



This snowboarder flying through the air shows an example of motion in two dimensions. In the absence of air resistance, the path would be a perfect parabola. The gold arrow represents the downward acceleration of gravity,  $\vec{g}$ . Galileo analyzed the motion of objects in 2 dimensions under the action of gravity near the Earth's surface (now called "projectile motion") into its horizontal and vertical components.

We will discuss vectors and how to add them. Besides analyzing projectile motion, we will also see how to work with relative velocity.

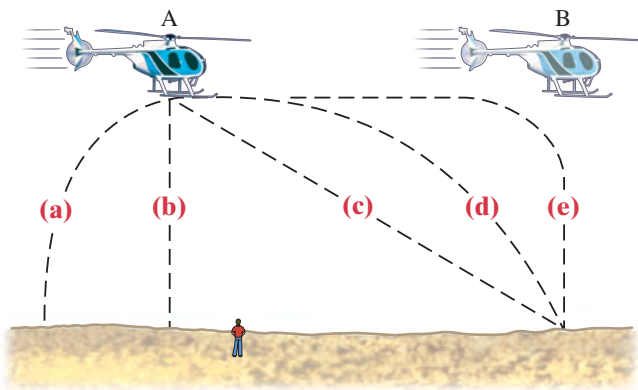
# Kinematics in Two Dimensions; Vectors

## CHAPTER 3

### CHAPTER-OPENING QUESTION—Guess now!

[Don't worry about getting the right answer now—you will get another chance later in the Chapter. See also p. 1 of Chapter 1 for more explanation.]

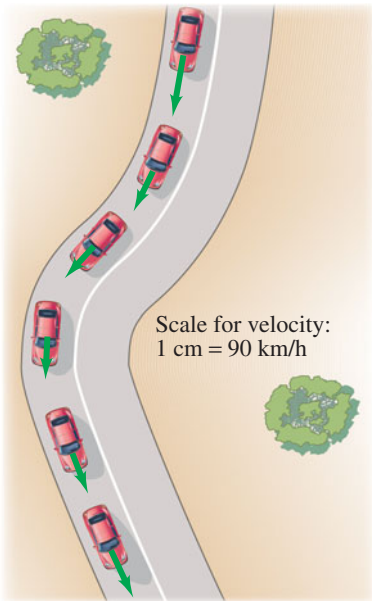
A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



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In Chapter 2 we dealt with motion along a straight line. We now consider the motion of objects that move in paths in two (or three) dimensions. In particular, we discuss an important type of motion known as *projectile motion*: objects projected outward near the Earth's surface, such as struck baseballs and golf balls, kicked footballs, and other projectiles. Before beginning our discussion of motion in two dimensions, we will need a new tool, vectors, and how to add them.



**FIGURE 3-1** Car traveling on a road, slowing down to round the curve. The green arrows represent the velocity vector at each position.

## 3-1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast an object is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities have no direction associated with them, such as mass, time, and temperature. They are specified completely by a number and units. Such quantities are called **scalar** quantities.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector quantity it represents. The length of the arrow is drawn proportional to the magnitude of the vector quantity. For example, in Fig. 3-1, green arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off Fig. 3-1 by measuring the length of the corresponding arrow and using the scale shown ( $1 \text{ cm} = 90 \text{ km/h}$ ).

When we write the symbol for a vector, we will always use boldface type, with a tiny arrow over the symbol. Thus for velocity we write  $\vec{v}$ . If we are concerned only with the magnitude of the vector, we will write simply  $v$ , in italics, as we do for other symbols.

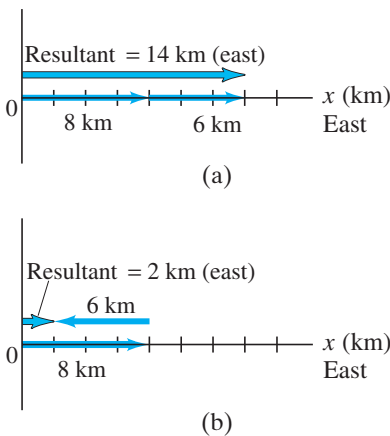
## 3-2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this Chapter, we will deal mainly with displacement vectors, for which we now use the symbol  $\vec{D}$ , and velocity vectors,  $\vec{v}$ . But the results will apply for other vectors we encounter later.

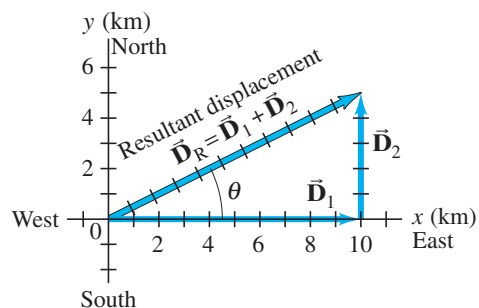
We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be  $8 \text{ km} + 6 \text{ km} = 14 \text{ km}$  east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3-2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3-2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction:  $8 \text{ km} - 6 \text{ km} = 2 \text{ km}$ .

But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive  $y$  axis points north and the positive  $x$  axis points east, Fig. 3-3. On this graph, we draw an arrow, labeled  $\vec{D}_1$ , to represent the 10.0-km displacement to the east. Then we draw a second arrow,  $\vec{D}_2$ , to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3-3.

**FIGURE 3-2** Combining vectors in one dimension.



**FIGURE 3-3** A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors  $\vec{D}_1$  and  $\vec{D}_2$ , which are shown as arrows. Also shown is the resultant displacement vector,  $\vec{D}_R$ , which is the vector sum of  $\vec{D}_1$  and  $\vec{D}_2$ . Measurement on the graph with ruler and protractor shows that  $\vec{D}_R$  has a magnitude of 11.2 km and points at an angle  $\theta = 27^\circ$  north of east.



After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled  $\vec{D}_R$  in Fig. 3–3. (The subscript R stands for resultant.) Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle  $\theta = 27^\circ$  north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle  $\theta = 27^\circ$  with the positive  $x$  axis. The magnitude (length) of  $\vec{D}_R$  can also be obtained using the theorem of Pythagoras in this case, because  $D_1$ ,  $D_2$ , and  $D_R$  form a right triangle with  $D_R$  as the hypotenuse. Thus

$$\begin{aligned} D_R &= \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0 \text{ km})^2 + (5.0 \text{ km})^2} \\ &= \sqrt{125 \text{ km}^2} = 11.2 \text{ km}. \end{aligned}$$

You can use the Pythagorean theorem only when the vectors are *perpendicular* to each other.

The resultant displacement vector,  $\vec{D}_R$ , is the sum of the vectors  $\vec{D}_1$  and  $\vec{D}_2$ . That is,

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum. That is,

$$D_R \leq (D_1 + D_2),$$

where the equals sign applies only if the two vectors point in the same direction. In our example (Fig. 3–3),  $D_R = 11.2$  km, whereas  $D_1 + D_2$  equals 15 km, which is the total distance traveled. Note also that we cannot set  $\vec{D}_R$  equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though:  $\vec{D}_R = \vec{D}_1 + \vec{D}_2 = (11.2 \text{ km}, 27^\circ \text{ N of E})$ .

