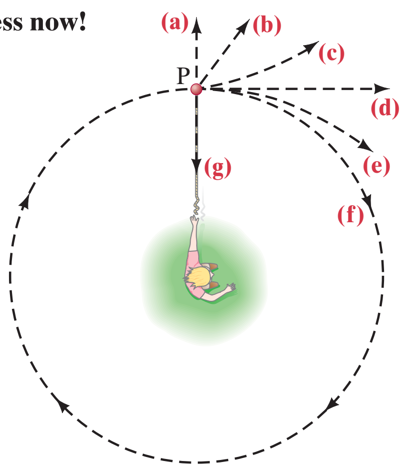
The astronauts in the upper left of this photo are working on a space shuttle. As they orbit the Earth—at a rather high speed—they experience apparent weightlessness. The Moon, in the background seen against the blackness of space, also is orbiting the Earth at high speed. Both the Moon and the space shuttle move in nearly circular orbits, and each undergoes a centripetal acceleration. What keeps the Moon and the space shuttle (and its astronauts) from moving off in a straight line away from Earth? It is the force of gravity. Newton's law of universal gravitation states that all objects attract all other objects with a force that depends on their masses and the square of the distance between them.

CHAPTER 5

Circular Motion; Gravitation

CHAPTER-OPENING QUESTIONS—Guess now!

1. You revolve a ball around you in a horizontal circle at constant speed on a string, as shown here from above. Which path will the ball follow if you let go of the string when the ball is at point P?



2. A space station revolves around the Earth as a satellite, 100 km above Earth's surface. What is the net force on an astronaut at rest inside the space station?

(a) Equal to her weight on Earth.
(b) A little less than her weight on Earth.
(c) Less than half her weight on Earth.
(d) Zero (she is weightless).
(e) Somewhat larger than her weight on Earth.

CONTENTS

An object moves in a straight line if the net force on it acts along the direction of motion, or the net force is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string being swung in a circle above one's head, or the nearly circular motion of the Moon about the Earth.

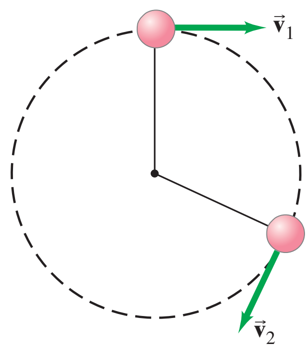


FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.

In this Chapter, we study the circular motion of objects, and how Newton's laws of motion apply. We also discuss how Newton conceived of another great law by applying the concepts of circular motion to the motion of the Moon and the planets. This is the law of universal gravitation, which was the capstone of Newton's analysis of the physical world.

5–1 Kinematics of Uniform Circular Motion

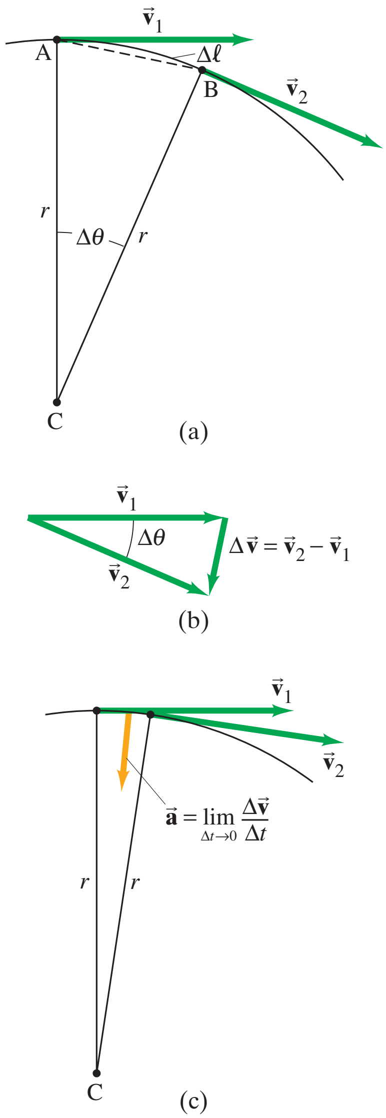
An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity continuously changes as the object moves around the circle (Fig. 5–1). Because acceleration is defined as the rate of change of velocity, a change in direction of velocity is an acceleration, just as a change in its magnitude is. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant $(v_1 = v_2 = v$ in Fig. 5–1). We now investigate this acceleration quantitatively.

Acceleration is defined as

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t},$$

where $\Delta \vec{\mathbf{v}}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation in which Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing, Fig. 5–2, we consider a nonzero time interval. During the time interval Δt, the particle in Fig. 5–2a moves from point A to point B, covering a distance $\Delta \ell$ *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \Delta \vec{\mathbf{v}}$, and is shown in Fig. 5–2b (note that $\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 + \Delta \vec{\mathbf{v}}$).

FIGURE 5–2 Determining the change in velocity, $\Delta \vec{\mathbf{v}}$, for a particle moving in a circle. The length $\Delta \ell$ is the distance along the arc, from A to B.



Now we let Δt be very small, approaching zero. Then $\Delta \ell$ and $\Delta\theta$ are also very small, and $\vec{\mathbf{v}}_2$ will be almost parallel to $\vec{\mathbf{v}}_1$, Fig. 5–2c; $\Delta \vec{\mathbf{v}}$ will be essentially perpendicular to them. Thus $\Delta \vec{\mathbf{v}}$ points toward the center of the circle. Since $\vec{\mathbf{a}}$, by definition, is in the same direction as $\Delta \vec{\mathbf{v}}$ (equation above), it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-pointing" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\vec{\mathbf{a}}_R$.

Now that we have determined the direction, next we find the magnitude of the radial (centripetal) acceleration, a_R. Because the line CA in Fig. 5–2a is perpendicular to $\vec{\mathbf{v}}_1$, and line CB is perpendicular to $\vec{\mathbf{v}}_2$, then the angle $\Delta\theta$ between CA and CB is also the angle between $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$. Hence the vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, and $\Delta \vec{\mathbf{v}}$ in Fig. 5–2b form a triangle that is geometrically similar† to triangle ACB in Fig. 5–2a. If we take $\Delta\theta$ to be very small (letting Δt be very small) and set $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change, we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta \ell}{r}.$$

This is an exact equality when Δt approaches zero, for then the arc length $\Delta \ell$ equals the chord length AB. We want to find the instantaneous acceleration, so we let Δt approach zero, write the above expression as an equality, and then solve for Δv:

$$\Delta v = \frac{v}{r}\,\Delta \ell. \qquad [\Delta t \to 0]$$

To get the centripetal acceleration, a_R, we divide Δv by Δt:

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v}{r}\,\frac{\Delta \ell}{\Delta t}. \qquad [\Delta t \to 0]$$

But $\Delta \ell / \Delta t$ is the linear speed, v, of the object, so the radial (centripetal)

†Appendix A contains a review of geometry.

acceleration is

$$a_R = \frac{v^2}{r}. \qquad \text{[radial (centripetal) acceleration]} \qquad \textbf{(5–1)}$$

[Equation 5–1 is valid at any instant in circular motion, and even when v is not constant.]

To summarize, *an object moving in a circle of radius r at constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is* $a_R = v^2/r$. It is not surprising that this acceleration depends on v and r. The greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

CAUTION
In uniform circular motion, the speed is constant, but the acceleration is not zero

The acceleration vector points toward the center of the circle when v is constant. But the velocity vector always points in the direction of motion, which is tangential to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion (Fig. 5–3). This is another example that illustrates the error in thinking that acceleration and velocity are always in the same direction. For an object falling in a vertical path, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are indeed parallel. But in uniform circular motion, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are perpendicular, not parallel (nor were they parallel in projectile motion, Section 3–5).

CAUTION
The direction of motion ($\vec{\mathbf{v}}$) *and the acceleration* ($\vec{\mathbf{a}}$) *are not in the same direction; instead,* $\vec{\mathbf{a}} \perp \vec{\mathbf{v}}$

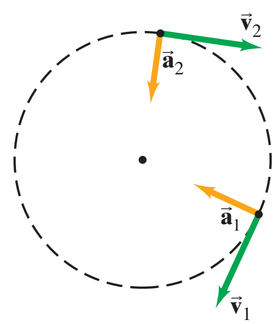


FIGURE 5–3 For uniform circular motion, $\vec{\mathbf{a}}$ is always perpendicular to $\vec{\mathbf{v}}$.

Circular motion is often described in terms of the **frequency** f, the number of revolutions per second. The **period** T of an object revolving in a circle is the time required for one complete revolution. Period and frequency are related by

$$T = \frac{1}{f}. \qquad \textbf{(5–2)}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution (= rev) takes $\frac{1}{3}$ s. An object revolving in a circle (of circumference $2\pi r$) at constant speed v travels a distance $2\pi r$ in one revolution which takes a time T. Thus

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}.$$

EXAMPLE 5–1 **Acceleration of a revolving ball.** A 150-g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m, as in Fig. 5–1 or 5–3. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

APPROACH The centripetal acceleration is $a_R = v^2/r$. We are given r, and we can find the speed of the ball, v, from the given radius and frequency.

SOLUTION If the ball makes 2.00 complete revolutions per second, then the ball travels in a complete circle in a time interval equal to 0.500 s, which is its period T. The distance traveled in this time is the circumference of the circle, $2\pi r$, where r is the radius of the circle. Therefore, the ball has speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.600\text{ m})}{(0.500\text{ s})} = 7.54\text{ m/s}.$$

The centripetal acceleration† is

$$a_R = \frac{v^2}{r} = \frac{(7.54\text{ m/s})^2}{(0.600\text{ m})} = 94.7\text{ m/s}^2.$$

EXERCISE A In Example 5–1, if the radius is doubled to 1.20 m, but the period stays the same, the centripetal acceleration will change by a factor of:
(*a*) 2; (*b*) 4; (*c*) $\frac{1}{2}$; (*d*) $\frac{1}{4}$; (*e*) none of these.

†Differences in the final digit can depend on whether you keep all digits in your calculator for v (which gives $a_R = 94.7\text{ m/s}^2$), or if you use $v = 7.54\text{ m/s}$ (which gives $a_R = 94.8\text{ m/s}^2$). Both results are valid since our assumed accuracy is about ± 0.1 m/s (see Section 1–4).

EXAMPLE 5–2 Moon's centripetal acceleration. The Moon's nearly circular orbit around the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

APPROACH Again we need to find the velocity v in order to find a_R.

SOLUTION In one orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8$ m is the radius of its circular path. The time required for one complete orbit is the Moon's period of 27.3 d. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period T in seconds is $T = (27.3\text{ d})(24.0\text{ h/d})(3600\text{ s/h}) = 2.36 \times 10^6$ s. Therefore,

$$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.84 \times 10^8\text{ m})}{(2.36 \times 10^6\text{ s})^2}$$
$$= 0.00272\text{ m/s}^2 = 2.72 \times 10^{-3}\text{ m/s}^2.$$

We can write this acceleration in terms of $g = 9.80\text{ m/s}^2$ (the acceleration of gravity at the Earth's surface) as

$$a_R = 2.72 \times 10^{-3}\text{ m/s}^2\left(\frac{g}{9.80\text{ m/s}^2}\right) = 2.78 \times 10^{-4}\, g$$
$$\approx 0.0003\, g.$$

CAUTION

Distinguish the Moon's gravity on objects at its surface from the Earth's gravity acting on the Moon (this Example)

NOTE The centripetal acceleration of the Moon, $a_R = 2.78 \times 10^{-4}\, g$, is *not* the acceleration of gravity for objects at the Moon's surface due to the Moon's gravity. Rather, it is the acceleration due to the *Earth's* gravity for any object (such as the Moon) that is 384,000 km from the Earth. Notice how small this acceleration is compared to the acceleration of objects near the Earth's surface.

5–2 Dynamics of Uniform Circular Motion

According to Newton's second law $(\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}})$, an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in that circle. That is, a net force is necessary to give it centripetal acceleration. The magnitude of the required force can be calculated using Newton's second law for the radial component, $\Sigma F_R = ma_R$, where a_R is the centripetal acceleration, $a_R = v^2/r$, and ΣF_R is the total (or net) force in the radial direction:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}. \qquad \text{[circular motion]} \qquad \textbf{(5–3)}$$

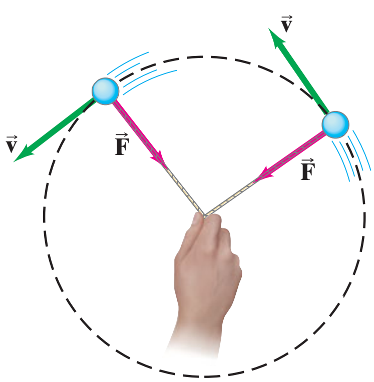


FIGURE 5–4 A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the circle's center.

For uniform circular motion (v = constant), the acceleration is a_R, which is directed toward the center of the circle at all times. Thus the *net force too must be directed toward the center of the circle* (Fig. 5–4). A net force is necessary because if no net force were exerted on the object, it would not move in a circle but in a straight line, as Newton's first law tells us. The direction of the net force is continually changing so that it is always directed toward the center of the circle. This force is sometimes called a centripetal ("pointing toward the center") force. But be aware that "centripetal force" does not indicate some new kind of force. The term "centripetal force" merely describes the *direction* of the net force needed to provide a circular path: the net force is directed toward the circle's center. The force *must be applied by other objects*. For example, to swing a ball in a circle on the end of a string, you pull on the string and the string exerts the force on the ball. (Try it.) Here, the "centripetal force" that provides the centripetal acceleration is tension in the string. In other cases it can be gravity (on the Moon, for example), a normal force, or even an electric force.

CAUTION

Centripetal force is not a new kind of force (Every force must be exerted by an object)

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: *there is no outward force* on the revolving object. Consider, for example, a person swinging a ball on the end of a string around her head (Fig. 5–5). If you have ever done this yourself, you know that you feel a force pulling outward on your hand. The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to your hand. This is not what is happening at all. To keep the ball moving in a circle, you pull *inwardly* on the string, and the string exerts this inward force on the ball. The ball exerts an equal and opposite force on the string (Newton's third law), and *this* is the outward force your hand feels (see Fig. 5–5).

The force *on the ball* in Fig. 5–5 is the one exerted *inwardly* on it by you, via the string. To see even more convincing evidence that a "centrifugal force" does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly outward, as shown in Fig. 5–6a. But it doesn't; the ball flies off tangentially (Fig. 5–6b), in the direction of the velocity it had at the moment it was released, because the inward force no longer acts. Try it and see!

CAUTION

There is no real "centrifugal force"

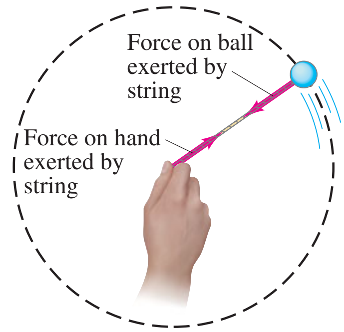


FIGURE 5–5 Swinging a ball on the end of a string (looking down from above).

EXERCISE B Return to Chapter-Opening Question 1, page 109, and answer it again now. Try to explain why you may have answered differently the first time.

EXAMPLE 5–3 ESTIMATE Force on revolving ball (horizontal). Estimate the force a person must exert on a string attached to a 0.150-kg ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second $(T = 0.500\,\text{s})$, as in Example 5–1. Ignore the string's mass.

