What percent of this light is transmitted out through the third polarizer?

65. (II) A piece of material, suspected of being a stolen diamond $(n = 2.42)$, is submerged in oil of refractive index 1.43 and illuminated by unpolarized light. It is found that the reflected light is completely polarized at an angle of 62°. Is it diamond? Explain.

66. (II) Two Polaroids are aligned so that the initially unpolarized light passing through them is a maximum. At what angle should one of them be placed so the transmitted intensity is subsequently reduced by half?

67. (II) What is Brewster's angle for a diamond submerged in water?

68. (II) The critical angle for total internal reflection at a boundary between two materials is 58°. What is Brewster's angle at this boundary? Give two answers, one for each material.

69. (II) What would Brewster's angle be for reflections off the surface of water for light coming from beneath the surface? Compare to the angle for total internal reflection, and to Brewster's angle from above the surface.

70. (II) Unpolarized light of intensity I_0 passes through six successive Polaroid sheets each of whose axis makes a 35° angle with the previous one. What is the intensity of the transmitted beam?

71. (III) Two polarizers are oriented at 48° to each other and plane-polarized light is incident on them. If only 35% of the light gets through both of them, what was the initial polarization direction of the incident light?

72. (III) Four polarizers are placed in succession with their axes vertical, at 30.0° to the vertical, at 60.0° to the vertical, and at 90.0° to the vertical. (*a*) Calculate what fraction of the incident unpolarized light is transmitted by the four polarizers. (*b*) Can the transmitted light be *decreased* by removing one of the polarizers? If so, which one? (*c*) Can the transmitted light intensity be extinguished by removing polarizers? If so, which one(s)?

General Problems

73. Light of wavelength 5.0×10^{-7} m passes through two parallel slits and falls on a screen 5.0 m away. Adjacent bright bands of the interference pattern are 2.0 cm apart. (*a*) Find the distance between the slits. (*b*) The same two slits are next illuminated by light of a different wavelength, and the fifth-order minimum for this light occurs at the same point on the screen as the fourth-order minimum for the previous light. What is the wavelength of the second source of light?

74. Television and radio waves reflecting from mountains or airplanes can interfere with the direct signal from the station. (*a*) What kind of interference will occur when 75-MHz television signals arrive at a receiver directly from a distant station, and are reflected from a nearby airplane 122 m directly above the receiver? Assume $\frac{1}{2}\lambda$ change in phase of the signal upon reflection. (*b*) What kind of interference will occur if the plane is 22 m closer to the receiver?

75. Red light from three separate sources passes through a diffraction grating with 3.60×10^5 slits/m. The wavelengths of the three lines are 6.56×10^{-7} m (hydrogen), 6.50×10^{-7} m (neon), and 6.97×10^{-7} m (argon). Calculate the angles for the first-order diffraction line of each source.

76. What is the index of refraction of a clear material if a minimum thickness of 125 nm, when laid on glass, is needed to reduce reflection to nearly zero when light of 675 nm is incident normally upon it? Do you have a choice for an answer?

77. Light of wavelength 650 nm passes through two narrow slits 0.66 mm apart. The screen is 2.40 m away. A second source of unknown wavelength produces its second-order fringe 1.23 mm closer to the central maximum than the 650-nm light. What is the wavelength of the unknown light?

78. Monochromatic light of variable wavelength is incident normally on a thin sheet of plastic film in air. The reflected light is a maximum only for $\lambda = 491.4$ nm and $\lambda = 688.0$ nm in the visible spectrum. What is the thickness of the film ($n = 1.58$)? [*Hint*: Assume successive values of m.]

79. Show that the second- and third-order spectra of white light produced by a diffraction grating always overlap. What wavelengths overlap?

80. A radio station operating at 90.3 MHz broadcasts from two identical antennas at the same elevation but separated by a 9.0-m horizontal distance d, Fig. 24–62. A maximum signal is found along the midline, perpendicular to d at its midpoint and extending horizontally in both directions. If the midline is taken as 0°, at what other angle(s) θ is a maximum signal detected? A minimum signal? Assume all measurements are made much farther than 9.0 m from the antenna towers.

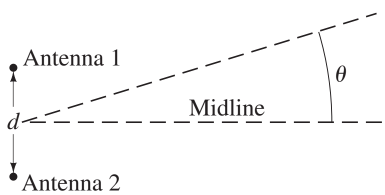


FIGURE 24–62 Problem 80.

81. Calculate the minimum thickness needed for an antireflective coating ($n = 1.38$) applied to a glass lens in order to eliminate (*a*) blue (450 nm), or (*b*) red (720 nm) reflections for light at normal incidence.

82. Stealth aircraft are designed to not reflect radar, whose wavelength is typically 2 cm, by using an antireflecting coating. Ignoring any change in wavelength in the coating, estimate its thickness.

83. A laser beam passes through a slit of width 1.0 cm and is pointed at the Moon, which is approximately 380,000 km from the Earth. Assume the laser emits waves of wavelength 633 nm (the red light of a He–Ne laser). Estimate the width of the beam when it reaches the Moon due to diffraction.

84. A thin film of soap ($n = 1.34$) coats a piece of flat glass ($n = 1.52$). How thick is the film if it reflects 643-nm red light most strongly when illuminated normally by white light?