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

1. On a diagram, draw one of the vectors—call it  $\vec{D}_1$ —to scale.
2. Next draw the second vector,  $\vec{D}_2$ , to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second vector represents the *sum*, or **resultant**, of the two vectors.

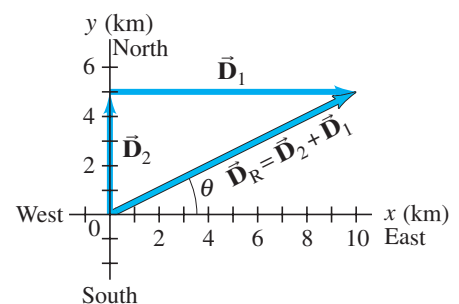
The length of the resultant vector represents its magnitude. Note that vectors can be moved parallel to themselves on paper (maintaining the same length and angle) to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors**.

The resultant is not affected by the order in which the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle  $\theta = 27^\circ$  (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). That is, now using  $\vec{V}$  to represent any type of vector,

$$\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1.$$

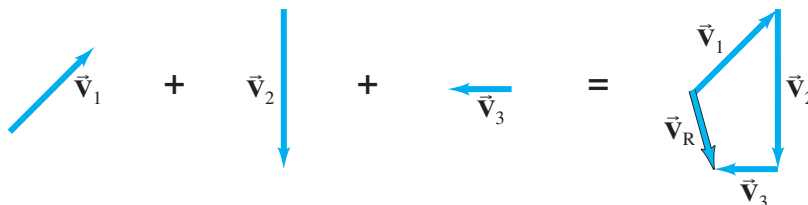
[Mathematicians call this equation the *commutative* property of vector addition.]

**FIGURE 3–4** If the vectors are added in reverse order, the resultant is the same. (Compare to Fig. 3–3.)

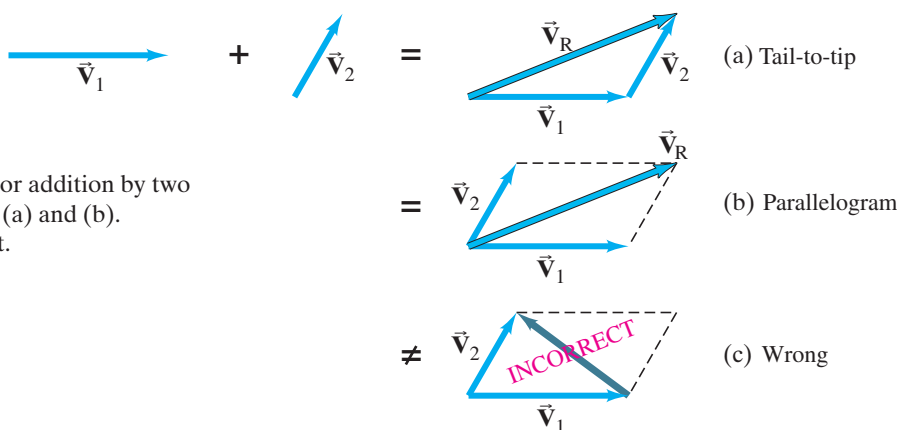


The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 3-5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors.

**FIGURE 3-5** The resultant of three vectors:  $\vec{V}_R = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$ .



A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3-6b. The resultant is the diagonal drawn from the common origin. In Fig. 3-6a, the tail-to-tip method is shown, and we can see that both methods yield the same result.



**FIGURE 3-6** Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

**CAUTION**

Be sure to use the correct diagonal on the parallelogram to get the resultant

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3-6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference,  $\vec{V}_2 - \vec{V}_1$ , as we will see in the next Section.)

**CONCEPTUAL EXAMPLE 3-1 Range of vector lengths.** Suppose two vectors each have length 3.0 units. What is the range of possible lengths for the vector representing the sum of the two?

**RESPONSE** The sum can take on any value from 6.0 ( $= 3.0 + 3.0$ ) where the vectors point in the same direction, to 0 ( $= 3.0 - 3.0$ ) when the vectors are antiparallel. Magnitudes between 0 and 6.0 occur when the two vectors are at an angle other than  $0^\circ$  and  $180^\circ$ .

**EXERCISE A** If the two vectors of Example 3-1 are perpendicular to each other, what is the resultant vector length?

## 3-3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

**FIGURE 3-7** The negative of a vector is a vector having the same length but opposite direction.



Given a vector  $\vec{V}$ , we define the *negative* of this vector ( $-\vec{V}$ ) to be a vector with the same magnitude as  $\vec{V}$  but opposite in direction, Fig. 3-7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

We can now define the subtraction of one vector from another: the difference between two vectors  $\vec{V}_2 - \vec{V}_1$  is defined as

$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

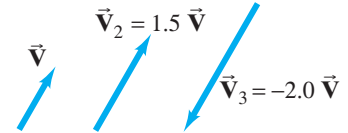


**FIGURE 3–8** Subtracting two vectors:  $\vec{V}_2 - \vec{V}_1$ .

A vector  $\vec{V}$  can be multiplied by a scalar  $c$ . We define their product so that  $c\vec{V}$  has the same direction as  $\vec{V}$  and has magnitude  $cV$ . That is, multiplication of a vector by a positive scalar  $c$  changes the magnitude of the vector by a factor  $c$  but doesn't alter the direction. If  $c$  is a negative scalar (such as  $-2.0$ ), the magnitude of the product  $c\vec{V}$  is changed by the factor  $|c|$  (where  $|c|$  means the magnitude of  $c$ ), but the direction is precisely opposite to that of  $\vec{V}$ . See Fig. 3–9.

**FIGURE 3–9** Multiplying a vector  $\vec{V}$  by a scalar  $c$  gives a vector whose magnitude is  $c$  times greater and in the same direction as  $\vec{V}$  (or opposite direction if  $c$  is negative).

**EXERCISE B** What does the “incorrect” vector in Fig. 3–6c represent? (a)  $\vec{V}_2 - \vec{V}_1$ ; (b)  $\vec{V}_1 - \vec{V}_2$ ; (c) something else (specify).



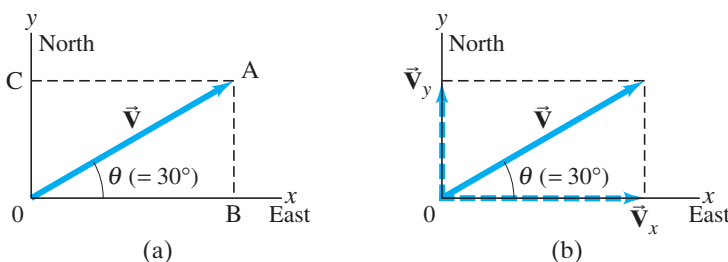
## 3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are useful for visualizing, for checking your math, and thus for getting the correct result.