APPROACH First we need to draw the free-body diagram for the ball. The forces acting on the ball are the force of gravity, $m\vec{\mathbf{g}}$ downward, and the tension force $\vec{\mathbf{F}}_\text{T}$ that the string exerts toward the hand at the center (which occurs because the person exerts that same force on the string). The free-body diagram for the ball is shown in Fig. 5–7. The ball's weight complicates matters and makes it impossible to revolve a ball with the cord perfectly horizontal. We estimate the force assuming the weight is small, and letting $\phi \approx 0$ in Fig. 5–7. Then $\vec{\mathbf{F}}_\text{T}$ will act nearly horizontally and, in any case, provides the force necessary to give the ball its centripetal acceleration.

SOLUTION We apply Newton's second law to the radial direction, which we assume is horizontal:

$$(\Sigma F)_\text{R} = ma_\text{R},$$

where $a_\text{R} = v^2/r$ and $v = 2\pi r/T = 2\pi(0.600\,\text{m})/(0.500\,\text{s}) = 7.54\,\text{m/s}$. Thus

$$F_\text{T} = m\frac{v^2}{r}$$
$$= (0.150\,\text{kg})\frac{(7.54\,\text{m/s})^2}{(0.600\,\text{m})} \approx 14\,\text{N}.$$

NOTE We keep only two significant figures in the answer because we ignored the ball's weight; it is $mg = (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 1.5\,\text{N}$, about $\frac{1}{10}$ of our result, which is small but maybe not so small as to justify stating a more precise answer for F_T.

FIGURE 5–6 If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b). In (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.

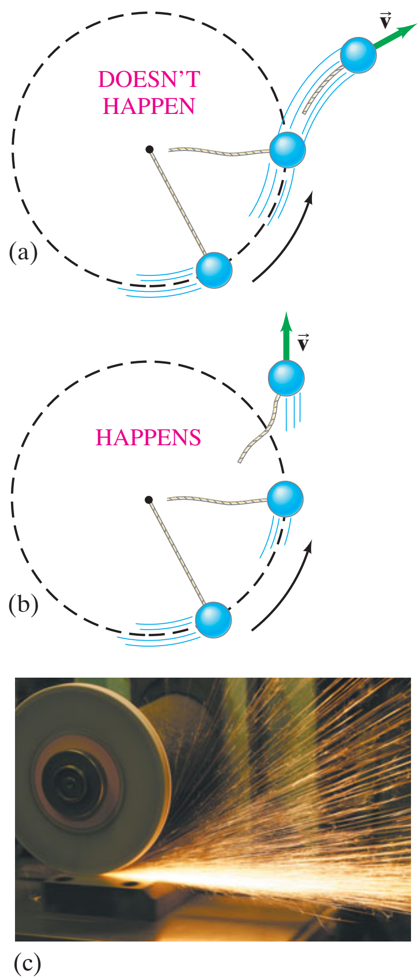
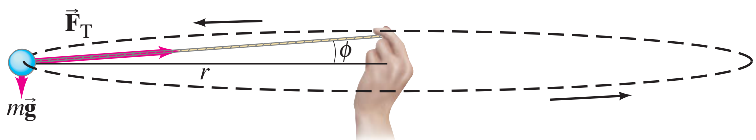


FIGURE 5–7 Example 5–3.



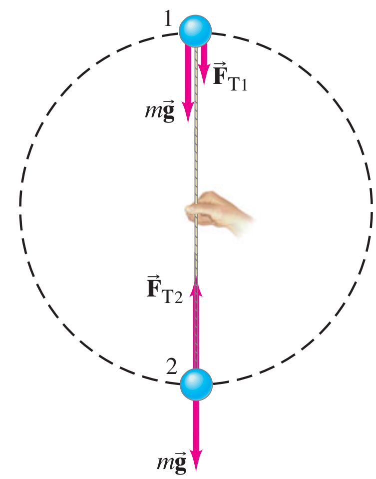


FIGURE 5–8 Example 5–4. Free-body diagrams for positions 1 and 2.

EXAMPLE 5–4 **Revolving ball (vertical circle).** A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. (*a*) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (*a*).

APPROACH The ball moves in a vertical circle and is *not* undergoing uniform circular motion. The radius is assumed constant, but the speed v changes because of gravity. Nonetheless, Eq. 5–1 $(a_R = v^2/r)$ is valid at each point along the circle, and we use it at the top and bottom points. The free-body diagram is shown in Fig. 5–8 for both positions.

SOLUTION (*a*) At the top (point 1), two forces act on the ball: $m\vec{\mathbf{g}}$, the force of gravity, and $\vec{\mathbf{F}}_{T1}$, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law, for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m\frac{v_1^2}{r}. \qquad \text{[at top]}$$

CAUTION
Circular motion only if cord is under tension

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$ (the ball at the topmost point), for which the equation above becomes

$$mg = m\frac{v_1^2}{r}. \qquad \text{[minimum speed at top]}$$

We solve for v_1, keeping an extra digit for use in (*b*):

$$v_1 = \sqrt{gr} = \sqrt{(9.80\,\text{m/s}^2)(1.10\,\text{m})}$$
$$= 3.283\,\text{m/s} \approx 3.28\,\text{m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(*b*) When the ball is at the bottom of the circle (point 2 in Fig. 5–8), the cord exerts its tension force F_{T2} upward, whereas the force of gravity, $m\vec{\mathbf{g}}$, still acts downward. Choosing *upward* as positive, Newton's second law gives:

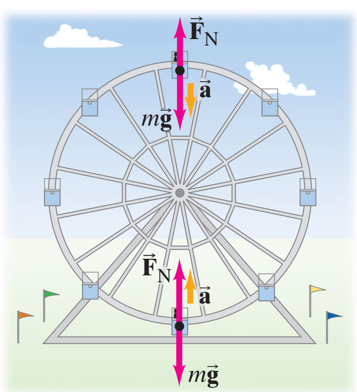
$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m\frac{v_2^2}{r}. \qquad \text{[at bottom]}$$

The speed v_2 is given as twice that in (*a*), namely 6.566 m/s. We solve for F_{T2}:

$$F_{T2} = m\frac{v_2^2}{r} + mg$$
$$= (0.150\,\text{kg})\frac{(6.566\,\text{m/s})^2}{(1.10\,\text{m})} + (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 7.35\,\text{N}.$$

FIGURE 5–9 Exercise C.



EXERCISE C A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5–9). Is the normal force that the seat exerts on the rider at the top of the wheel (*a*) less than, (*b*) more than, or (*c*) the same as, the force the seat exerts at the bottom of the wheel?

CONCEPTUAL EXAMPLE 5–5 **Tetherball.** The game of tetherball is played with a ball tied to a pole with a cord. After the ball is struck, it revolves around the pole as shown in Fig. 5–10. In what direction is the acceleration of the ball, and what force causes the acceleration, assuming constant speed?

RESPONSE If the ball revolves in a horizontal plane as shown, then the acceleration points horizontally toward the center of the ball's circular path (not toward the top of the pole). The force responsible for the acceleration may not be obvious at first, since there seems to be no force pointing directly horizontally. But it is the *net* force (the sum of $m\vec{\mathbf{g}}$ and $\vec{\mathbf{F}}_\mathrm{T}$ here) that must point in the direction of the acceleration. The vertical component of the cord tension, $F_{\mathrm{T}y}$, balances the ball's weight, $m\vec{\mathbf{g}}$. The horizontal component of the cord tension, $F_{\mathrm{T}x}$, is the force that produces the centripetal acceleration toward the center.

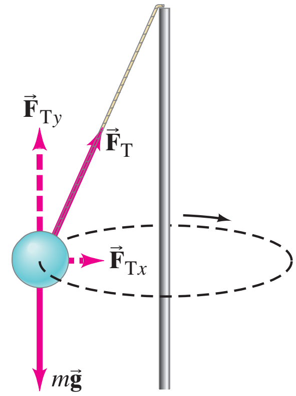


FIGURE 5–10 Example 5–5.

PROBLEM SOLVING

Uniform Circular Motion

1. **Draw a free-body diagram**, showing all the forces acting on each object under consideration. Be sure you can identify the source of each force (tension in a cord, Earth's gravity, friction, normal force, and so on). Don't put in something that doesn't belong (like a centrifugal force).
2. **Determine** which of the forces, or which of their components, act to provide the centripetal acceleration—that is, all the **forces or components that act radially**, toward or away from the center of the circular path. The sum of these forces (or components) provides the centripetal acceleration, $a_\mathrm{R} = v^2/r$.
3. **Choose a convenient coordinate system**, preferably with one axis along the acceleration direction.
4. **Apply Newton's second law** to the radial component:

$$\Sigma F_\mathrm{R} = ma_\mathrm{R} = m\frac{v^2}{r}. \qquad \text{[radial direction]}$$

5–3 Highway Curves: Banked and Unbanked

PHYSICS APPLIED
Driving around a curve

An example of circular dynamics occurs when an automobile rounds a curve, say to the left. In such a situation, you may feel that you are thrust outward toward the right side door. But there is no mysterious centrifugal force pulling on you. What is happening is that you tend to move in a straight line, whereas the car has begun to follow a curved path. To make you go in the curved path, the seat (friction) or the door of the car (direct contact) exerts a force on you (Fig. 5–11). The car also must have a force exerted on it toward the center of the curve if it is to move in that curve. On a flat road, this force is supplied by friction between the tires and the pavement.

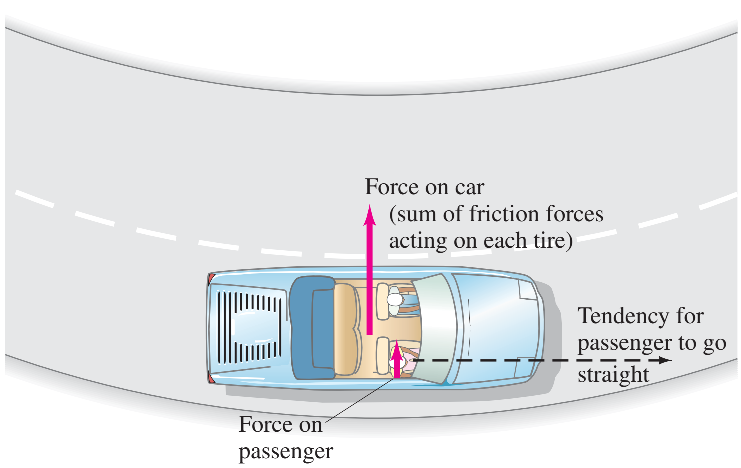


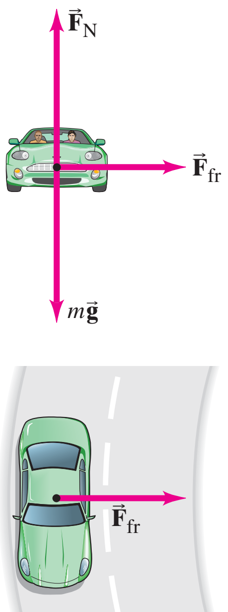
FIGURE 5–11 The road exerts an inward force (friction against the tires) on a car to make it move in a circle. The car exerts an inward force on the passenger.

FIGURE 5–12 Race car heading into a curve. From the tire marks we see that most cars experienced a sufficient friction force to give them the needed centripetal acceleration for rounding the curve safely. But, we also see tire tracks of cars on which there was not sufficient force—and which unfortunately followed more nearly straight-line paths.

If the wheels and tires of the car are rolling normally without slipping or sliding, the bottom of the tire is at rest against the road at each instant. So the friction force the road exerts on the tires is static friction. But if static friction is not great enough, as under icy conditions or high speed, the static friction force is less than mv^2/r and the car will skid out of a circular path into a more nearly straight path. See Fig. 5–12. Once a car skids or slides, the friction force becomes kinetic friction, which is smaller than static friction.

FIGURE 5–13 Example 5–6. Forces on a car rounding a curve on a flat road. (a) Front view, (b) top view.

(a) (b)



EXAMPLE 5–6 **Skidding on a curve.** A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 15 m/s (54 km/h). Will the car follow the curve, or will it skid? Assume: (*a*) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$; (*b*) the pavement is icy and $\mu_s = 0.25$.

APPROACH The forces on the car are gravity mg downward, the normal force F_N exerted upward by the road, and a horizontal friction force due to the road. They are shown in Fig. 5–13, which is the free-body diagram for the car. The car will follow the curve if the maximum static friction force is greater than the mass times the centripetal acceleration.

SOLUTION In the vertical direction (y) there is no acceleration. Newton's second law tells us that the normal force F_N on the car is equal to the weight mg since the road is flat:

$$0 = \Sigma F_y = F_N - mg$$

so

$$F_N = mg = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}.$$

In the horizontal direction the only force is friction, and we must compare it to the force needed to produce the centripetal acceleration to see if it is sufficient. The net horizontal force required to keep the car moving in a circle around the curve is

$$(\Sigma F)_R = ma_R = m\frac{v^2}{r} = (1000\,\text{kg})\frac{(15\,\text{m/s})^2}{(50\,\text{m})} = 4500\,\text{N}.$$

Now we compute the maximum total static friction force (the sum of the friction forces acting on each of the four tires) to see if it can be large enough to provide a safe centripetal acceleration. For (*a*), $\mu_s = 0.60$, and the maximum friction force attainable (recall from Section 4–8 that $F_{fr} \leq \mu_s F_N$) is

$$(F_{fr})_{max} = \mu_s F_N = (0.60)(9800\,\text{N}) = 5880\,\text{N}.$$

Since a force of only 4500 N is needed, and that is, in fact, how much will be exerted by the road as a static friction force, the car can follow the curve. But in (*b*) the maximum static friction force possible is

$$(F_{fr})_{max} = \mu_s F_N = (0.25)(9800\,\text{N}) = 2450\,\text{N}.$$

The car will skid because the ground cannot exert sufficient force (4500 N is needed) to keep it moving in a curve of radius 50 m at a speed of 54 km/h.

The possibility of skidding is worse if the wheels lock (stop rotating) when the brakes are applied too hard. When the tires are rolling, static friction exists. But if the wheels lock (stop rotating), the tires slide and the friction force, which is now kinetic friction, is less. More importantly, the *direction* of the friction force changes suddenly if the wheels lock. Static friction can point perpendicular to the velocity, as in Fig. 5–13b; but if the car slides, kinetic friction points *opposite* to the velocity. The force no longer points toward the center of the circle, and the car cannot continue in a curved path (see Fig. 5–12). Even worse, if the road is wet or icy, locking of the wheels occurs with less force on the brake pedal since there is less road friction to keep the wheels turning rather than sliding. Antilock brakes (ABS) are designed to limit brake pressure just before the point where sliding would occur, by means of delicate sensors and a fast computer.

PHYSICS APPLIED
Antilock brakes

EXERCISE D To negotiate a flat (unbanked) curve at a *faster* speed, a driver puts a couple of sand bags in his van aiming to increase the force of friction between the tires and the road. Will the sand bags help?

The banking of curves can reduce the chance of skidding. The normal force exerted by a banked road, acting perpendicular to the road, will have a component toward the center of the circle (Fig. 5–14), thus reducing the reliance on friction. For a given banking angle θ, there will be one speed for which no friction at all is required. This will be the case when the horizontal component of the normal force toward the center of the curve, $F_N \sin\theta$ (see Fig. 5–14), is just equal to the force required to give a vehicle its centripetal acceleration—that is, when

$$F_N \sin\theta = m\frac{v^2}{r}. \qquad \text{[no friction required]}$$

The banking angle of a road, θ, is chosen so that this condition holds for a particular speed, called the "design speed."