85. When violet light of wavelength 415 nm falls on a single slit, it creates a central diffraction peak that is 8.20 cm wide on a screen that is 3.15 m away. How wide is the slit?

86. A series of polarizers are each rotated 10° from the previous polarizer. Unpolarized light is incident on this series of polarizers. How many polarizers does the light have to go through before it is $\frac{1}{5}$ of its original intensity?

87. The wings of a certain beetle have a series of parallel lines across them. When normally incident 480-nm light is reflected from the wing, the wing appears bright when viewed at an angle of 56°. How far apart are the lines?

88. A teacher stands well back from an outside doorway 0.88 m wide, and blows a whistle of frequency 950 Hz. Ignoring reflections, estimate at what angle(s) it is *not* possible to hear the whistle clearly on the playground outside the doorway. Assume 340 m/s for the speed of sound.

89. Light is incident on a diffraction grating with 7200 slits/cm and the pattern is viewed on a screen located 2.5 m from the grating. The incident light beam consists of two wavelengths, $\lambda_1 = 4.4 \times 10^{-7}$ m and $\lambda_2 = 6.8 \times 10^{-7}$ m. Calculate the linear distance between the first-order bright fringes of these two wavelengths on the screen.

90. How many slits per centimeter must a grating have if there is to be no second-order spectrum for any visible wavelength?

91. When yellow sodium light, $\lambda = 589$ nm, falls on a diffraction grating, its first-order peak on a screen 72.0 cm away falls 3.32 cm from the central peak. Another source produces a line 3.71 cm from the central peak. What is its wavelength? How many slits/cm are on the grating?

92. Two of the lines of the atomic hydrogen spectrum have wavelengths of 656 nm and 410 nm. If these fall at normal incidence on a grating with 7700 slits/cm, what will be the angular separation of the two wavelengths in the first-order spectrum?

93. A tungsten–halogen bulb emits a continuous spectrum of ultraviolet, visible, and infrared light in the wavelength range 360 nm to 2000 nm. Assume that the light from a tungsten–halogen bulb is incident on a diffraction grating with slit spacing d and that the first-order brightness maximum for the wavelength of 1200 nm occurs at angle θ. What other wavelengths within the spectrum of incident light will produce a brightness maximum at this same angle θ? [Optical filters are used to deal with this bothersome effect when a continuous spectrum of light is measured by a spectrometer.]

94. At what angle above the horizon is the Sun when light reflecting off a smooth lake is polarized most strongly?

95. Unpolarized light falls on two polarizer sheets whose axes are at right angles. (*a*) What fraction of the incident light intensity is transmitted? (*b*) What fraction is transmitted if a third polarizer is placed between the first two so that its axis makes a 56° angle with the axis of the first polarizer? (*c*) What if the third polarizer is in front of the other two?

96. At what angle should the axes of two Polaroids be placed so as to reduce the intensity of the incident unpolarized light by an additional factor (after the first Polaroid cuts it in half) of (*a*) 4, (*b*) 10, (*c*) 100?

Search and Learn

1. Compare Figs. 24–5, 24–6, and 24–7, which are different representations of the double-slit experiment. For each figure state the direction the light is traveling. Where are the wave crests in terms of this direction? How are they represented in each figure? Give one advantage of each figure in helping you understand the double-slit experiment and interference.

2. Discuss the similarities, and differences, of double-slit interference and single-slit diffraction.

3. Describe why the various colors of visible light appear as they do in Fig. 24–16, where red is at the top and violet at the bottom, and in Fig. 24–26, where violet is closest to the central maximum and red is farthest from the central maximum.

4. When can we use geometric optics as in Chapter 23, and when do we need to use the more complicated wave model of light discussed in Chapter 24? In particular, what are the physical characteristics that matter in making this decision?

5. A parallel beam of light containing two wavelengths, 420 nm and 650 nm, enters a borate flint glass equilateral prism (Fig. 24–63). (*a*) What is the angle between the two beams leaving the prism? (*b*) Repeat part (*a*) for a diffraction grating with 5800 slits/cm. (*c*) Discuss two advantages of a diffraction grating, including one that you see from your results.

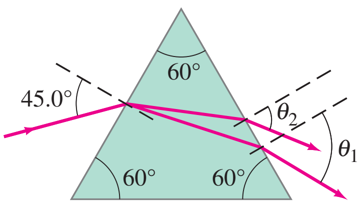


FIGURE 24–63
Search and Learn 5.

6. Suppose you viewed the light *transmitted* through a thin coating layered on a flat piece of glass. Draw a diagram, similar to Fig. 24–30 or 24–36, and describe the conditions required for maxima and minima. Consider all possible values of index of refraction. Discuss the relative intensity of the minima compared to the maxima and to zero.

7. What percent of visible light is reflected from plain glass? Assume your answer refers to transmission through each surface, front and back. How does the presence of multiple lenses in a good camera degrade the image? What is suggested in Section 24–8 to reduce this reflection? Explain in words, and sketch how this solution works. For a glass lens in air, about how much improvement does this solution provide?

ANSWERS TO EXERCISES

A: 333 nm; 6.0×10^{14} Hz; 2.0×10^{8} m/s.
B: 2.5 mm.
C: Narrower.
D: A.
E: (*e*).
F: Zero for both (*a*) and (*b*), because the two successive polarizers at 90° cancel all light. The 45° Polaroid must be inserted *between* the other two if transmission is to occur.