### Components

Consider first a vector  $\vec{V}$  that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions, such as the  $x$  and  $y$  axes. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector  $\vec{V}$  could be a displacement vector that points at an angle  $\theta = 30^\circ$  north of east, where we have chosen the positive  $x$  axis to be to the east and the positive  $y$  axis north. This vector  $\vec{V}$  is resolved into its  $x$  and  $y$  components by drawing dashed lines (AB and AC) out from the tip (A) of the vector, making them perpendicular to the  $x$  and  $y$  axes. Then the lines 0B and 0C represent the  $x$  and  $y$  components of  $\vec{V}$ , respectively, as shown in Fig. 3–10b. These *vector components* are written  $\vec{V}_x$  and  $\vec{V}_y$ . In this book we usually show vector components as arrows, like vectors, but dashed. The *scalar components*,  $V_x$  and  $V_y$ , are the magnitudes of the vector components, with units, accompanied by a positive or negative sign depending on whether they point along the positive or negative  $x$  or  $y$  axis. As can be seen in Fig. 3–10,  $\vec{V}_x + \vec{V}_y = \vec{V}$  by the parallelogram method of adding vectors.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are  $\vec{V}_x$ ,  $\vec{V}_y$ , and  $\vec{V}_z$ .

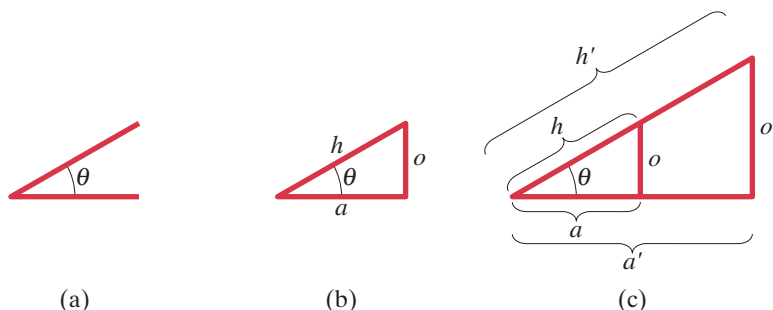


**FIGURE 3–10** Resolving a vector  $\vec{V}$  into its components along a chosen set of  $x$  and  $y$  axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.

To add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.

Given any angle  $\theta$ , as in Fig. 3–11a, a right triangle can be constructed by drawing a line perpendicular to one of its sides, as in Fig. 3–11b. The longest side of a right triangle, opposite the right angle, is called the hypotenuse, which we label  $h$ . The side opposite the angle  $\theta$  is labeled  $o$ , and the side adjacent is labeled  $a$ . We let  $h$ ,  $o$ , and  $a$  represent the lengths of these sides, respectively.

**FIGURE 3–11** Starting with an angle  $\theta$  as in (a), we can construct right triangles of different sizes, (b) and (c), but the ratio of the lengths of the sides does not depend on the size of the triangle.



We now define the three trigonometric functions, sine, cosine, and tangent (abbreviated sin, cos, tan), in terms of the right triangle, as follows:

$$\begin{aligned}\sin \theta &= \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h} \\ \cos \theta &= \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h} \\ \tan \theta &= \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}.\end{aligned}\tag{3-1}$$

If we make the triangle bigger, but keep the same angles, then the ratio of the length of one side to the other, or of one side to the hypotenuse, remains the same. That is, in Fig. 3–11c we have:  $a/h = a'/h'$ ;  $o/h = o'/h'$ ; and  $o/a = o'/a'$ . Thus the values of sine, cosine, and tangent do not depend on how big the triangle is. They depend only on the size of the angle. The values of sine, cosine, and tangent for different angles can be found using a scientific calculator, or from the Table in Appendix A.

A useful trigonometric identity is

$$\sin^2 \theta + \cos^2 \theta = 1\tag{3-2}$$

which follows from the Pythagorean theorem ( $o^2 + a^2 = h^2$  in Fig. 3–11). That is:

$$\sin^2 \theta + \cos^2 \theta = \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2 + a^2}{h^2} = \frac{h^2}{h^2} = 1.$$

(See Appendix A and inside the rear cover for other details on trigonometric functions and identities.)

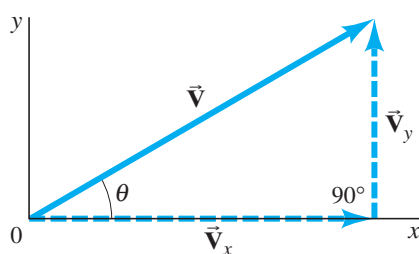
The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–12, where a vector and its two components are thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in Fig. 3–12, where  $\theta$  is the angle  $\vec{V}$  makes with the  $+x$  axis. If we multiply the definition of  $\sin \theta = V_y/V$  by  $V$  on both sides, we get

$$V_y = V \sin \theta.\tag{3-3a}$$

Similarly, from the definition of  $\cos \theta$ , we obtain

$$V_x = V \cos \theta.\tag{3-3b}$$

Note that if  $\theta$  is not the angle the vector makes with the positive  $x$  axis, Eqs. 3–3 are not valid.



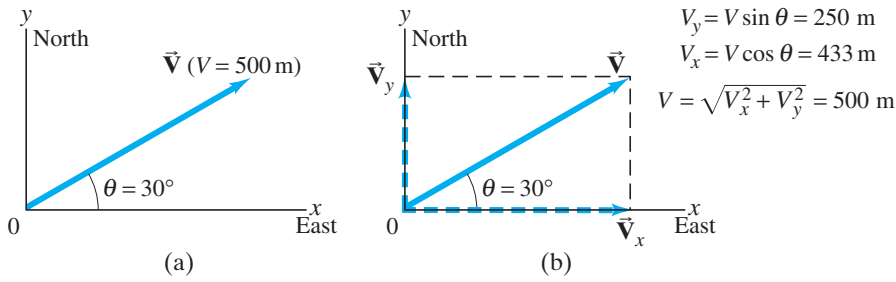
$$\sin \theta = \frac{V_y}{V}$$

$$\cos \theta = \frac{V_x}{V}$$

$$\tan \theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

**FIGURE 3–12** Finding the components of a vector using trigonometric functions. The equations are valid only if  $\theta$  is the angle  $\vec{V}$  makes with the positive  $x$  axis.



**FIGURE 3-13** (a) Vector  $\vec{V}$  represents a displacement of 500 m at a  $30^\circ$  angle north of east. (b) The components of  $\vec{V}$  are  $\vec{V}_x$  and  $\vec{V}_y$ , whose magnitudes are given on the right in the diagram.