PHYSICS APPLIED
Banked curves

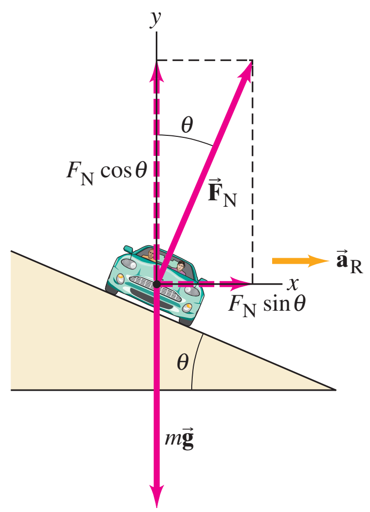


FIGURE 5–14 Normal force on a car rounding a banked curve, resolved into its horizontal and vertical components. The centripetal acceleration is horizontal (*not* parallel to the sloping road). The friction force on the tires, not shown, could point up or down along the slope, depending on the car's speed. The friction force will be zero for one particular speed.

EXAMPLE 5–7 **Banking angle.** (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for a road which has a curve of radius 50 m with a design speed of 50 km/h?

APPROACH Even though the road is banked, the car is still moving along a horizontal circle, so the centripetal acceleration needs to be horizontal. We choose our x and y axes as horizontal and vertical so that a_R, which is horizontal, is along the x axis. The forces on the car are the Earth's gravity mg downward, and the normal force F_N exerted by the road perpendicular to its surface. See Fig. 5–14, where the components of F_N are also shown. We don't need to consider the friction of the road because we are designing a road to be banked so as to eliminate dependence on friction.

SOLUTION (*a*) Since there is no vertical motion, $a_y = 0$ and $\Sigma F_y = ma_y$ gives

$$F_N \cos\theta - mg = 0$$

or

$$F_N = \frac{mg}{\cos\theta}.$$

[Note in this case that $F_N \geq mg$ because $\cos\theta \leq 1$.]

CAUTION
F_N is not always equal to mg

We substitute this relation for F_N into the equation for the horizontal motion,

$$F_N \sin\theta = m\frac{v^2}{r},$$

which becomes

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

or

$$\tan\theta = \frac{v^2}{rg}.$$

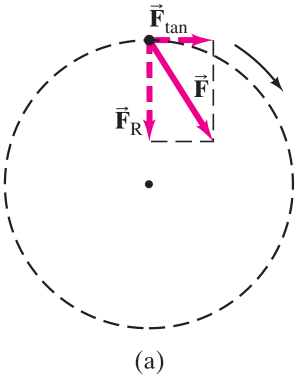
This is the formula for the banking angle θ: no friction needed at this speed v.
(*b*) For $r = 50$ m and $v = 50$ km/h $(= 14$ m/s$)$,

$$\tan\theta = \frac{(14\text{ m/s})^2}{(50\text{ m})(9.8\text{ m/s}^2)} = 0.40,$$

so $\theta = \tan^{-1}(0.40) = 22°$.

We have been using the centripetal acceleration $a = v^2/r$ where r is the radius of a circle. For a road, and in many other situations, we don't have a full circle, but only a portion of a circle: $a = v^2/r$ still works and we often call r the **radius of curvature** of that portion of a circle we are dealing with.

*5–4 Nonuniform Circular Motion



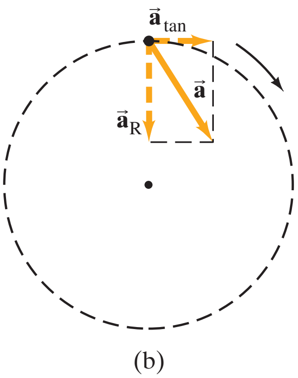


FIGURE 5–15 The speed of an object moving in a circle changes if the force on it has a tangential component, F_{tan}. Part (a) shows the force $\vec{F}$ and its vector components; part (b) shows the acceleration vector and its vector components.

Circular motion at constant speed occurs when the net force on an object is exerted toward the center of the circle. If the net force is not directed toward the center but is at an angle, as shown in Fig. 5–15a, the force has two components. The component directed toward the center of the circle, $\vec{F}_R$, gives rise to the centripetal acceleration, $\vec{a}_R$, and keeps the object moving in a circle. The component tangent to the circle, $\vec{F}_{tan}$, acts to increase (or decrease) the speed, and thus gives rise to a component of the acceleration tangent to the circle, $\vec{a}_{tan}$. When the speed of the object is changing, a tangential component of force is acting.

When you first start revolving a ball on the end of a string around your head, you must give it tangential acceleration. You do this by pulling on the string with your hand displaced from the center of the circle. In athletics, a hammer thrower accelerates the hammer tangentially in a similar way so that it reaches a high speed before release.

The tangential component of the acceleration, a_{tan}, has magnitude equal to the rate of change of the *magnitude* of the object's velocity:

$$a_{tan} = \frac{\Delta v}{\Delta t}.$$

The radial (centripetal) acceleration arises from the change in *direction* of the velocity and, as we have seen (Eq. 5–1), has magnitude

$$a_R = \frac{v^2}{r}.$$

The tangential acceleration always points in a direction tangent to the circle, and is in the direction of motion (parallel to $\vec{v}$, which is always tangent to the circle) if the speed is increasing, as shown in Fig. 5–15b. If the speed is decreasing, $\vec{a}_{tan}$ points antiparallel to $\vec{v}$. In either case, $\vec{a}_{tan}$ and $\vec{a}_R$ are always perpendicular to each other; and *their directions change* continually as the object moves along its circular path. The total vector acceleration $\vec{a}$ is the sum of the two components:

$$\vec{a} = \vec{a}_{tan} + \vec{a}_R.$$

Since $\vec{a}_R$ and $\vec{a}_{tan}$ are always perpendicular to each other, the magnitude of $\vec{a}$ at any moment is

$$a = \sqrt{a_{tan}^2 + a_R^2}.$$

EXAMPLE 5–8 **Two components of acceleration.** A race car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35 m/s in 11 s, moving on a circular track of radius 500 m. Assuming constant tangential acceleration, find (*a*) the tangential acceleration, and (*b*) the radial acceleration, at the instant when the speed is $v = 15\,\mathrm{m/s}$.

APPROACH The tangential acceleration relates to the change in speed of the car, and can be calculated as $a_{tan} = \Delta v/\Delta t$. The centripetal acceleration relates to the change in the *direction* of the velocity vector and is calculated using $a_R = v^2/r$.

SOLUTION (*a*) During the 11-s time interval, we assume the tangential acceleration a_{tan} is constant. Its magnitude is

$$a_{tan} = \frac{\Delta v}{\Delta t} = \frac{(35\,\mathrm{m/s} - 0\,\mathrm{m/s})}{11\,\mathrm{s}} = 3.2\,\mathrm{m/s^2}.$$

(*b*) When $v = 15\,\mathrm{m/s}$, the centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(15\,\mathrm{m/s})^2}{(500\,\mathrm{m})} = 0.45\,\mathrm{m/s^2}.$$

NOTE The radial (centripetal) acceleration increases continually, whereas the tangential acceleration stays constant.

5–5 Newton's Law of Universal Gravitation

Besides developing the three laws of motion, Isaac Newton also examined the motion of the planets and the Moon. In particular, he wondered about the nature of the force that must act to keep the Moon in its nearly circular orbit around the Earth.

Newton was also thinking about the problem of gravity. Since falling objects accelerate, Newton had concluded that they must have a force exerted on them, a force we call the force of gravity. Whenever an object has a force exerted *on* it, that force is exerted *by* some other object. But what *exerts* the force of gravity? Every object on the surface of the Earth feels the force of gravity F_G, and no matter where the object is, the force is directed toward the center of the Earth (Fig. 5–16). Newton concluded that it must be the Earth itself that exerts the gravitational force on objects at its surface.

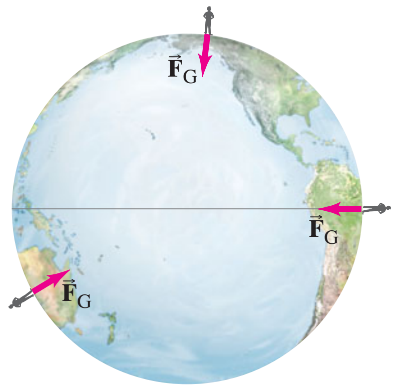


FIGURE 5–16 Anywhere on Earth, whether in Alaska, Peru, or Australia, the force of gravity acts downward toward the Earth's center.

According to legend, Newton noticed an apple drop from a tree. He is said to have been struck with a sudden inspiration: If gravity acts at the tops of trees, and even at the tops of mountains, then perhaps it acts all the way to the Moon! With this idea that it is the Earth's gravity that holds the Moon in its orbit, Newton developed his great theory of gravitation. But there was controversy at the time. Many thinkers had trouble accepting the idea of a force "acting at a distance." Typical forces act through contact—your hand pushes a cart and pulls a wagon, a bat hits a ball, and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the Moon, even though there is no contact, and the two objects may even be very far apart.†

Newton set about determining the magnitude of the gravitational force that the Earth exerts on the Moon as compared to the gravitational force on objects at the Earth's surface. The centripetal acceleration of the Moon, as we calculated in Example 5–2, is $a_R = 0.00272\ \text{m/s}^2$. In terms of the acceleration of gravity at the Earth's surface, $g = 9.80\ \text{m/s}^2$,

$$a_R = \frac{0.00272\ \text{m/s}^2}{9.80\ \text{m/s}^2}\, g \approx \frac{1}{3600}\, g.$$

That is, the acceleration of the Moon toward the Earth is about $\frac{1}{3600}$ as great as the acceleration of objects at the Earth's surface. The Moon is 384,000 km from the Earth, which is about 60 times the Earth's radius of 6380 km. That is, the Moon is 60 times farther from the Earth's center than are objects at the Earth's surface. But $60 \times 60 = 60^2 = 3600$. Again that number 3600! Newton concluded that the gravitational force F_{grav} or F_G exerted by the Earth on any object decreases with the square of its distance r from the Earth's center:

$$F_G \propto \frac{1}{r^2}.$$

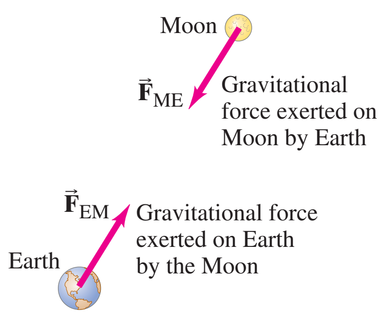
The Moon is 60 Earth radii away, so it feels a gravitational force only $\frac{1}{60^2} = \frac{1}{3600}$ times as strong as it would if it were at a point on the Earth's surface.

Newton realized that the force of gravity on an object depends not only on distance but also on the object's mass. In fact, it is directly proportional to its mass, as we have seen (Eq. 4–3). According to Newton's third law, when the Earth exerts its gravitational force on any object, such as the Moon, that object exerts an equal and opposite force on the Earth (Fig. 5–17). Because of this *symmetry*, Newton reasoned, the magnitude of the force of gravity must be proportional to *both* masses:

$$F_G \propto \frac{m_E m_{Obj}}{r^2},$$

where m_E and m_{Obj} are the masses of the Earth and the other object, respectively, and r is the distance from the Earth's center to the center of the other object.

FIGURE 5–17 The gravitational force one object exerts on a second object is directed toward the first object; and, by Newton's third law, is equal and opposite to the force exerted by the second object on the first. In the case shown, the gravitational force on the Moon due to Earth, $\vec{F}_{ME}$, is equal and opposite to the gravitational force on Earth due to the Moon, $\vec{F}_{EM}$. That is, $\vec{F}_{ME} = -\vec{F}_{EM}$.



†To deal with the conceptual difficulty of "action at a distance," the idea of a *gravitational field* was introduced many years later: every object that has mass produces a gravitational field in space. The force one object exerts on a second object is then due to the gravitational field produced by the first object at the position of the second object. We discuss fields in Section 16–7.

Newton went a step further in his analysis of gravity. In his examination of the orbits of the planets, he concluded that the force required to hold the different planets in their orbits around the Sun seems to diminish as the inverse square of their distance from the Sun. This led him to believe that it is also the gravitational force that acts between the Sun and each of the planets to keep them in their orbits. And if gravity acts between these objects, why not between all objects? Thus he proposed his **law of universal gravitation**, which we can state as follows:

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

The magnitude of the gravitational force can be written as

$$F_G = G\frac{m_1 m_2}{r^2}, \tag{5–4}$$

where m_1 and m_2 are the masses of the two particles, r is the distance between them, and G is a universal constant which must be measured experimentally.

The value of G must be very small, since we are not aware of any force of attraction between ordinary-sized objects, such as between two baseballs. The force between two ordinary objects was first measured by Henry Cavendish in 1798, over 100 years after Newton published his law. To detect and measure the incredibly small force between ordinary objects, he used an apparatus like that shown in Fig. 5–18. Cavendish confirmed Newton's hypothesis that two objects attract one another and that Eq. 5–4 accurately describes this force. In addition, because Cavendish could measure F_G, m_1, m_2, and r accurately, he was able to determine the value of the constant G as well. The accepted value today is

$$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}.$$

(See Table inside front cover for values of all constants to highest known precision.) Equation 5–4 is called an **inverse square law** because the force is inversely proportional to r^2.

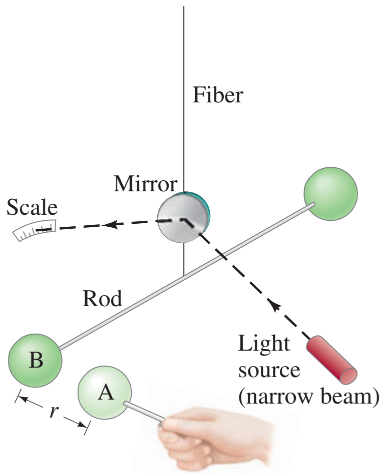


FIGURE 5–18 Schematic diagram of Cavendish's apparatus. Two spheres are attached to a light horizontal rod, which is suspended at its center by a thin fiber. When a third sphere (labeled A) is brought close to one of the suspended spheres (labeled B), the gravitational force causes the latter to move, and this twists the fiber slightly. The tiny movement is magnified by the use of a narrow light beam directed at a mirror mounted on the fiber. The beam reflects onto a scale. Previous determination of how large a force will twist the fiber a given amount then allows the experimenter to determine the magnitude of the gravitational force between the two objects, A and B.

[Strictly speaking, Eq. 5–4 gives the magnitude of the gravitational force that one particle exerts on a second particle that is a distance r away. For an extended object (that is, not a point), we must consider how to measure the distance r. A correct calculation treats each extended body as a collection of particles, and the total force is the sum of the forces due to all the particles. The sum over all these particles is often done using integral calculus, which Newton himself invented. When extended bodies are small compared to the distance between them (as for the Earth–Sun system), little inaccuracy results from considering them as point particles. Newton was able to show that the *gravitational force exerted on a particle outside a uniform sphere is the same as if the entire mass of the sphere was concentrated at its center*.† Thus Eq. 5–4 gives the correct force between two uniform spheres where r is the distance between their centers.]