Using Eqs. 3-3, we can calculate  $V_x$  and  $V_y$  for any vector, such as that illustrated in Fig. 3-10 or Fig. 3-12. Suppose  $\vec{V}$  represents a displacement of 500 m in a direction  $30^\circ$  north of east, as shown in Fig. 3-13. Then  $V = 500 \text{ m}$ . From a calculator or Tables,  $\sin 30^\circ = 0.500$  and  $\cos 30^\circ = 0.866$ . Then

$$V_x = V \cos \theta = (500 \text{ m})(0.866) = 433 \text{ m (east)},$$

$$V_y = V \sin \theta = (500 \text{ m})(0.500) = 250 \text{ m (north)}.$$

There are two ways to specify a vector in a given coordinate system:

1. We can give its components,  $V_x$  and  $V_y$ .
2. We can give its magnitude  $V$  and the angle  $\theta$  it makes with the positive  $x$  axis.

We can shift from one description to the other using Eqs. 3-3, and, for the reverse, by using the theorem of Pythagoras<sup>†</sup> and the definition of tangent:

$$V = \sqrt{V_x^2 + V_y^2} \quad (3-4a)$$

$$\tan \theta = \frac{V_y}{V_x} \quad (3-4b)$$

as can be seen in Fig. 3-12.

### Adding Vectors

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 3-14, that the addition of any two vectors  $\vec{V}_1$  and  $\vec{V}_2$  to give a resultant,  $\vec{V}_R = \vec{V}_1 + \vec{V}_2$ , implies that

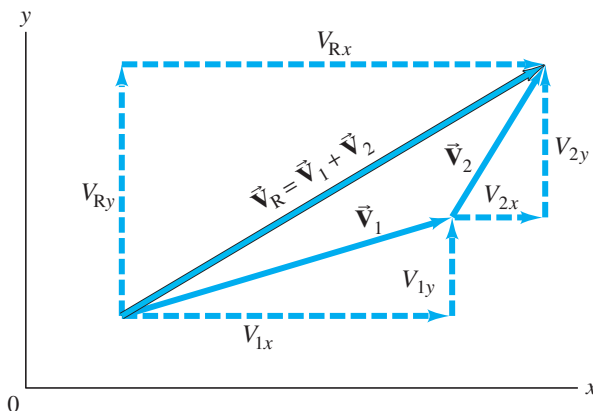
$$V_{Rx} = V_{1x} + V_{2x}$$

$$V_{Ry} = V_{1y} + V_{2y}. \quad (3-5)$$

That is, the sum of the  $x$  components equals the  $x$  component of the resultant vector, and the sum of the  $y$  components equals the  $y$  component of the resultant, as can be verified by a careful examination of Fig. 3-14. Note that we do *not* add  $x$  components to  $y$  components.

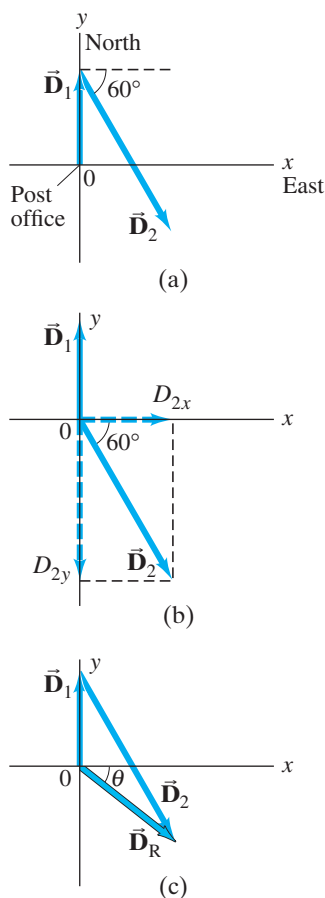
If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3-4.

<sup>†</sup>In three dimensions, the theorem of Pythagoras becomes  $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$ , where  $V_z$  is the component along the third, or  $z$ , axis.



**FIGURE 3-14** The components of  $\vec{V}_R = \vec{V}_1 + \vec{V}_2$  are  $V_{Rx} = V_{1x} + V_{2x}$  and  $V_{Ry} = V_{1y} + V_{2y}$ .

The components of a given vector depend on the choice of coordinate axes. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.



**FIGURE 3-15** Example 3-2. (a) The two displacement vectors,  $\vec{D}_1$  and  $\vec{D}_2$ . (b)  $\vec{D}_2$  is resolved into its components. (c)  $\vec{D}_1$  and  $\vec{D}_2$  are added to obtain the resultant  $\vec{D}_R$ . The component method of adding the vectors is explained in the Example.

**EXAMPLE 3-2 Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction  $60.0^\circ$  south of east for 47.0 km (Fig. 3-15a). What is her displacement from the post office?

**APPROACH** We choose the positive  $x$  axis to be east and the positive  $y$  axis to be north, since those are the compass directions used on most maps. The origin of the  $xy$  coordinate system is at the post office. We resolve each vector into its  $x$  and  $y$  components. We add the  $x$  components together, and then the  $y$  components together, giving us the  $x$  and  $y$  components of the resultant.

**SOLUTION** Resolve each displacement vector into its components, as shown in Fig. 3-15b. Since  $\vec{D}_1$  has magnitude 22.0 km and points north, it has only a  $y$  component:

$$D_{1x} = 0, \quad D_{1y} = 22.0 \text{ km.}$$

$\vec{D}_2$  has both  $x$  and  $y$  components:

$$D_{2x} = +(47.0 \text{ km})(\cos 60^\circ) = +(47.0 \text{ km})(0.500) = +23.5 \text{ km}$$

$$D_{2y} = -(47.0 \text{ km})(\sin 60^\circ) = -(47.0 \text{ km})(0.866) = -40.7 \text{ km.}$$

Notice that  $D_{2y}$  is negative because this vector component points along the negative  $y$  axis. The resultant vector,  $\vec{D}_R$ , has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0 \text{ km} + 23.5 \text{ km} = +23.5 \text{ km}$$

$$D_{Ry} = D_{1y} + D_{2y} = 22.0 \text{ km} + (-40.7 \text{ km}) = -18.7 \text{ km.}$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5 \text{ km}, \quad D_{Ry} = -18.7 \text{ km.}$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3-4:

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5 \text{ km})^2 + (-18.7 \text{ km})^2} = 30.0 \text{ km}$$

$$\tan \theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7 \text{ km}}{23.5 \text{ km}} = -0.796.$$

A calculator with a key labeled INV TAN, or ARC TAN, or  $\text{TAN}^{-1}$  gives  $\theta = \text{tan}^{-1}(-0.796) = -38.5^\circ$ . The negative sign means  $\theta = 38.5^\circ$  below the  $x$  axis, Fig. 3-15c. So, the resultant displacement is 30.0 km directed at  $38.5^\circ$  in a southeasterly direction.

**NOTE** Always be attentive about the quadrant in which the resultant vector lies. An electronic calculator does not fully give this information, but a good diagram does.