EXAMPLE 5–9 ESTIMATE Can you attract another person gravitationally? A 50-kg person and a 70-kg person are sitting on a bench close to each other. Estimate the magnitude of the gravitational force each exerts on the other.

APPROACH This is an estimate: we let the distance between the centers of the two people be $\frac{1}{2}$m (about as close as you can get).

SOLUTION We use Eq. 5–4, which gives

$$F_G = G\frac{m_1 m_2}{r^2} \approx \frac{(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(50\,\mathrm{kg})(70\,\mathrm{kg})}{(0.5\,\mathrm{m})^2} \approx 10^{-6}\,\mathrm{N},$$

rounded off to an order of magnitude. Such a force is unnoticeably small unless extremely sensitive instruments are used (< 1/100,000 of a pound).

†We demonstrate this result in Section 16–12.

EXAMPLE 5–10 **Spacecraft at $2r_E$.** What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance $r_E = 6380$ km above the Earth's surface, Fig. 5–19)? The mass of the Earth is $m_E = 5.98 \times 10^{24}$ kg.

APPROACH We could plug all the numbers into Eq. 5–4, but there is a simpler approach. The spacecraft is twice as far from the Earth's center as when it is at the surface of the Earth. Therefore, since the force of gravity F_G decreases as the square of the distance (and $\frac{1}{2^2} = \frac{1}{4}$), the force of gravity on the satellite will be only one-fourth its weight at the Earth's surface.

SOLUTION At the surface of the Earth, $F_G = mg$. At a distance from the Earth's center of $2r_E$, F_G is $\frac{1}{4}$ as great:

$$F_G = \tfrac{1}{4}mg = \tfrac{1}{4}(2000\,\text{kg})(9.80\,\text{m/s}^2) = 4900\,\text{N}.$$

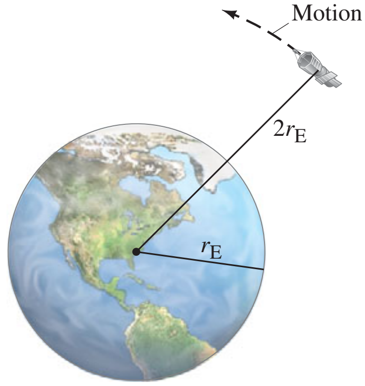


FIGURE 5–19 Example 5–10; a spacecraft in orbit at $r = 2r_E$.

Note carefully that the law of universal gravitation describes a *particular* force (gravity), whereas Newton's second law of motion $(F = ma)$ tells how an object accelerates due to *any* type of force.

CAUTION
Distinguish Newton's second law from the law of universal gravitation

5–6 Gravity Near the Earth's Surface

When Eq. 5–4 is applied to the gravitational force between the Earth and an object at its surface, m_1 becomes the mass of the Earth m_E, m_2 becomes the mass of the object m, and r becomes the distance of the object from the Earth's center, which is the radius of the Earth r_E. This force of gravity due to the Earth is the weight of the object on Earth, which we have been writing as mg. Thus,

$$mg = G\frac{mm_E}{r_E^2}.$$

We can solve this for g, the acceleration of gravity at the Earth's surface:

$$g = G\frac{m_E}{r_E^2}. \qquad \textbf{(5–5)}$$

Thus, the acceleration of gravity at the surface of the Earth, g, is determined by m_E and r_E. (Don't confuse G with g; they are very different quantities, but are related by Eq. 5–5.)

CAUTION
Distinguish G from g

Until G was measured, the mass of the Earth was not known. But once G was measured, Eq. 5–5 could be used to calculate the Earth's mass, and Cavendish was the first to do so. Since $g = 9.80\,\text{m/s}^2$ and the radius of the Earth is $r_E = 6.38 \times 10^6$ m, then, from Eq. 5–5, we obtain the mass of the Earth to be

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\,\text{m/s}^2)(6.38 \times 10^6\,\text{m})^2}{6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98 \times 10^{24}\,\text{kg}.$$

Equation 5–5 can be applied to other planets, where g, m, and r would refer to that planet.

FIGURE 5–20 Example 5–11. Mount Everest, 8850 m (29,035 ft) above sea level; in the foreground, the author with sherpas at 5500 m (18,000 ft).

EXAMPLE 5–11 **ESTIMATE** **Gravity on Everest.** Estimate the effective value of g on the top of Mt. Everest, 8850 m (29,035 ft) above sea level (Fig. 5–20). That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude? Ignore the mass of the mountain itself.

APPROACH The force of gravity (and the acceleration due to gravity g) depends on the distance from the center of the Earth, so there will be an effective value g' on top of Mt. Everest which will be smaller than g at sea level. We assume the Earth is a uniform sphere (a reasonable "estimate").

SOLUTION We use Eq. 5–5, with r_E replaced by $r = 6380\,\text{km} + 8.9\,\text{km} = 6389\,\text{km} = 6.389 \times 10^6$ m:

$$g = G\frac{m_E}{r^2} = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})}{(6.389 \times 10^6\,\text{m})^2} = 9.77\,\text{m/s}^2,$$

which is a reduction of about 3 parts in a thousand (0.3%).

TABLE 5–1 Acceleration Due to Gravity at Various Locations

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

Note that Eq. 5–5 does not give precise values for g at different locations because the Earth is not a perfect sphere. The Earth not only has mountains and valleys, and it bulges at the equator, but also its mass is not distributed precisely uniformly. (See Table 5–1.) The Earth's rotation also affects the value of g. However, for most practical purposes, when an object is near the Earth's surface, we will simply use $g = 9.80\ m/s^2$ and write the weight of an object as mg.

EXERCISE E Suppose you could double the mass of a planet but keep its volume the same. How would the acceleration of gravity, g, at the surface change?

5–7 Satellites and "Weightlessness"

Satellite Motion

PHYSICS APPLIED
Artificial Earth satellites

Artificial satellites circling the Earth are now commonplace (Fig. 5–21). A satellite is put into orbit by accelerating it to a sufficiently high tangential speed with the use of rockets, as shown in Fig. 5–22. If the speed is too high, the spacecraft will not be confined by the Earth's gravity and will escape, never to return. If the speed is too low, it will return to Earth. Satellites are typically put into circular (or nearly circular) orbits, because such orbits require the least takeoff speed.

FIGURE 5–21 A satellite, the International Space Station, circling the Earth.

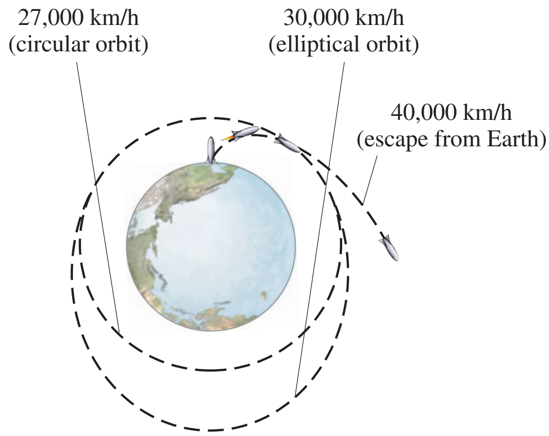
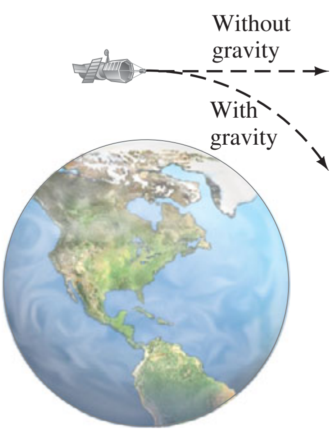


FIGURE 5–22 Artificial satellites launched at different speeds.

It is sometimes asked: "What keeps a satellite up?" The answer is: its high speed. If a satellite in orbit stopped moving, it would fall directly to Earth. But at the very high speed a satellite has, it would quickly fly out into space (Fig. 5–23) if it weren't for the gravitational force of the Earth pulling it into orbit. In fact, a satellite in orbit *is* falling (accelerating) toward Earth, but its high tangential speed keeps it from hitting Earth.

FIGURE 5–23 A moving satellite "falls" out of a straight-line path toward the Earth.



For satellites that move in a circle (at least approximately), the needed acceleration is centripetal and equals v^2/r. The force that gives a satellite this acceleration is the force of gravity exerted by the Earth, and since a satellite may be at a considerable distance from the Earth, we must use Newton's law of universal gravitation (Eq. 5–4) for the force acting on it. When we apply Newton's second law, $\Sigma F_R = ma_R$ in the radial direction, we find

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r}, \tag{5–6}$$

where m is the mass of the satellite. This equation relates the distance of the satellite from the Earth's center, r, to its speed, v, in a circular orbit. Note that only one force—gravity—is acting on the satellite, and that r is the sum of the Earth's radius r_E plus the satellite's height h above the Earth: $r = r_E + h$.

If we solve Eq. 5–6 for v, we find $v = \sqrt{Gm_E/r}$ and we see that a satellite's speed does not depend on its own mass. Satellites of different mass orbiting at the same distance above Earth have the same speed and period.

PHYSICS APPLIED
Geosynchronous satellites

EXAMPLE 5–12 Geosynchronous satellite. A *geosynchronous* satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Why? Because the center of a satellite orbit is always at the center of the Earth; so it is not possible to have a satellite orbiting above a fixed point on the Earth at any latitude other than 0°. Geosynchronous satellites are commonly used for TV and radio transmission, for weather forecasting, and as communication relays.[†] Determine (*a*) the height above the Earth's surface such a satellite must orbit, and (*b*) such a satellite's speed. (*c*) Compare to the speed of a satellite orbiting 200 km above Earth's surface.

APPROACH To remain above the same point on Earth as the Earth rotates, the satellite must have a period of 24 hours. We can apply Newton's second law, $F = ma$, where $a = v^2/r$ if we assume the orbit is circular.

SOLUTION (*a*) The only force on the satellite is the gravitational force due to the Earth. (We can ignore the gravitational force exerted by the Sun. Why?) We apply Eq. 5–6 assuming the satellite moves in a circle:

$$G\frac{m_{\text{Sat}}m_E}{r^2} = m_{\text{Sat}}\frac{v^2}{r}.$$

This equation has two unknowns, r and v. So we need a second equation. The satellite revolves around the Earth with the same period that the Earth rotates on its axis, namely once in 24 hours. Thus the speed of the satellite must be

$$v = \frac{2\pi r}{T},$$

where $T = 1\text{ day} = (24\text{ h})(3600\text{ s/h}) = 86{,}400\text{ s}$. We substitute this into the "satellite equation" above and obtain (after cancelling m_{Sat} on both sides):

$$G\frac{m_E}{r^2} = \frac{(2\pi r)^2}{rT^2}.$$

After cancelling an r, we can solve for r^3:

$$r^3 = \frac{Gm_ET^2}{4\pi^2} = \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})(86{,}400\,\text{s})^2}{4\pi^2}$$
$$= 7.54\times10^{22}\,\text{m}^3.$$

We take the cube root and find

$$r = 4.22\times10^7\,\text{m},$$

or 42,200 km from the Earth's center. We subtract the Earth's radius of 6380 km to find that a geosynchronous satellite must orbit about 36,000 km (about $6r_E$) above the Earth's surface.

(*b*) We solve for v in the satellite equation, Eq. 5–6:

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(4.22\times10^7\,\text{m})}} = 3070\,\text{m/s},$$

or about 11,000 km/h (≈ 7000 mi/h). We get the same result if we use $v = 2\pi r/T$.

(*c*) The equation in part (*b*) for v shows $v \propto \sqrt{1/r}$. So for $r = r_E + h = 6380\text{ km} + 200\text{ km} = 6580\text{ km}$, we get

$$v' = v\sqrt{\frac{r}{r'}} = (3070\,\text{m/s})\sqrt{\frac{(42{,}200\,\text{km})}{(6580\,\text{km})}} = 7770\,\text{m/s},$$

or about 28,000 km/h (≈ 17,000 mi/h).

[†]Geosynchronous satellites are useful because receiving and transmitting antennas at a given place on Earth can stay fixed on such a satellite (no tracking and no switching satellites is needed).

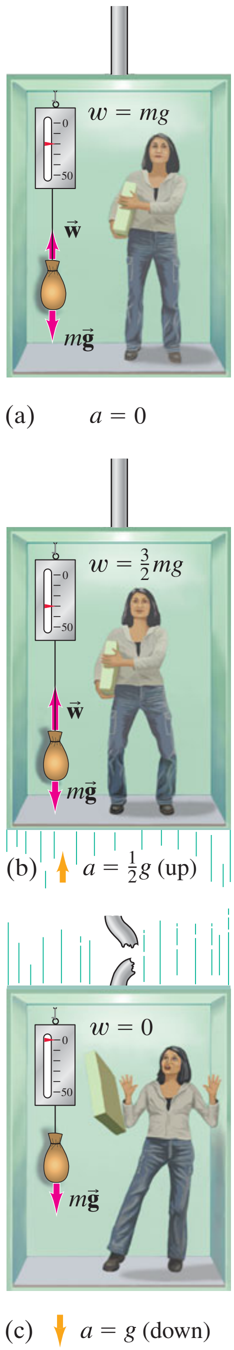


FIGURE 5–24 (a) A bag in an elevator at rest exerts a force on a spring scale equal to its weight. (b) In an elevator accelerating upward at $\frac{1}{2}g$, the bag's apparent weight is $1\frac{1}{2}$ times larger than its true weight. (c) In a freely falling elevator, the bag experiences "weightlessness": the scale reads zero.

Weightlessness

People and other objects in a satellite circling the Earth are said to experience apparent weightlessness. Let us first look at a simpler case: a falling elevator. In Fig. 5–24a, an elevator is at rest with a bag hanging from a spring scale. The scale reading indicates the downward force exerted on it by the bag. This force, exerted *on* the scale, is equal and opposite to the force exerted *by* the scale upward on the bag, and we call its magnitude w (for "weight"). Two forces act on the bag: the downward gravitational force and the upward force exerted by the scale equal to w. Because the bag is not accelerating $(a = 0)$, when we apply $\Sigma F = ma$ to the bag in Fig. 5–24a we obtain

$$w - mg = 0,$$

where mg is the weight of the bag. Thus, $w = mg$, and since the scale indicates the force w exerted on it by the bag, it registers a force equal to the weight of the bag, as we expect.

Now let the elevator have an acceleration, a. Applying Newton's second law, $\Sigma F = ma$, to the bag as seen from an inertial reference frame (the elevator itself is not now an inertial frame) we have

$$w - mg = ma.$$

Solving for w, we have

$$w = mg + ma. \qquad [a \text{ is + upward}]$$

We have chosen the positive direction up. Thus, if the acceleration a is up, a is positive; and the scale, which measures w, will read more than mg. We call w the *apparent weight* of the bag, which in this case would be greater than its actual weight (mg). If the elevator accelerates downward, a will be negative and w, the apparent weight, will be less than mg. The direction of the velocity $\vec{\mathbf{v}}$ doesn't matter. Only the direction of the acceleration $\vec{\mathbf{a}}$ (and its magnitude) influences the scale reading.