**PROBLEM SOLVING**

Identify the correct quadrant by drawing a careful diagram

As we saw in Example 3-2, any component that points along the negative  $x$  or  $y$  axis gets a minus sign. The signs of trigonometric functions depend on which “quadrant” the angle falls in: for example, the tangent is positive in the first and third quadrants (from  $0^\circ$  to  $90^\circ$ , and  $180^\circ$  to  $270^\circ$ ), but negative in the second and fourth quadrants; see Appendix A, Fig. A-7. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram, like Fig. 3-15. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

The following Problem Solving Strategy should not be considered a prescription. Rather it is a summary of things to do to get you thinking and involved in the problem at hand.



## Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

- 1. Draw a diagram**, adding the vectors graphically by either the parallelogram or tail-to-tip method.
- 2. Choose  $x$  and  $y$  axes.** Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors, which then will have only one component.)
- 3. Resolve** each vector into its  $x$  and  $y$  **components**, showing each component along its appropriate ( $x$  or  $y$ ) axis as a (dashed) arrow.
- 4. Calculate each component** (when not given) using sines and cosines. If  $\theta_1$  is the angle that vector  $\vec{V}_1$  makes with the positive  $x$  axis, then:

$$V_{1x} = V_1 \cos \theta_1, \quad V_{1y} = V_1 \sin \theta_1.$$

Pay careful attention to **signs**: any component that points along the negative  $x$  or  $y$  axis gets a minus sign.

- 5. Add the  $x$  components** together to get the  $x$  component of the resultant. Similarly for  $y$ :

$$V_{Rx} = V_{1x} + V_{2x} + \text{any others}$$

$$V_{Ry} = V_{1y} + V_{2y} + \text{any others.}$$

This is the answer: the components of the resultant vector. Check signs to see if they fit the quadrant shown in your diagram (point 1 above).

- If you want to know the **magnitude and direction** of the resultant vector, use Eqs. 3–4:

$$V_R = \sqrt{V_{Rx}^2 + V_{Ry}^2}, \quad \tan \theta = \frac{V_{Ry}}{V_{Rx}}.$$

The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle  $\theta$ .

**EXAMPLE 3–3 Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–16a. The first leg is due east for 620 km; the second leg is southeast ( $45^\circ$ ) for 440 km; and the third leg is at  $53^\circ$  south of west, for 550 km, as shown. What is the plane’s total displacement?

**APPROACH** We follow the steps in the Problem Solving Strategy above.

### SOLUTION

- 1. Draw a diagram** such as Fig. 3–16a, where  $\vec{D}_1$ ,  $\vec{D}_2$ , and  $\vec{D}_3$  represent the three legs of the trip, and  $\vec{D}_R$  is the plane’s total displacement.
- 2. Choose axes:** Axes are also shown in Fig. 3–16a:  $x$  is east,  $y$  north.
- 3. Resolve components:** It is imperative to draw a good diagram. The components are drawn in Fig. 3–16b. Instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–15b, here we draw them “tail-to-tip” style, which is just as valid and may make it easier to see.
- 4. Calculate the components:**

$$\vec{D}_1: D_{1x} = +D_1 \cos 0^\circ = D_1 = 620 \text{ km}$$

$$D_{1y} = +D_1 \sin 0^\circ = 0 \text{ km}$$

$$\vec{D}_2: D_{2x} = +D_2 \cos 45^\circ = +(440 \text{ km})(0.707) = +311 \text{ km}$$

$$D_{2y} = -D_2 \sin 45^\circ = -(440 \text{ km})(0.707) = -311 \text{ km}$$

$$\vec{D}_3: D_{3x} = -D_3 \cos 53^\circ = -(550 \text{ km})(0.602) = -331 \text{ km}$$

$$D_{3y} = -D_3 \sin 53^\circ = -(550 \text{ km})(0.799) = -439 \text{ km.}$$

We have given a minus sign to each component that in Fig. 3–16b points in the  $-x$  or  $-y$  direction. The components are shown in the Table in the margin.

- 5. Add the components:** We add the  $x$  components together, and we add the  $y$  components together to obtain the  $x$  and  $y$  components of the resultant:

$$D_{Rx} = D_{1x} + D_{2x} + D_{3x} = 620 \text{ km} + 311 \text{ km} - 331 \text{ km} = 600 \text{ km}$$

$$D_{Ry} = D_{1y} + D_{2y} + D_{3y} = 0 \text{ km} - 311 \text{ km} - 439 \text{ km} = -750 \text{ km.}$$

The  $x$  and  $y$  components of the resultant are 600 km and  $-750$  km, and point respectively to the east and south. This is one way to give the answer.

- 6. Magnitude and direction:** We can also give the answer as

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(600)^2 + (-750)^2} \text{ km} = 960 \text{ km}$$

$$\tan \theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-750 \text{ km}}{600 \text{ km}} = -1.25, \quad \text{so } \theta = -51^\circ.$$

Thus, the total displacement has magnitude 960 km and points  $51^\circ$  below the  $x$  axis (south of east), as was shown in our original sketch, Fig. 3–16a.

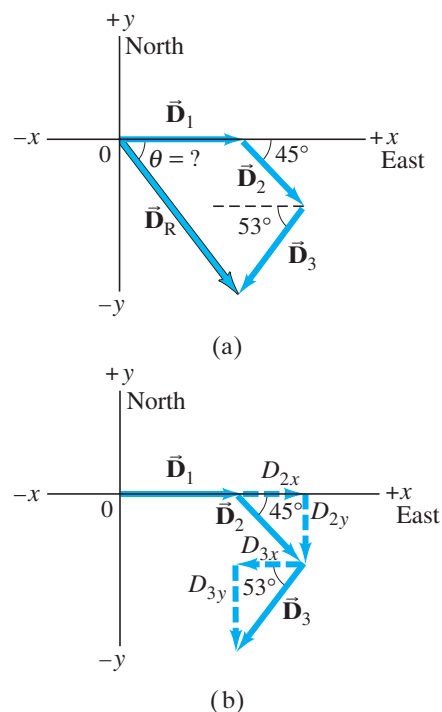
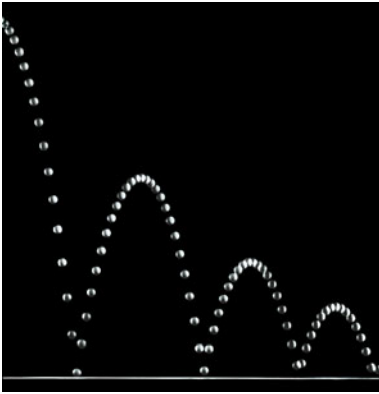


FIGURE 3–16 Example 3–3.

Vector	Components	
	$x$ (km)	$y$ (km)
$\vec{D}_1$	620	0
$\vec{D}_2$	311	-311
$\vec{D}_3$	-331	-439
$\vec{D}_R$	600	-750

## 3–5 Projectile Motion



(a)



(b)

**FIGURE 3–17** Photographs of (a) a bouncing ball and (b) a thrown basketball, each showing the characteristic “parabolic” path of projectile motion.

In Chapter 2, we studied the one-dimensional motion of an object in terms of displacement, velocity, and acceleration, including purely vertical motion of a falling object undergoing acceleration due to gravity. Now we examine the more general translational motion of objects moving through the air in two dimensions near the Earth’s surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–17), which we can describe as taking place in two dimensions if there is no wind.

Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude  $g = 9.80 \text{ m/s}^2$ , and we assume it is constant.<sup>†</sup>

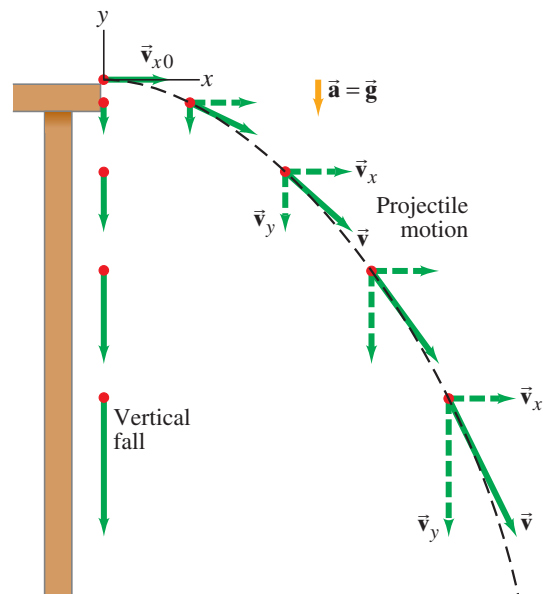
Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time  $t = 0$  at the origin of an  $xy$  coordinate system (so  $x_0 = y_0 = 0$ ).

Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal ( $x$ ) direction,  $\vec{v}_{x0}$ . See Fig. 3–18, where an object falling vertically is also shown for comparison. The velocity vector  $\vec{v}$  at each instant points in the direction of the ball’s motion at that instant and is thus always tangent to the path. Following Galileo’s ideas, we treat the horizontal and vertical components of velocity and acceleration separately, and we can apply the kinematic equations (Eqs. 2–11a through 2–11c) to the  $x$  and  $y$  components of the motion.

First we examine the vertical ( $y$ ) component of the motion. At the instant the ball leaves the table’s top ( $t = 0$ ), it has only an  $x$  component of velocity. Once the ball leaves the table (at  $t = 0$ ), it experiences a vertically downward acceleration  $g$ , the acceleration due to gravity. Thus  $v_y$  is initially zero ( $v_{y0} = 0$ ) but increases continually in the downward direction (until the ball hits the ground). Let us take  $y$  to be positive upward. Then the acceleration due to gravity is in the  $-y$  direction, so  $a_y = -g$ . From Eq. 2–11a (using  $y$  in place of  $x$ ) we can write  $v_y = v_{y0} + a_y t = -gt$  since we set  $v_{y0} = 0$ . The vertical displacement is given by Eq. 2–11b written in terms of  $y$ :  $y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$ . Given  $y_0 = 0$ ,  $v_{y0} = 0$ , and  $a_y = -g$ , then  $y = -\frac{1}{2} g t^2$ .

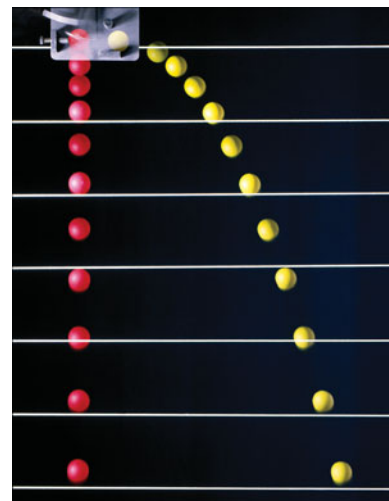
<sup>†</sup>This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth’s radius (6400 km).

**FIGURE 3–18** Projectile motion of a small ball projected horizontally with initial velocity  $\vec{v} = \vec{v}_{x0}$ . The dashed black line represents the path of the object. The velocity vector  $\vec{v}$  is in the direction of motion at each point, and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting from rest at the same place and time is shown at the left for comparison;  $v_y$  is the same at each instant for the falling object and the projectile.)



In the horizontal direction, on the other hand, there is no acceleration (we are ignoring air resistance). With  $a_x = 0$ , the horizontal component of velocity,  $v_x$ , remains constant, equal to its initial value,  $v_{x0}$ , and thus has the same magnitude at each point on the path. The horizontal displacement (with  $a_x = 0$ ) is given by  $x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$ . The two vector components,  $\vec{v}_x$  and  $\vec{v}_y$ , can be added vectorially at any instant to obtain the velocity  $\vec{v}$  at that time (that is, for each point on the path), as shown in Fig. 3–18.

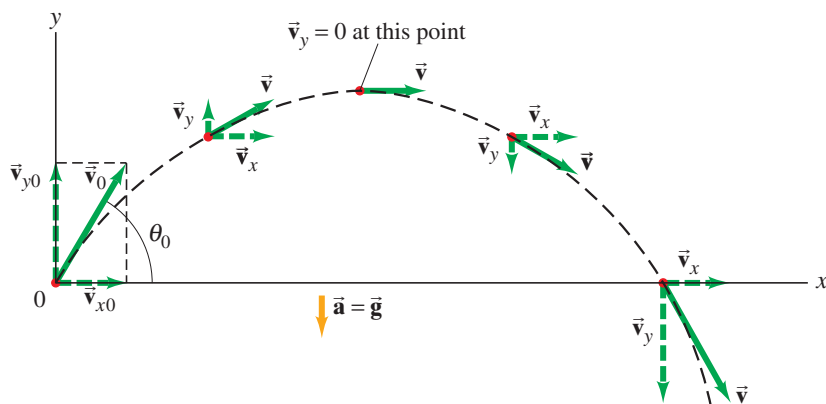
One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 3–18. Figure 3–19 is a multiple-exposure photograph of an experiment that confirms this.



**FIGURE 3–19** Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

**EXERCISE C** Two balls having different speeds roll off the edge of a horizontal table at the same time. Which hits the floor sooner, the faster ball or the slower one?

If an object is projected at an upward angle, as in Fig. 3–20, the analysis is similar, except that now there is an initial vertical component of velocity,  $v_{y0}$ . Because of the downward acceleration of gravity, the upward component of velocity  $v_y$  gradually decreases with time until the object reaches the highest point on its path, at which point  $v_y = 0$ . Subsequently the object moves downward (Fig. 3–20) and  $v_y$  increases in the downward direction, as shown (that is, becoming more negative). As before,  $v_x$  remains constant.