Suppose the elevator's acceleration is $\frac{1}{2}g$ upward; then we find

$$\begin{aligned} w &= mg + m\left(\tfrac{1}{2}g\right) \\ &= \tfrac{3}{2}mg. \end{aligned}$$

That is, the scale reads $1\frac{1}{2}$ times the actual weight of the bag (Fig. 5–24b). The apparent weight of the bag is $1\frac{1}{2}$ times its real weight. The same is true of the person: her apparent weight (equal to the normal force exerted on her by the elevator floor) is $1\frac{1}{2}$ times her real weight. We can say that she is experiencing $1\frac{1}{2}$ g's, just as astronauts experience so many g's at a rocket's launch.

If, instead, the elevator's acceleration is $a = -\frac{1}{2}g$ (downward), then $w = mg - \frac{1}{2}mg = \frac{1}{2}mg$. That is, the scale reads half the actual weight. If the elevator is in *free fall* (for example, if the cables break), then $a = -g$ and $w = mg - mg = 0$. The scale reads zero. See Fig. 5–24c. The bag appears weightless. If the person in the elevator accelerating at $-g$ let go of a box, it would not fall to the floor. True, the box would be falling with acceleration g. But so would the floor of the elevator and the person. The box would hover right in front of the person. This phenomenon is called **apparent weightlessness** because in the reference frame of the person, objects don't fall or seem to have weight—yet gravity does not disappear. Gravity is still acting on each object, whose weight is still mg.

The "weightlessness" experienced by people in a satellite orbit close to the Earth (Fig. 5–25) is the same apparent weightlessness experienced in a freely falling elevator. It may seem strange, at first, to think of a satellite as freely falling. But a satellite is indeed falling toward the Earth, as was shown in Fig. 5–23. The force of gravity causes it to "fall" out of its natural straight-line path. The acceleration of the satellite must be the acceleration due to gravity at that point, because the only force acting on it is gravity. Thus, although the force of gravity acts on objects within the satellite, the objects experience an apparent weightlessness because they, and the satellite, are accelerating together as in free fall.

Figure 5–26 shows some examples of "free fall," or apparent weightlessness, experienced by people on Earth for brief moments.

A completely different situation occurs if a spacecraft is out in space far from the Earth, the Moon, and other attracting bodies. The force of gravity due to the Earth and other celestial bodies will then be quite small because of the distances involved, and persons in such a spacecraft would experience real weightlessness.

EXERCISE F Return to Chapter-Opening Question 2, page 109, and answer it again now. Try to explain why you may have answered differently the first time.

FIGURE 5–25 This astronaut is outside the International Space Station. He must feel very free because he is experiencing apparent weightlessness.

(a) (b) (c)

FIGURE 5–26 Experiencing "weightlessness" on Earth.

5–8 Planets, Kepler's Laws, and Newton's Synthesis

Where did we first get the idea of planets? Have you ever escaped the lights of the city to gaze late at night at the multitude of stars in the night sky? It is a moving experience. Thousands of years ago, the ancients saw this sight every cloudless night, and were fascinated. They noted that the vast majority of stars, bright or dim, seemed to maintain fixed positions relative to each other. The ancients imagined these **fixed stars** as being attached to a huge inverted bowl, or sphere. This **celestial sphere** revolved around the Earth almost exactly once a day (Fig. 5–27), from east to west. Among all the stars that were visible to the naked eye (there were no telescopes until much later, about 1600), the ancients saw five stars that changed position relative to the fixed stars over weeks and months. These five wandering stars were called **planets** (Greek for wandering). Planets were thus visible at night as tiny points of light like other stars.

FIGURE 5–27 Time exposure showing movement of stars over a period of several hours.

The ancient idea that the Sun, Moon, and planets revolve around the Earth is called the **geocentric** view (geo = Earth in Greek). It was developed into a fine theoretical system by Ptolemy in the second century B.C. Today we believe in a **heliocentric** system (helios = Sun in Greek), where the Earth is just another planet, between Venus and Mars, orbiting around the Sun. Although a heliocentric view was proposed in ancient times, it was largely ignored until Renaissance Italy of the fifteenth century. The real theory change (see Section 1–1 and Fig. 1–2) began with the heliocentric theory of Nicolaus Copernicus (1473–1543) and then was greatly advanced by the experimental observations of Galileo around 1610 using his newly developed 30× telescope. Galileo observed that the planet Jupiter has moons (like a miniature solar system) and that Venus has phases like our Moon, not explainable by Ptolemy's geocentric system. [Galileo's famous encounter with the Church had little to do with religious faith, but rather with politics, personality conflict, and authority. Today it is generally understood that science and faith are different approaches that are not in conflict.]

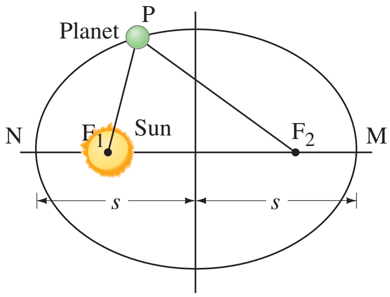


FIGURE 5–28 *Kepler's first law.* An ellipse is a closed curve such that the sum of the distances from any point P on the curve to two fixed points (called the foci, F_1 and F_2) remains constant. That is, the sum of the distances, $F_1P + F_2P$, is the same for all points on the curve. A circle is a special case of an ellipse in which the two foci coincide, at the center of the circle.

FIGURE 5–29 *Kepler's second law.* The two shaded regions have equal areas. The planet moves from point 1 to point 2 in the same time it takes to move from point 3 to point 4. Planets move fastest when closest to the Sun.

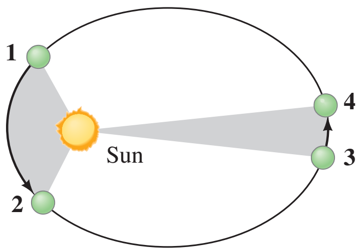


TABLE 5–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance to Sun, s (10^6 km)	Period, T (Earth yr)	s^3/T^2 $\left(10^{24}\,\frac{\text{km}^3}{\text{yr}^2}\right)$
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.000	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
(Pluto)†	5900	248	3.34

†Pluto, since its discovery in 1930, was considered a ninth planet. But its small mass and the recent discovery of other objects beyond Neptune with similar masses has led to calling these smaller objects, including Pluto, "dwarf planets." We keep it in the Table to indicate its great distance, and its consistency with Kepler's third law.

Kepler's Laws

Also about 1600, more than a half century before Newton proposed his three laws of motion and his law of universal gravitation, the German astronomer Johannes Kepler (1571–1630) had worked out a detailed description of the motion of the planets around the Sun. Kepler's work resulted in part from the many years he spent examining data collected (without a telescope) by Tycho Brahe (1546–1601) on the positions of the planets in their motion through the night sky.

Among Kepler's writings were three empirical findings that we now refer to as **Kepler's laws of planetary motion**. These are summarized as follows, with additional explanation in Figs. 5–28 and 5–29.

Kepler's first law: The path of each planet around the Sun is an ellipse with the Sun at one focus (Fig. 5–28).

Kepler's second law: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time (Fig. 5–29).

Kepler's third law: The ratio of the squares of the periods T of any two planets revolving around the Sun is equal to the ratio of the cubes of their mean distances from the Sun. [The mean distance equals the semimajor axis s (= half the distance from the planet's near point N and far point M from the Sun, Fig. 5–28).] That is, if T_1 and T_2 represent the periods (the time needed for one revolution about the Sun) for any two planets, and s_1 and s_2 represent their mean distances from the Sun, then

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3.$$

We can rewrite Kepler's third law as

$$\frac{s_1^3}{T_1^2} = \frac{s_2^3}{T_2^2},$$

meaning that s^3/T^2 should be the same for each planet. Present-day data are given in Table 5–2; see the last column.

In Examples and Problems we usually will assume the orbits are circles, although it is not quite true in general.

EXAMPLE 5–13 Where is Mars? Mars' period (its "year") was noted by Kepler to be about 687 days (Earth days), which is (687 d/365 d) = 1.88 yr (Earth years). Determine the mean distance of Mars from the Sun using the Earth as a reference.

APPROACH We are given the ratio of the periods of Mars and Earth. We can find the distance from Mars to the Sun using Kepler's third law, given the Earth–Sun distance as 1.50×10^{11} m (Table 5–2; also Table inside front cover).

SOLUTION Let the distance of Mars from the Sun be s_{MS}, and the Earth–Sun distance be $s_{ES} = 1.50 \times 10^{11}$ m. From Kepler's third law:

$$\frac{s_{MS}}{s_{ES}} = \left(\frac{T_M}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1.88\ \text{yr}}{1\ \text{yr}}\right)^{\frac{2}{3}} = 1.52.$$

So Mars is 1.52 times the Earth's distance from the Sun, or 2.28×10^{11} m.

Kepler's Third Law Derived, Sun's Mass, Perturbations

We will derive Kepler's third law for the special case of a circular orbit, in which case the mean distance s is the radius r of the circle. (Most planetary orbits are close to a circle.) First, we write Newton's second law of motion, $\Sigma F = ma$. For F we use the law of universal gravitation (Eq. 5–4) for the force between the Sun and a planet of mass m_1, and for a the centripetal acceleration, v^2/r. We assume the mass of the Sun M_S is much greater than the mass of its planets, so we ignore the effects of the planets on each other. Then

$$\Sigma F = ma$$

$$G\frac{m_1 M_S}{r_1^2} = m_1 \frac{v_1^2}{r_1}.$$

Here m_1 is the mass of a particular planet, r_1 its distance from the Sun, and v_1 its speed in orbit; M_S is the mass of the Sun, since it is the gravitational attraction of the Sun that keeps each planet in its orbit. The period T_1 of the planet is the time required for one complete orbit, which is a distance equal to $2\pi r_1$, the circumference of a circle. Thus

$$v_1 = \frac{2\pi r_1}{T_1}.$$

We substitute this formula for v_1 into the previous equation:

$$G\frac{m_1 M_S}{r_1^2} = m_1 \frac{4\pi^2 r_1}{T_1^2}.$$

We rearrange this to get

$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S}. \qquad \textbf{(5–7a)}$$

We derived this for planet 1 (say, Mars). The same derivation would apply for a second planet (say, Saturn) orbiting the Sun,

$$\frac{T_2^2}{r_2^3} = \frac{4\pi^2}{GM_S},$$

where T_2 and r_2 are the period and orbit radius, respectively, for the second planet. Since the right sides of the two previous equations are equal, we have $T_1^2/r_1^3 = T_2^2/r_2^3$ or, rearranging,

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3, \qquad \textbf{(5–7b)}$$

Kepler's third law

which is Kepler's third law. Equations 5–7a and 5–7b are valid also for elliptical orbits if we replace *r* with the semimajor axis *s*.

PHYSICS APPLIED
Determining the Sun's mass

EXAMPLE 5–14 **The Sun's mass determined.** Determine the mass of the Sun given the Earth's distance from the Sun as $r_{ES} = 1.5 \times 10^{11}$ m.

APPROACH Equation 5–7a relates the mass of the Sun M_S to the period and distance of any planet. We use the Earth.

SOLUTION The Earth's period is $T_E = 1\text{ yr} = \left(365\tfrac{1}{4}\text{ d}\right)(24\text{ h/d})(3600\text{ s/h}) = 3.16 \times 10^7$ s. We solve Eq. 5–7a for M_S:

$$M_S = \frac{4\pi^2 r_{ES}^3}{GT_E^2} = \frac{4\pi^2\left(1.5 \times 10^{11}\text{ m}\right)^3}{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(3.16 \times 10^7\text{ s}\right)^2} = 2.0 \times 10^{30}\text{ kg}.$$

Accurate measurements on the orbits of the planets indicated that they did not precisely follow Kepler's laws. For example, slight deviations from perfectly elliptical orbits were observed. Newton was aware that this was to be expected because any planet would be attracted gravitationally not only by the Sun but also (to a much lesser extent) by the other planets. Such deviations, or **perturbations**, in the orbit of Saturn were a hint that helped Newton formulate the law of universal gravitation, that all objects attract each other gravitationally. Observation of other perturbations later led to the discovery of Neptune. Deviations in the orbit of Uranus could not all be accounted for by perturbations due to the other known planets. Careful calculation in the nineteenth century indicated that these deviations could be accounted for if another planet existed farther out in the solar system. The position of this planet was predicted from the deviations in the orbit of Uranus, and telescopes focused on that region of the sky quickly found it; the new planet was called Neptune. Similar but much smaller perturbations of Neptune's orbit led to the discovery of Pluto in 1930.

PHYSICS APPLIED
Perturbations and discovery of planets

Other Centers for Kepler's Laws

The derivation of Eq. 5–7b, Kepler's third law, compared two planets revolving around the Sun. But the derivation is general enough to be applied to other systems. For example, we could apply Eq. 5–7b to compare an artificial satellite and our Moon, both revolving around Earth (then M_S would be replaced by M_E, the mass of the Earth). Or we could apply Eq. 5–7b to compare two moons revolving around Jupiter. But Kepler's third law, Eq. 5–7b, applies only to objects orbiting the same attracting center. Do not use Eq. 5–7b to compare, say, the Moon's orbit around Earth to the orbit of Mars around the Sun: they depend on different attracting centers.

CAUTION
Compare orbits of objects only around the same center

Distant Planetary Systems

PHYSICS APPLIED
Planets around other stars

Starting in the mid-1990s, planets revolving around distant stars (Fig. 5–30) were inferred from the regular "wobble" in position of each star due to the gravitational attraction of the revolving planet(s). Many such "extrasolar" planets are now known.

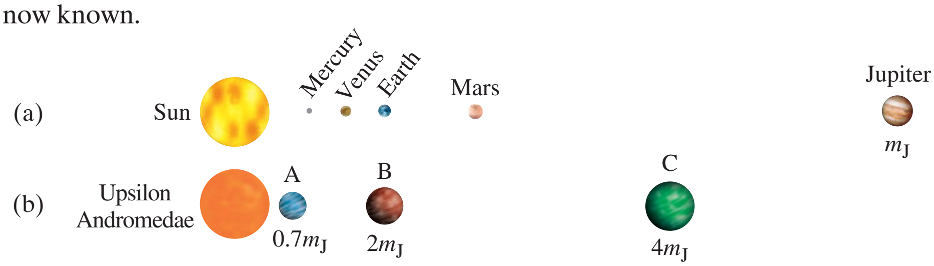


FIGURE 5–30 Our solar system (a) is compared to recently discovered planets orbiting (b) the star Upsilon Andromedae with at least three planets. m_J is the mass of Jupiter. (Sizes are not to scale.)

Newton's Synthesis

Kepler arrived at his laws through careful analysis of experimental data. Fifty years later, Newton was able to show that Kepler's laws could be derived mathematically from the law of universal gravitation and the laws of motion. Newton also showed that for any reasonable form for the gravitational force law, only one that depends on the inverse square of the distance is fully consistent with Kepler's laws. He thus used Kepler's laws as evidence in favor of his law of universal gravitation, Eq. 5–4.

The development by Newton of the law of universal gravitation and the three laws of motion was a major intellectual achievement. With these laws, he was able to describe the motion of objects on Earth and of the far-away planets seen in the night sky. The motions of the planets through the heavens and of objects on Earth were seen to follow the same laws (not recognized previously). For this reason, and also because Newton integrated the results of earlier scientists into his system, we sometimes speak of **Newton's synthesis**.

The laws formulated by Newton are referred to as **causal laws**. By **causality** we mean that one occurrence can cause another. When a rock strikes a window, we infer the rock *causes* the window to break. This idea of "cause and effect" relates to Newton's laws: the acceleration of an object was seen to be *caused* by the net force acting on it.

As a result of Newton's theories, the universe came to be viewed by many as a machine whose parts move in a **deterministic** way. This deterministic view of the universe had to be modified in the twentieth century (Chapter 28).

Sun/Earth Reference Frames

The geocentric–heliocentric controversy (page 125) may be seen today as a matter of frame of reference. From the reference frame of Earth, we see the Sun and Moon as revolving around us with average periods of 24 h (= definition of 1 day) and almost 25 h, respectively, roughly in circles. The orbits of the planets as seen from Earth are very complicated, however.

In the Sun's reference frame, Earth makes one revolution (= definition of the year) in 365.256 days, in an ellipse that is nearly a circle. The Sun's reference frame has the advantage that the other planets also have simple elliptical orbits. (Or nearly so—each planet's gravity pulls on the others, causing small perturbations.) The Sun's vastly greater mass ($>10^5 \times$ Earth's) allows it to be an easier reference frame to use.

The Sun itself (and the Earth with it) revolves around the center of our Galaxy (see Fig. 33–2 or 5–49) which itself moves relative to other galaxies. Indeed, there is no one reference frame that we can consider as preferred or central.

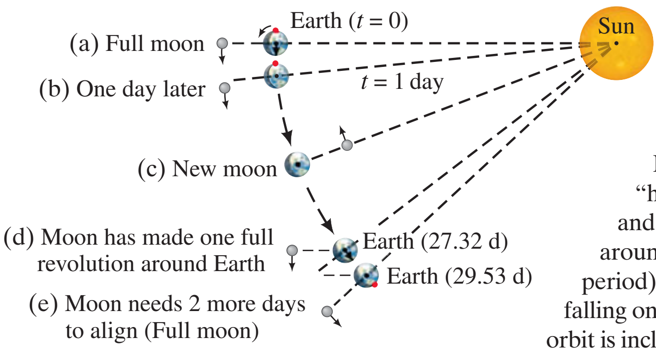


FIGURE 5–31 Looking down on the plane of Earth's orbit around the Sun (not to scale), above Earth's north pole, showing our Moon making one revolution about Earth: (a) at a Full moon (the red dot is an observer at about 6 PM who can just see the Full moon rise); (b) exactly one day later (for the red dot to see the Moon rise, the Earth must rotate another 50 min); (c) after making a "half revolution" the Moon is in line with the Sun, on the Sun's side, and is a New moon; (d) after the Moon makes one complete revolution around Earth (sidereal period); (e) at the next Full moon (synodic period). At (a) and (e) there could be a **lunar eclipse** (Earth's shadow falling on the Moon) but this rarely happens because the plane of Moon's orbit is inclined to the plane of Earth's orbit, so the Moon is usually above or below the Earth's orbital plane. At (c) there could be a **solar eclipse**, also rare.

5–9 Moon Rises an Hour Later Each Day

From the Earth's reference frame, our Moon revolves on average in 24 h, 50 min, which means the Moon rises nearly an hour later each day; and it is at its highest point in the sky about an hour later each day. When the Moon is on the direct opposite side of Earth from the Sun, the Sun's light fully illuminates the Moon and we call it a **Full moon** (Fig. 5–31a). When the Moon is on the same side of the Earth as the Sun, and nearly aligned with both, we see the Moon as a thin sliver—most or all of it is in shadow (= a **New moon**). The phases of the Moon (new, first quarter, full, third quarter) take it from one Full moon to the next Full moon in 29.53 days (= **synodic period**) on average, as seen from the Earth as reference frame (Fig. 5–31e). In the Sun's frame of reference, the Moon revolves around the Earth in 27.32 days (**sidereal period**, Fig. 5–31d). This small difference arises because, when the Moon has made one complete revolution around the Earth, the Earth itself has moved in its orbit relative to the Sun. So the Moon needs more time (≈ 2 days) to be fully aligned with the Sun and Earth and be a Full moon, Fig. 5–31e. The red dot in Figs. 5–31a, b, and e represents an observer at the same location on Earth, which in (a) is when the Full moon is rising and the Sun is just setting.

5–10 Types of Forces in Nature

We have already discussed that Newton's law of universal gravitation, Eq. 5–4, describes how a particular type of force—gravity—depends on the masses of the objects involved and the distance between them. Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, on the other hand, tells how an object will accelerate due to *any* type of force. But what are the types of forces that occur in nature besides gravity?

In the twentieth century, physicists came to recognize four fundamental forces in nature: (1) the gravitational force; (2) the electromagnetic force (we shall see later that electric and magnetic forces are intimately related); (3) the strong nuclear force (which holds protons and neutrons together to form atomic nuclei); and (4) the weak nuclear force (involved in radioactivity). In this Chapter, we discussed the gravitational force in detail. The nature of the electromagnetic force will be discussed in Chapters 16 to 22. The strong and weak nuclear forces, which are discussed in Chapters 30 to 32, operate at the level of the atomic nucleus and are much less obvious in our daily lives.

Physicists have been working on theories that would unify these four forces—that is, to consider some or all of these forces as different manifestations of the same basic force. So far, the electromagnetic and weak nuclear forces have been theoretically united to form *electroweak* theory, in which the electromagnetic and weak forces are seen as two aspects of a single *electroweak force*. Attempts to further unify the forces, such as in *grand unified theories* (GUT), are hot research topics today.

But where do everyday forces fit? Ordinary forces, other than gravity, such as pushes, pulls, and other contact forces like the normal force and friction, are today considered to be due to the electromagnetic force acting at the atomic level. For example, the force your fingers exert on a pencil is the result of electrical repulsion between the outer electrons of the atoms of your finger and those of the pencil.

Summary

An object moving in a circle of radius r with constant speed v is said to be in **uniform circular motion**. It has a **radial acceleration** a_R that is directed radially toward the center of the circle (also called **centripetal acceleration**), and has magnitude

$$a_R = \frac{v^2}{r}. \qquad (5\text{–}1)$$

The velocity vector and the acceleration vector $\vec{\mathbf{a}}_R$ are continually changing in direction, but are perpendicular to each other at each moment.

A force is needed to keep an object revolving in a circle, and the direction of this force is toward the center of the circle. This force could be due to gravity (as for the Moon), to tension in a cord, to a component of the normal force, or to another type of force or combination of forces.

[*When the speed of circular motion is not constant, the acceleration has two components, tangential as well as centripetal.]

Newton's **law of universal gravitation** states that every particle in the universe attracts every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$F_G = G\frac{m_1 m_2}{r^2}. \qquad (5\text{–}4)$$

The direction of this force is along the line joining the two particles, and the force is always attractive. It is this gravitational force that keeps the Moon revolving around the Earth, and the planets revolving around the Sun.

Satellites revolving around the Earth are acted on by gravity, but "stay up" because of their high tangential speed.

Newton's three laws of motion, plus his law of universal gravitation, constituted a wide-ranging theory of the universe. With them, motion of objects on Earth and in space could be accurately described. And they provided a theoretical base for **Kepler's laws** of planetary motion.

The four fundamental forces in nature are (1) the gravitational force, (2) the electromagnetic force, (3) the strong nuclear force, and (4) the weak nuclear force. The first two fundamental forces are responsible for nearly all "everyday" forces.

Questions

1. How many "accelerators" do you have in your car? There are at least three controls in the car which can be used to cause the car to accelerate. What are they? What accelerations do they produce?

2. A car rounds a curve at a steady 50 km/h. If it rounds the same curve at a steady 70 km/h, will its acceleration be any different? Explain.

3. Will the acceleration of a car be the same when a car travels around a sharp curve at a constant 60 km/h as when it travels around a gentle curve at the same speed? Explain.

4. Describe all the forces acting on a child riding a horse on a merry-go-round. Which of these forces provides the centripetal acceleration of the child?

5. A child on a sled comes flying over the crest of a small hill, as shown in Fig. 5–32. His sled does not leave the ground, but he feels the normal force between his chest and the sled decrease as he goes over the hill. Explain this decrease using Newton's second law.

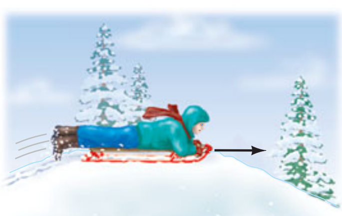


FIGURE 5–32 Question 5.

6. Sometimes it is said that water is removed from clothes in the spin dryer by centrifugal force throwing the water outward. Is this correct? Discuss.

7. A girl is whirling a ball on a string around her head in a horizontal plane. She wants to let go at precisely the right time so that the ball will hit a target on the other side of the yard. When should she let go of the string?

8. A bucket of water can be whirled in a vertical circle without the water spilling out, even at the top of the circle when the bucket is upside down. Explain.

9. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface (Fig. 5–33). Explain how this simulates gravity. Consider (*a*) how objects fall, (*b*) the force we feel on our feet, and (*c*) any other aspects of gravity you can think of.

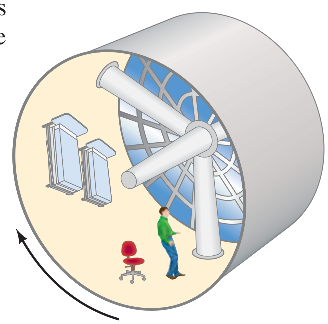


FIGURE 5–33 Question 9.

10. A car maintains a constant speed v as it traverses the hill and valley shown in Fig. 5–34. Both the hill and valley have a radius of curvature R. At which point, A, B, or C, is the normal force acting on the car (*a*) the largest, (*b*) the smallest? Explain. (*c*) Where would the driver feel heaviest and (*d*) lightest? Explain. (*e*) How fast can the car go without losing contact with the road at A?

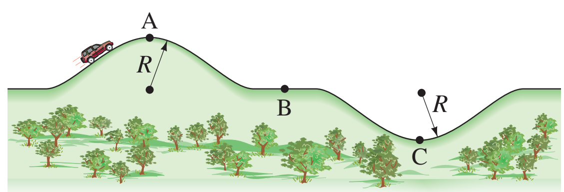


FIGURE 5–34 Question 10.

11. Can a particle with constant speed be accelerating? What if it has constant velocity? Explain.

12. Why do airplanes bank when they turn? How would you compute the banking angle given the airspeed and radius of the turn? [*Hint*: Assume an aerodynamic "lift" force acts perpendicular to the wings. See also Example 5–7.]

13. Does an apple exert a gravitational force on the Earth? If so, how large a force? Consider an apple (*a*) attached to a tree and (*b*) falling.

14. Why is more fuel required for a spacecraft to travel from the Earth to the Moon than to return from the Moon to the Earth?

15. Would it require less speed to launch a satellite (*a*) toward the east or (*b*) toward the west? Consider the Earth's rotation direction and explain your choice.

16. An antenna loosens and becomes detached from a satellite in a circular orbit around the Earth. Describe the antenna's subsequent motion. If it will land on the Earth, describe where; if not, describe how it could be made to land on the Earth.

17. The Sun is below us at midnight, nearly in line with the Earth's center. Are we then heavier at midnight, due to the Sun's gravitational force on us, than we are at noon? Explain.

18. When will your apparent weight be the greatest, as measured by a scale in a moving elevator: when the elevator (*a*) accelerates downward, (*b*) accelerates upward, (*c*) is in free fall, or (*d*) moves upward at constant speed? (*e*) In which case would your apparent weight be the least? (*f*) When would it be the same as when you are on the ground? Explain.

19. The source of the Mississippi River is closer to the center of the Earth than is its outlet in Louisiana (because the Earth is fatter at the equator than at the poles). Explain how the Mississippi can flow "uphill."

20. People sometimes ask, "What keeps a satellite up in its orbit around the Earth?" How would you respond?

21. Is the centripetal acceleration of Mars in its orbit around the Sun larger or smaller than the centripetal acceleration of the Earth? Explain.

22. The mass of the "planet" Pluto was not known until it was discovered to have a moon. Explain how this enabled an estimate of Pluto's mass.

23. The Earth moves faster in its orbit around the Sun in January than in July. Is the Earth closer to the Sun in January, or in July? Explain. [*Note*: This is not much of a factor in producing the seasons—the main factor is the tilt of the Earth's axis relative to the plane of its orbit.]

MisConceptual Questions

1. While driving fast around a sharp right turn, you find yourself pressing against the car door. What is happening?
(*a*) Centrifugal force is pushing you into the door.
(*b*) The door is exerting a rightward force on you.
(*c*) Both of the above.
(*d*) Neither of the above.

2. Which of the following point towards the center of the circle in uniform circular motion?
(*a*) Acceleration.
(*b*) Velocity, acceleration, net force.
(*c*) Velocity, acceleration.
(*d*) Velocity, net force.
(*e*) Acceleration, net force.

3. A Ping-Pong ball is shot into a circular tube that is lying flat (horizontal) on a tabletop. When the Ping-Pong ball exits the tube, which path will it follow in Fig. 5–35?

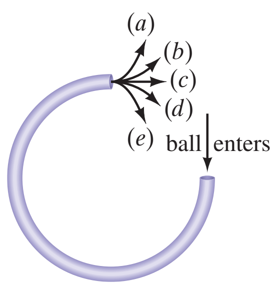


FIGURE 5–35 MisConceptual Question 3.

4. A car drives at steady speed around a perfectly circular track.
(*a*) The car's acceleration is zero.
(*b*) The net force on the car is zero.
(*c*) Both the acceleration and net force on the car point outward.
(*d*) Both the acceleration and net force on the car point inward.
(*e*) If there is no friction, the acceleration is outward.

5. A child whirls a ball in a vertical circle. Assuming the speed of the ball is constant (an approximation), when would the tension in the cord connected to the ball be greatest?
(*a*) At the top of the circle.
(*b*) At the bottom of the circle.
(*c*) A little after the bottom of the circle when the ball is climbing.
(*d*) A little before the bottom of the circle when the ball is descending quickly.
(*e*) Nowhere; the cord is stretched the same amount at all points.

6. In a rotating vertical cylinder (Rotor-ride) a rider finds herself pressed with her back to the rotating wall. Which is the correct free-body diagram for her (Fig. 5–36)?

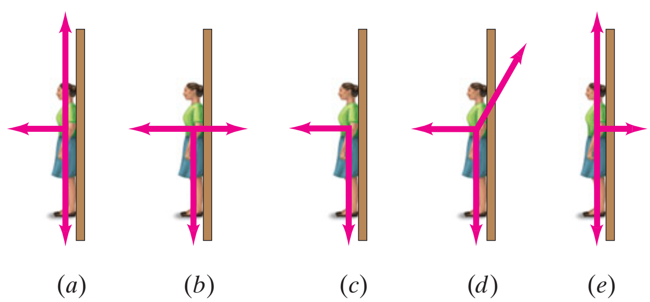


FIGURE 5–36 MisConceptual Question 6.

7. The Moon does not crash into the Earth because:
(*a*) the net force on it is zero.
(*b*) it is beyond the main pull of the Earth's gravity.
(*c*) it is being pulled by the Sun as well as by the Earth.
(*d*) it is freely falling but it has a high tangential velocity.

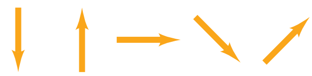
8. Which pulls harder gravitationally, the Earth on the Moon, or the Moon on the Earth? Which accelerates more?
(*a*) The Earth on the Moon; the Earth.
(*b*) The Earth on the Moon; the Moon.
(*c*) The Moon on the Earth; the Earth.
(*d*) The Moon on the Earth; the Moon.
(*e*) Both the same; the Earth.
(*f*) Both the same; the Moon.