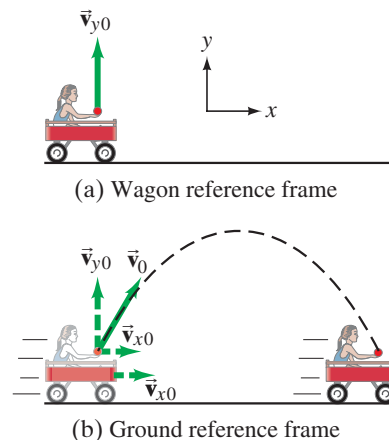


**FIGURE 3–20** Path of a projectile launched with initial velocity  $\vec{v}_0$  at angle  $\theta_0$  to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The figure does not show where the projectile hits the ground (at that point, projectile motion ceases).

**EXERCISE D** Where in Fig. 3–20 is (i)  $\vec{v} = 0$ , (ii)  $v_y = 0$ , and (iii)  $v_x = 0$ ?

**CONCEPTUAL EXAMPLE 3–4** **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 3–21. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 3–21a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (a) behind the wagon, (b) in the wagon, or (c) in front of the wagon?

**FIGURE 3–21** Example 3–4.



**RESPONSE** The child throws the apple straight up from her own reference frame with initial velocity  $\vec{v}_{y0}$  (Fig. 3–21a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon,  $\vec{v}_{x0}$ . Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 3–21b. The apple experiences no horizontal acceleration, so  $\vec{v}_{x0}$  will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (b).

**EXERCISE E** Return to the Chapter-Opening Question, page 49, and answer it again now. Try to explain why you may have answered differently the first time. Describe the role of the helicopter in this example of projectile motion.

## 3–6 Solving Projectile Motion Problems

We now work through several Examples of projectile motion quantitatively. We use the kinematic equations (2–11a through 2–11c) separately for the vertical and horizontal components of the motion. These equations are shown separately for the  $x$  and  $y$  components of the motion in Table 3–1, for the general case of two-dimensional motion at constant acceleration. Note that  $x$  and  $y$  are the respective displacements, that  $v_x$  and  $v_y$  are the components of the velocity, and that  $a_x$  and  $a_y$  are the components of the acceleration, each of which is constant. The subscript 0 means “at  $t = 0$ .”

**TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions**

x component (horizontal)		y component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set  $a_x = 0$ . See Table 3–2, which assumes  $y$  is positive upward, so  $a_y = -g = -9.80 \text{ m/s}^2$ .

**TABLE 3–2 Kinematic Equations for Projectile Motion**  
( $y$  positive upward;  $a_x = 0$ ,  $a_y = -g = -9.80 \text{ m/s}^2$ )

Horizontal Motion ( $a_x = 0$ , $v_x = \text{constant}$ )	Vertical Motion <sup>†</sup> ( $a_y = -g = \text{constant}$ )
$v_x = v_{x0}$	(Eq. 2–11a) $v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	(Eq. 2–11b) $y = y_0 + v_{y0} t - \frac{1}{2} gt^2$
	(Eq. 2–11c) $v_y^2 = v_{y0}^2 - 2g(y - y_0)$

<sup>†</sup> If  $y$  is taken positive downward, the minus (–) signs in front of  $g$  become + signs.

If the projection angle  $\theta_0$  is chosen relative to the  $+x$  axis (Fig. 3–20), then

$$v_{x0} = v_0 \cos \theta_0, \quad \text{and} \quad v_{y0} = v_0 \sin \theta_0.$$



### PROBLEM SOLVING Choice of time interval

In doing Problems involving projectile motion, we must consider a time interval for which our chosen object is in the air, influenced only by gravity. We do not consider the throwing (or projecting) process, nor the time after the object lands or is caught, because then other influences act on the object, and we can no longer set  $\vec{a} = \vec{g}$ .

## PROBLEM SOLVING

### Projectile Motion

Our approach to solving Problems in Section 2–6 also applies here. Solving Problems involving projectile motion can require creativity, and cannot be done just by following some rules. Certainly you must avoid just plugging numbers into equations that seem to “work.”

1. As always, **read** carefully; **choose** the **object** (or objects) you are going to analyze.
2. **Draw** a careful **diagram** showing what is happening to the object.
3. **Choose** an origin and an  $xy$  **coordinate system**.
4. Decide on the **time interval**, which for projectile motion can only include motion under the effect of gravity alone, not throwing or landing. The time interval must be the same for the  $x$  and  $y$  analyses. The  $x$  and  $y$  motions are connected by the common time,  $t$ .

5. **Examine** the horizontal ( $x$ ) and vertical ( $y$ ) **motions** separately. If you are given the initial velocity, you may want to resolve it into its  $x$  and  $y$  components.
6. List the **known** and **unknown** quantities, choosing  $a_x = 0$  and  $a_y = -g$  or  $+g$ , where  $g = 9.80 \text{ m/s}^2$ , and using the  $+$  or  $-$  sign, depending on whether you choose  $y$  positive up or down. Remember that  $v_x$  never changes throughout the trajectory, and that  $v_y = 0$  at the highest point of any trajectory that returns downward. The velocity just before landing is generally not zero.
7. Think for a minute before jumping into the equations. A little planning goes a long way. **Apply** the **relevant equations** (Table 3–2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–4).

**EXAMPLE 3–5 Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.

**APPROACH** We explicitly follow the steps of the Problem Solving Strategy on the previous page.

**SOLUTION**

- and **2. Read, choose the object, and draw a diagram.** Our object is the motorcycle and driver, taken as a single unit. The diagram is shown in Fig. 3–22.
- Choose a coordinate system.** We choose the  $y$  direction to be positive upward, with the top of the cliff as  $y_0 = 0$ . The  $x$  direction is horizontal with  $x_0 = 0$  at the point where the motorcycle leaves the cliff.
- Choose a time interval.** We choose our time interval to begin ( $t = 0$ ) just as the motorcycle leaves the cliff top at position  $x_0 = 0$ ,  $y_0 = 0$ . Our time interval ends just before the motorcycle touches the ground below.
- Examine  $x$  and  $y$  motions.** In the horizontal ( $x$ ) direction, the acceleration  $a_x = 0$ , so the velocity is constant. The value of  $x$  when the motorcycle reaches the ground is  $x = +90.0$  m. In the vertical direction, the acceleration is the acceleration due to gravity,  $a_y = -g = -9.80$  m/s<sup>2</sup>. The value of  $y$  when the motorcycle reaches the ground is  $y = -50.0$  m. The initial velocity is horizontal and is our unknown,  $v_{x0}$ ; the initial vertical velocity is zero,  $v_{y0} = 0$ .
- List knowns and unknowns.** See the Table in the margin. Note that in addition to not knowing the initial horizontal velocity  $v_{x0}$  (which stays constant until landing), we also do not know the time  $t$  when the motorcycle reaches the ground.
- Apply relevant equations.** The motorcycle maintains constant  $v_x$  as long as it is in the air. The time it stays in the air is determined by the  $y$  motion—when it reaches the ground. So we first find the time using the  $y$  motion, and then use this time value in the  $x$  equations. To find out how long it takes the motorcycle to reach the ground below, we use Eq. 2–11b (Tables 3–1 and 3–2) for the vertical ( $y$ ) direction with  $y_0 = 0$  and  $v_{y0} = 0$ :

$$y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$$

$$= 0 + 0 + \frac{1}{2}(-g)t^2$$

or

$$y = -\frac{1}{2}gt^2.$$

We solve for  $t$  and set  $y = -50.0$  m:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 3.19 \text{ s}.$$