9. In the International Space Station which orbits Earth, astronauts experience apparent weightlessness because
(*a*) the station is so far away from the center of the Earth.
(*b*) the station is kept in orbit by a centrifugal force that counteracts the Earth's gravity.
(*c*) the astronauts and the station are in free fall towards the center of the Earth.
(*d*) there is no gravity in space.
(*e*) the station's high speed nullifies the effects of gravity.

10. Two satellites orbit the Earth in circular orbits of the same radius. One satellite is twice as massive as the other. Which statement is true about the speeds of these satellites?
(*a*) The heavier satellite moves twice as fast as the lighter one.
(*b*) The two satellites have the same speed.
(*c*) The lighter satellite moves twice as fast as the heavier one.
(*d*) The ratio of their speeds depends on the orbital radius.

11. A space shuttle in orbit around the Earth carries its payload with its mechanical arm. Suddenly, the arm malfunctions and releases the payload. What will happen to the payload?
(*a*) It will fall straight down and hit the Earth.
(*b*) It will follow a curved path and eventually hit the Earth.
(*c*) It will remain in the same orbit with the shuttle.
(*d*) It will drift out into deep space.

*12. A penny is placed on a turntable which is spinning clockwise as shown in Fig. 5–37. If the power to the turntable is turned off, which arrow best represents the direction of the acceleration of the penny at point P while the turntable is still spinning but slowing down?



(*a*) (*b*) (*c*) (*d*) (*e*)

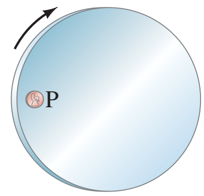


FIGURE 5–37 MisConceptual Question 12.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

5–1 to 5–3 Uniform Circular Motion

1. (I) A child sitting 1.20 m from the center of a merry-go-round moves with a speed of 1.10 m/s. Calculate (*a*) the centripetal acceleration of the child and (*b*) the net horizontal force exerted on the child (mass = 22.5 kg).

2. (I) A jet plane traveling 1890 km/h (525 m/s) pulls out of a dive by moving in an arc of radius 5.20 km. What is the plane's acceleration in *g*'s?

3. (I) A horizontal force of 310 N is exerted on a 2.0-kg ball as it rotates (at arm's length) uniformly in a horizontal circle of radius 0.90 m. Calculate the speed of the ball.

4. (II) What is the magnitude of the acceleration of a speck of clay on the edge of a potter's wheel turning at 45 rpm (revolutions per minute) if the wheel's diameter is 35 cm?

5. (II) A 0.55-kg ball, attached to the end of a horizontal cord, is revolved in a circle of radius 1.3 m on a frictionless horizontal surface. If the cord will break when the tension in it exceeds 75 N, what is the maximum speed the ball can have?

6. (II) How fast (in rpm) must a centrifuge rotate if a particle 7.00 cm from the axis of rotation is to experience an acceleration of 125,000 *g*'s?

7. (II) A car drives straight down toward the bottom of a valley and up the other side on a road whose bottom has a radius of curvature of 115 m. At the very bottom, the normal force on the driver is twice his weight. At what speed was the car traveling?

8. (II) How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve of radius 125 m at a speed of 95 km/h?

9. (II) What is the maximum speed with which a 1200-kg car can round a turn of radius 90.0 m on a flat road if the coefficient of friction between tires and road is 0.65? Is this result independent of the mass of the car?

10. (II) A bucket of mass 2.00 kg is whirled in a vertical circle of radius 1.20 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N. (*a*) Find the speed of the bucket. (*b*) How fast must the bucket move at the top of the circle so that the rope does not go slack?

11. (II) How many revolutions per minute would a 25-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

12. (II) A jet pilot takes his aircraft in a vertical loop (Fig. 5–38). (*a*) If the jet is moving at a speed of 840 km/h at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at the lowest point does not exceed 6.0 *g*'s. (*b*) Calculate the 78-kg pilot's effective weight (the force with which the seat pushes up on him) at the bottom of the circle, and (*c*) at the top of the circle (assume the same speed).

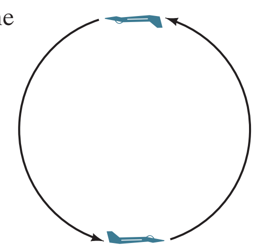


FIGURE 5–38 Problem 12.

13. (II) A proposed space station consists of a circular tube that will rotate about its center (like a tubular bicycle tire), Fig. 5–39. The circle formed by the tube has a diameter of 1.1 km. What must be the rotation speed (revolutions per day) if an effect nearly equal to gravity at the surface of the Earth (say, 0.90 *g*) is to be felt?

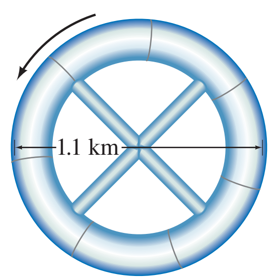


FIGURE 5–39 Problem 13.

14. (II) On an ice rink two skaters of equal mass grab hands and spin in a mutual circle once every 2.5 s. If we assume their arms are each 0.80 m long and their individual masses are 55.0 kg, how hard are they pulling on one another?

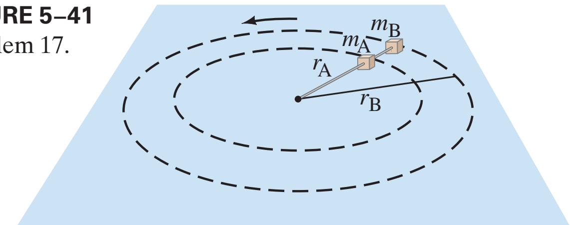
15. (II) A coin is placed 13.0 cm from the axis of a rotating turntable of variable speed. When the speed of the turntable is slowly increased, the coin remains fixed on the turntable until a rate of 38.0 rpm (revolutions per minute) is reached, at which point the coin slides off. What is the coefficient of static friction between the coin and the turntable?

16. (II) The design of a new road includes a straight stretch that is horizontal and flat but that suddenly dips down a steep hill at 18°. The transition should be rounded with what minimum radius so that cars traveling 95 km/h will not leave the road (Fig. 5–40)?

FIGURE 5–40 Problem 16.

17. (II) Two blocks, with masses m_A and m_B, are connected to each other and to a central post by thin rods as shown in Fig. 5–41. The blocks revolve about the post at the same frequency f (revolutions per second) on a frictionless horizontal surface at distances r_A and r_B from the post. Derive an algebraic expression for the tension in each rod.

FIGURE 5–41 Problem 17.



18. (II) Tarzan plans to cross a gorge by swinging in an arc from a hanging vine (Fig. 5–42). If his arms are capable of exerting a force of 1150 N on the vine, what is the maximum speed he can tolerate at the lowest point of his swing? His mass is 78 kg and the vine is 4.7 m long.

FIGURE 5–42 Problem 18.

19. (II) A 975-kg sports car (including driver) crosses the rounded top of a hill (radius = 88.0 m) at 18.0 m/s. Determine (*a*) the normal force exerted by the road on the car, (*b*) the normal force exerted by the car on the 62.0-kg driver, and (*c*) the car speed at which the normal force on the driver equals zero.

20. (II) Highway curves are marked with a suggested speed. If this speed is based on what would be safe in wet weather, estimate the radius of curvature for an unbanked curve marked 50 km/h. Use Table 4–2 (coefficients of friction).

21. (III) A pilot performs an evasive maneuver by diving vertically at 270 m/s. If he can withstand an acceleration of 8.0 *g*'s without blacking out, at what altitude must he begin to pull his plane out of the dive to avoid crashing into the sea?

22. (III) If a curve with a radius of 95 m is properly banked for a car traveling 65 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 95 km/h?

23. (III) A curve of radius 78 m is banked for a design speed of 85 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds can a car safely make the curve? [*Hint*: Consider the direction of the friction force when the car goes too slow or too fast.]

*5–4 Nonuniform Circular Motion

*24. (I) Determine the tangential and centripetal components of the net force exerted on the car (by the ground) in Example 5–8 when its speed is 15 m/s. The car's mass is 950 kg.

*25. (II) A car at the Indianapolis 500 accelerates uniformly from the pit area, going from rest to 270 km/h in a semicircular arc with a radius of 220 m. Determine the tangential and radial acceleration of the car when it is halfway through the arc, assuming constant tangential acceleration. If the curve were flat, what coefficient of static friction would be necessary between the tires and the road to provide this acceleration with no slipping or skidding?

*26. (II) For each of the cases described below, sketch and label the total acceleration vector, the radial acceleration vector, and the tangential acceleration vector. (*a*) A car is accelerating from 55 km/h to 70 km/h as it rounds a curve of constant radius. (*b*) A car is going a constant 65 km/h as it rounds a curve of constant radius. (*c*) A car slows down while rounding a curve of constant radius.

*27. (III) A particle revolves in a horizontal circle of radius 1.95 m. At a particular instant, its acceleration is 1.05 m/s^2, in a direction that makes an angle of 25.0° to its direction of motion. Determine its speed (*a*) at this moment, and (*b*) 2.00 s later, assuming constant tangential acceleration.

5–5 and 5–6 Law of Universal Gravitation

28. (I) Calculate the force of Earth's gravity on a spacecraft 2.00 Earth radii above the Earth's surface if its mass is 1850 kg.

29. (I) At the surface of a certain planet, the gravitational acceleration g has a magnitude of 12.0 m/s^2. A 24.0-kg brass ball is transported to this planet. What is (*a*) the mass of the brass ball on the Earth and on the planet, and (*b*) the weight of the brass ball on the Earth and on the planet?

30. (II) At what distance from the Earth will a spacecraft traveling directly from the Earth to the Moon experience zero net force because the Earth and Moon pull in opposite directions with equal force?

31. (II) Two objects attract each other gravitationally with a force of 2.5×10^{-10} N when they are 0.25 m apart. Their total mass is 4.00 kg. Find their individual masses.

32. (II) A hypothetical planet has a radius 2.0 times that of Earth, but has the same mass. What is the acceleration due to gravity near its surface?

33. (II) Calculate the acceleration due to gravity on the Moon, which has radius 1.74×10^6 m and mass 7.35×10^{22} kg.

34. (II) Estimate the acceleration due to gravity at the surface of Europa (one of the moons of Jupiter) given that its mass is 4.9×10^{22} kg and making the assumption that its mass per unit volume is the same as Earth's.

35. (II) Given that the acceleration of gravity at the surface of Mars is 0.38 of what it is on Earth, and that Mars' radius is 3400 km, determine the mass of Mars.

36. (II) Find the net force on the Moon $(m_M = 7.35 \times 10^{22}\,\text{kg})$ due to the gravitational attraction of both the Earth $(m_E = 5.98 \times 10^{24}\,\text{kg})$ and the Sun $(m_S = 1.99 \times 10^{30}\,\text{kg})$, assuming they are at right angles to each other, Fig. 5–43.

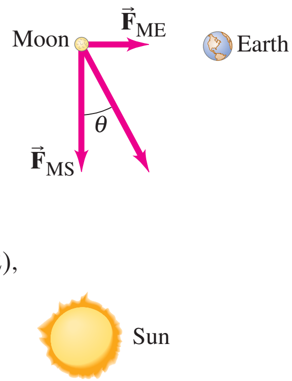


FIGURE 5–43 Problem 36. Orientation of Sun (S), Earth (E), and Moon (M) at right angles to each other (not to scale).

37. (II) A hypothetical planet has a mass 2.80 times that of Earth, but has the same radius. What is g near its surface?

38. (II) If you doubled the mass and tripled the radius of a planet, by what factor would g at its surface change?

39. (II) Calculate the effective value of g, the acceleration of gravity, at (*a*) 6400 m, and (*b*) 6400 km, above the Earth's surface.

40. (II) You are explaining to friends why an astronaut feels weightless orbiting in the space shuttle, and they respond that they thought gravity was just a lot weaker up there. Convince them that it isn't so by calculating how much weaker (in %) gravity is 380 km above the Earth's surface.

41. (II) Every few hundred years most of the planets line up on the same side of the Sun. Calculate the total force on the Earth due to Venus, Jupiter, and Saturn, assuming all four planets are in a line, Fig. 5–44. The masses are $m_V = 0.815\,m_E$, $m_J = 318\,m_E$, $m_{Sat} = 95.1\,m_E$, and the mean distances of the four planets from the Sun are 108, 150, 778, and 1430 million km. What fraction of the Sun's force on the Earth is this?

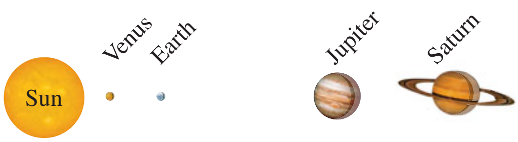


FIGURE 5–44 Problem 41 (not to scale).

42. (II) Four 7.5-kg spheres are located at the corners of a square of side 0.80 m. Calculate the magnitude and direction of the gravitational force exerted on one sphere by the other three.

43. (II) Determine the distance from the Earth's center to a point outside the Earth where the gravitational acceleration due to the Earth is $\frac{1}{10}$ of its value at the Earth's surface.

44. (II) A certain neutron star has five times the mass of our Sun packed into a sphere about 10 km in radius. Estimate the surface gravity on this monster.

5–7 Satellites and Weightlessness

45. (I) A space shuttle releases a satellite into a circular orbit 780 km above the Earth. How fast must the shuttle be moving (relative to Earth's center) when the release occurs?

46. (I) Calculate the speed of a satellite moving in a stable circular orbit about the Earth at a height of 4800 km.

47. (II) You know your mass is 62 kg, but when you stand on a bathroom scale in an elevator, it says your mass is 77 kg. What is the acceleration of the elevator, and in which direction?

48. (II) A 12.0-kg monkey hangs from a cord suspended from the ceiling of an elevator. The cord can withstand a tension of 185 N and breaks as the elevator accelerates. What was the elevator's minimum acceleration (magnitude and direction)?

49. (II) Calculate the period of a satellite orbiting the Moon, 95 km above the Moon's surface. Ignore effects of the Earth. The radius of the Moon is 1740 km.

50. (II) Two satellites orbit Earth at altitudes of 7500 km and 15,000 km above the Earth's surface. Which satellite is faster, and by what factor?

51. (II) What will a spring scale read for the weight of a 58.0-kg woman in an elevator that moves (*a*) upward with constant speed 5.0 m/s, (*b*) downward with constant speed 5.0 m/s, (*c*) with an upward acceleration 0.23 g, (*d*) with a downward acceleration 0.23 g, and (*e*) in free fall?

52. (II) Determine the time it takes for a satellite to orbit the Earth in a circular **near-Earth orbit**. A "near-Earth" orbit is at a height above the surface of the Earth that is very small compared to the radius of the Earth. [*Hint*: You may take the acceleration due to gravity as essentially the same as that on the surface.] Does your result depend on the mass of the satellite?

53. (II) What is the apparent weight of a 75-kg astronaut 2500 km from the center of the Moon in a space vehicle (*a*) moving at constant velocity and (*b*) accelerating toward the Moon at 1.8 m/s^2? State "direction" in each case.

54. (II) A Ferris wheel 22.0 m in diameter rotates once every 12.5 s (see Fig. 5–9). What is the ratio of a person's apparent weight to her real weight at (*a*) the top, and (*b*) the bottom?

55. (II) At what rate must a cylindrical spaceship rotate if occupants are to experience simulated gravity of 0.70 g? Assume the spaceship's diameter is 32 m, and give your answer as the time needed for one revolution. (See Question 9, Fig 5–33.)

56. (III) (*a*) Show that if a satellite orbits very near the surface of a planet with period T, the density (= mass per unit volume) of the planet is $\rho = m/V = 3\pi/GT^2$. (*b*) Estimate the density of the Earth, given that a satellite near the surface orbits with a period of 85 min. Approximate the Earth as a uniform sphere.

5–8 Kepler's Laws

57. (I) Neptune is an average distance of 4.5×10^9 km from the Sun. Estimate the length of the Neptunian year using the fact that the Earth is 1.50×10^8 km from the Sun on average.

58. (I) The *asteroid* Icarus, though only a few hundred meters across, orbits the Sun like the planets. Its period is 410 d. What is its mean distance from the Sun?

59. (I) Use Kepler's laws and the period of the Moon (27.4 d) to determine the period of an artificial satellite orbiting very near the Earth's surface.

60. (II) Determine the mass of the Earth from the known period and distance of the Moon.

61. (II) Our Sun revolves about the center of our Galaxy $(m_G \approx 4 \times 10^{41}\,\text{kg})$ at a distance of about 3×10^4 light-years $[1\,\text{ly} = (3.00 \times 10^8\,\text{m/s}) \cdot (3.16 \times 10^7\,\text{s/yr}) \cdot (1.00\,\text{yr})]$. What is the period of the Sun's orbital motion about the center of the Galaxy?

62. (II) Table 5–3 gives the mean distance, period, and mass for the four largest moons of Jupiter (those discovered by Galileo in 1609). Determine the mass of Jupiter: (*a*) using the data for Io; (*b*) using data for each of the other three moons. Are the results consistent?

TABLE 5–3 Principal Moons of Jupiter (Problems 62 and 63)

Moon	Mass (kg)	Period (Earth days)	Mean distance from Jupiter (km)
Io	8.9×10^{22}	1.77	422×10^3
Europa	4.9×10^{22}	3.55	671×10^3
Ganymede	15×10^{22}	7.16	1070×10^3
Callisto	11×10^{22}	16.7	1883×10^3

63. (II) Determine the mean distance from Jupiter for each of Jupiter's principal moons, using Kepler's third law. Use the distance of Io and the periods given in Table 5–3. Compare your results to the values in Table 5–3.

64. (II) Planet A and planet B are in circular orbits around a distant star. Planet A is 7.0 times farther from the star than is planet B. What is the ratio of their speeds v_A/v_B?

65. (II) *Halley's comet* orbits the Sun roughly once every 76 years. It comes very close to the surface of the Sun on its closest approach (Fig. 5–45). Estimate the greatest distance of the comet from the Sun. Is it still "in" the solar system? What planet's orbit is nearest when it is out there?

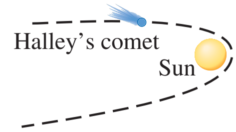


FIGURE 5–45 Problem 65.

66. (III) The *comet Hale–Bopp* has an orbital period of 2400 years. (*a*) What is its mean distance from the Sun? (*b*) At its closest approach, the comet is about 1.0 AU from the Sun (1 AU = distance from Earth to the Sun). What is the farthest distance? (*c*) What is the ratio of the speed at the closest point to the speed at the farthest point?

General Problems

67. Calculate the centripetal acceleration of the Earth in its orbit around the Sun, and the net force exerted on the Earth. What exerts this force on the Earth? Assume that the Earth's orbit is a circle of radius 1.50×10^{11} m.

68. A flat puck (mass M) is revolved in a circle on a frictionless air hockey table top, and is held in this orbit by a massless cord which is connected to a dangling mass (mass m) through a central hole as shown in Fig. 5–46. Show that the speed of the puck is given by $v = \sqrt{mgR/M}$.

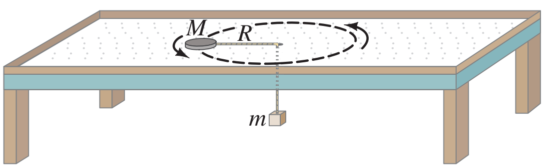


FIGURE 5–46 Problem 68.

69. A device for training astronauts and jet fighter pilots is designed to move the trainee in a horizontal circle of radius 11.0 m. If the force felt by the trainee is 7.45 times her own weight, how fast is she revolving? Express your answer in both m/s and rev/s.

70. A 1050-kg car rounds a curve of radius 72 m banked at an angle of 14°. If the car is traveling at 85 km/h, will a friction force be required? If so, how much and in what direction?

71. In a "Rotor-ride" at a carnival, people rotate in a vertical cylindrically walled "room." (See Fig. 5–47.) If the room radius is 5.5 m, and the rotation frequency 0.50 revolutions per second when the floor drops out, what minimum coefficient of static friction keeps the people from slipping down? People on this ride said they were "pressed against the wall." Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides nausea)? [*Hint:* Draw a free-body diagram for a person.]

FIGURE 5–47 Problem 71.

72. While fishing, you get bored and start to swing a sinker weight around in a circle below you on a 0.25-m piece of fishing line. The weight makes a complete circle every 0.75 s. What is the angle that the fishing line makes with the vertical? [*Hint*: See Fig. 5–10.]

73. At what minimum speed must a roller coaster be traveling so that passengers upside down at the top of the circle (Fig. 5–48) do not fall out? Assume a radius of curvature of 8.6 m.

FIGURE 5–48 Problem 73.

74. Consider a train that rounds a curve with a radius of 570 m at a speed of 160 km/h (approximately 100 mi/h). (*a*) Calculate the friction force needed on a train passenger of mass 55 kg if the track is not banked and the train does not tilt. (*b*) Calculate the friction force on the passenger if the train tilts at an angle of 8.0° toward the center of the curve.

75. Two equal-mass stars maintain a constant distance apart of 8.0×10^{11} m and revolve about a point midway between them at a rate of one revolution every 12.6 yr. (*a*) Why don't the two stars crash into one another due to the gravitational force between them? (*b*) What must be the mass of each star?

76. How far above the Earth's surface will the acceleration of gravity be half what it is at the surface?

77. Is it possible to whirl a bucket of water fast enough in a vertical circle so that the water won't fall out? If so, what is the minimum speed? Define all quantities needed.

78. How long would a day be if the Earth were rotating so fast that objects at the equator were apparently weightless?

79. The *rings of Saturn* are composed of chunks of ice that orbit the planet. The inner radius of the rings is 73,000 km, and the outer radius is 170,000 km. Find the period of an orbiting chunk of ice at the inner radius and the period of a chunk at the outer radius. Compare your numbers with Saturn's own rotation period of 10 hours and 39 minutes. The mass of Saturn is 5.7×10^{26} kg.

80. During an *Apollo* lunar landing mission, the command module continued to orbit the Moon at an altitude of about 100 km. How long did it take to go around the Moon once?

81. The **Navstar Global Positioning System** (GPS) utilizes a group of 24 satellites orbiting the Earth. Using "triangulation" and signals transmitted by these satellites, the position of a receiver on the Earth can be determined to within an accuracy of a few centimeters. The satellite orbits are distributed around the Earth, allowing continuous navigational "fixes." The satellites orbit at an altitude of approximately 11,000 nautical miles [1 nautical mile = 1.852 km = 6076 ft]. (*a*) Determine the speed of each satellite. (*b*) Determine the period of each satellite.

82. The *Near Earth Asteroid Rendezvous* (*NEAR*) spacecraft, after traveling 2.1 billion km, is meant to orbit the asteroid Eros with an orbital radius of about 20 km. Eros is roughly 40 km × 6 km × 6 km. Assume Eros has a density (mass/volume) of about $2.3 \times 10^3\ \mathrm{kg/m^3}$. (*a*) If Eros were a sphere with the same mass and density, what would its radius be? (*b*) What would g be at the surface of a spherical Eros? (*c*) Estimate the orbital period of *NEAR* as it orbits Eros, as if Eros were a sphere.

83. A train traveling at a constant speed rounds a curve of radius 215 m. A lamp suspended from the ceiling swings out to an angle of 16.5° throughout the curve. What is the speed of the train?

84. The Sun revolves around the center of the *Milky Way Galaxy* (Fig. 5–49) at a distance of about 30,000 light-years from the center $\left(1\ \text{ly} = 9.5 \times 10^{15}\ \text{m}\right)$. If it takes about 200 million years to make one revolution, estimate the mass of our Galaxy. Assume that the mass distribution of our Galaxy is concentrated mostly in a central uniform sphere. If all the stars had about the mass of our Sun $\left(2 \times 10^{30}\ \text{kg}\right)$, how many stars would there be in our Galaxy?

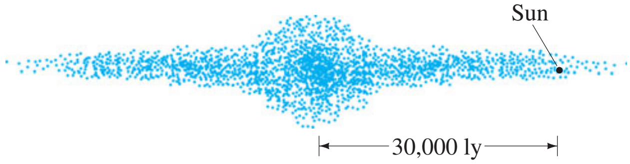


FIGURE 5–49 Edge-on view of our galaxy. Problem 84.

85. A satellite of mass 5500 kg orbits the Earth and has a period of 6600 s. Determine (*a*) the radius of its circular orbit, (*b*) the magnitude of the Earth's gravitational force on the satellite, and (*c*) the altitude of the satellite.

86. Astronomers using the Hubble Space Telescope deduced the presence of an extremely massive core in the distant *galaxy* M87, so dense that it could be a black hole (from which no light escapes). They did this by measuring the speed of gas clouds orbiting the core to be 780 km/s at a distance of 60 light-years $\left(= 5.7 \times 10^{17}\ \text{m}\right)$ from the core. Deduce the mass of the core, and compare it to the mass of our Sun.

87. Suppose all the mass of the Earth were compacted into a small spherical ball. What radius must the sphere have so that the acceleration due to gravity at the Earth's new surface would equal the acceleration due to gravity at the surface of the Sun?

88. A science-fiction tale describes an artificial "planet" in the form of a band completely encircling a sun (Fig. 5–50). The inhabitants live on the inside surface (where it is always noon). Imagine that this sun is exactly like our own, that the distance to the band is the same as the Earth–Sun distance (to make the climate livable), and that the ring rotates quickly enough to produce an apparent gravity of g as on Earth. What will be the period of revolution, this planet's year, in Earth days?

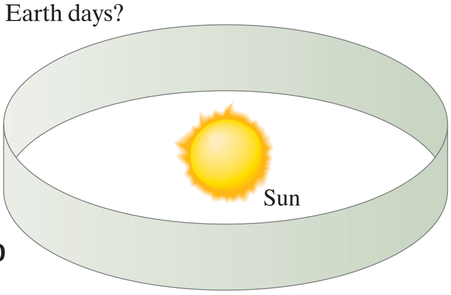


FIGURE 5–50 Problem 88.

89. An asteroid of mass m is in a circular orbit of radius r around the Sun with a speed v. It has an impact with another asteroid of mass M and is kicked into a new circular orbit with a speed of 1.5 v. What is the radius of the new orbit in terms of r?

*90. Use **dimensional analysis** (Section 1–8) to obtain the form for the centripetal acceleration, $a_R = v^2/r$.

Search and Learn

1. Reread each Example in this Chapter and identify (i) the object undergoing centripetal acceleration (if any), and (ii) the force, or force component, that causes the circular motion.
2. Redo Example 5–3, precisely this time, by not ignoring the weight of the ball which revolves on a string 0.600 m long. In particular, find the magnitude of $\vec{\mathbf{F}}_T$, and the angle it makes with the horizontal. [*Hint*: Set the horizontal component of $\vec{\mathbf{F}}_T$ equal to ma_R; also, since there is no vertical motion, what can you say about the vertical component of $\vec{\mathbf{F}}_T$?]
3. A banked curve of radius R in a new highway is designed so that a car traveling at speed v_0 can negotiate the turn safely on glare ice (zero friction). If a car travels too slowly, then it will slip toward the center of the circle. If it travels too fast, it will slip away from the center of the circle. If the coefficient of static friction increases, it becomes possible for a car to stay on the road while traveling at a speed within a range from v_{min} to v_{max}. Derive formulas for v_{min} and v_{max} as functions of μ_s, v_0, and R.
4. *Earth is not quite an inertial frame*. We often make measurements in a reference frame fixed on the Earth, assuming Earth is an inertial reference frame [Section 4–2]. But the Earth rotates, so this assumption is not quite valid. Show that this assumption is off by 3 parts in 1000 by calculating the acceleration of an object at Earth's equator due to Earth's daily rotation, and compare to $g = 9.80\ \text{m/s}^2$, the acceleration due to gravity.
5. A certain white dwarf star was once an average star like our Sun. But now it is in the last stage of its evolution and is the size of our Moon but has the mass of our Sun. (*a*) Estimate the acceleration due to gravity on the surface of this star. (*b*) How much would a 65-kg person weigh on this star? Give as a percentage of the person's weight on Earth. (*c*) What would be the speed of a baseball dropped from a height of 1.0 m when it hit the surface?
6. Jupiter is about 320 times as massive as the Earth. Thus, it has been claimed that a person would be crushed by the force of gravity on a planet the size of Jupiter because people cannot survive more than a few g's. Calculate the number of g's a person would experience at Jupiter's equator, using the following data for Jupiter: mass = 1.9×10^{27} kg, equatorial radius = 7.1×10^4 km, rotation period = 9 hr 55 min. Take the centripetal acceleration into account. [See Sections 5–2, 5–6, and 5–7.]
7. A plumb bob (a mass m hanging on a string) is deflected from the vertical by an angle θ due to a massive mountain nearby (Fig. 5–51). (*a*) Find an approximate formula for θ in terms of the mass of the mountain, m_M, the distance to its center, D_M, and the radius and mass of the Earth. (*b*) Make a rough estimate of the mass of Mt. Everest, assuming it has the shape of a cone 4000 m high and base of diameter 4000 m. Assume its mass per unit volume is 3000 kg per m^3. (*c*) Estimate the angle θ of the plumb bob if it is 5 km from the center of Mt. Everest.

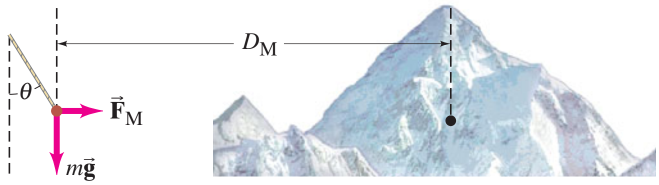


FIGURE 5–51 Search and Learn 7.

8. (*a*) Explain why a Full moon always rises at sunset. (*b*) Explain how the position of the Moon in Fig. 5–31b cannot be seen yet by the person at the red dot (shown at 6 PM). (*c*) Explain why the red dot is where it is in parts (b) and (e), and show where it should be in part (d). (*d*) PRETTY HARD. Determine the average period of the Moon around the Earth (sidereal period) starting with the synodic period of 29.53 days as observed from Earth. [*Hint*: First determine the angle of the Moon in Fig. 5–31e relative to "horizontal," as in part (a).]

ANSWERS TO EXERCISES

A: (*a*).
B: (*d*).
C: (*a*).
D: No.
E: g would double.
F: (*b*).