To calculate the initial velocity,  $v_{x0}$ , we again use Eq. 2–11b, but this time for the horizontal ( $x$ ) direction, with  $a_x = 0$  and  $x_0 = 0$ :

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$

$$= 0 + v_{x0}t + 0$$

or

$$x = v_{x0}t.$$

Then

$$v_{x0} = \frac{x}{t} = \frac{90.0 \text{ m}}{3.19 \text{ s}} = 28.2 \text{ m/s},$$

which is about 100 km/h (roughly 60 mi/h).

**NOTE** In the time interval of the projectile motion, the only acceleration is  $g$  in the negative  $y$  direction. The acceleration in the  $x$  direction is zero.

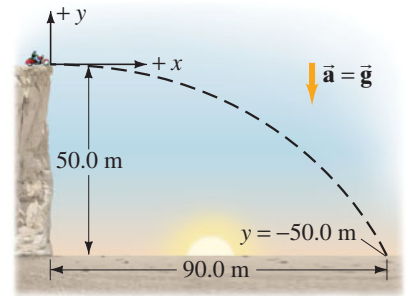
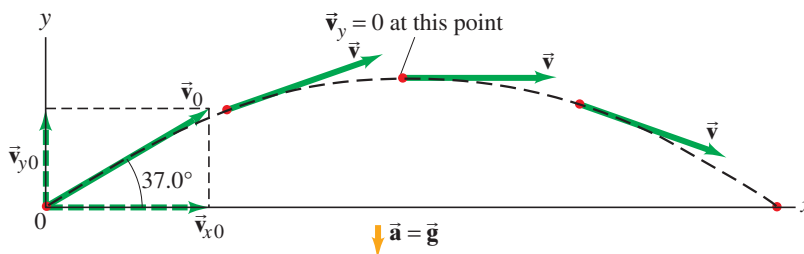


FIGURE 3–22 Example 3–5.

Known	Unknown
$x_0 = y_0 = 0$	$v_{x0}$
$x = 90.0$ m	$t$
$y = -50.0$ m	
$a_x = 0$	
$a_y = -g = -9.80$ m/s <sup>2</sup>	
$v_{y0} = 0$	

FIGURE 3–23 Example 3–6.



**PHYSICS APPLIED**

*Sports*

**EXAMPLE 3–6**

**A kicked football.** A kicked football leaves the ground at an angle  $\theta_0 = 37.0^\circ$  with a velocity of 20.0 m/s, as shown in Fig. 3–23. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, and (c) how far away it hits the ground. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.

**APPROACH** This may seem difficult at first because there are so many questions. But we can deal with them one at a time. We take the  $y$  direction as positive upward, and treat the  $x$  and  $y$  motions separately. The total time in the air is again determined by the  $y$  motion. The  $x$  motion occurs at constant velocity. The  $y$  component of velocity varies, being positive (upward) initially, decreasing to zero at the highest point, and then becoming negative as the football falls.

**SOLUTION** We resolve the initial velocity into its components (Fig. 3–23):

$$v_{x0} = v_0 \cos 37.0^\circ = (20.0 \text{ m/s})(0.799) = 16.0 \text{ m/s}$$

$$v_{y0} = v_0 \sin 37.0^\circ = (20.0 \text{ m/s})(0.602) = 12.0 \text{ m/s}.$$

(a) To find the maximum height, we consider a time interval that begins just after the football loses contact with the foot until the ball reaches its maximum height. During this time interval, the acceleration is  $g$  downward. At the maximum height, the velocity is horizontal (Fig. 3–23), so  $v_y = 0$ . This occurs at a time given by  $v_y = v_{y0} - gt$  with  $v_y = 0$  (see Eq. 2–11a in Table 3–2), so  $v_{y0} = gt$  and

$$t = \frac{v_{y0}}{g} = \frac{(12.0 \text{ m/s})}{(9.80 \text{ m/s}^2)} = 1.224 \text{ s} \approx 1.22 \text{ s}.$$

From Eq. 2–11b, with  $y_0 = 0$ , we can solve for  $y$  at this time ( $t = v_{y0}/g$ ):

$$y = v_{y0}t - \frac{1}{2}gt^2 = \frac{v_{y0}^2}{g} - \frac{1}{2} \frac{v_{y0}^2}{g} = \frac{v_{y0}^2}{2g} = \frac{(12.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 7.35 \text{ m}.$$

The maximum height is 7.35 m. [Solving Eq. 2–11c for  $y$  gives the same result.]

(b) To find the time it takes for the ball to return to the ground, we consider a different time interval, starting at the moment the ball leaves the foot ( $t = 0$ ,  $y_0 = 0$ ) and ending just before the ball touches the ground ( $y = 0$  again). We can use Eq. 2–11b with  $y_0 = 0$  and also set  $y = 0$  (ground level):

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$0 = 0 + v_{y0}t - \frac{1}{2}gt^2.$$

This equation can be factored:

$$t\left(\frac{1}{2}gt - v_{y0}\right) = 0.$$

There are two solutions,  $t = 0$  (which corresponds to the initial point,  $y_0$ ), and

$$t = \frac{2v_{y0}}{g} = \frac{2(12.0 \text{ m/s})}{(9.80 \text{ m/s}^2)} = 2.45 \text{ s},$$

which is the total travel time of the football.

(c) The total distance traveled in the  $x$  direction is found by applying Eq. 2–11b with  $x_0 = 0$ ,  $a_x = 0$ ,  $v_{x0} = 16.0 \text{ m/s}$ , and  $t = 2.45 \text{ s}$ :

$$x = v_{x0}t = (16.0 \text{ m/s})(2.45 \text{ s}) = 39.2 \text{ m}.$$

**NOTE** In (b), the time needed for the whole trip,  $t = 2v_{y0}/g = 2.45 \text{ s}$ , is double the time to reach the highest point, calculated in (a). That is, the time to go up equals the time to come back down to the same level (ignoring air resistance).



**PROBLEM SOLVING**

*Symmetry*

**EXERCISE F** In Example 3–6, what is (a) the velocity vector at the maximum height, and (b) the acceleration vector at maximum height?

In Example 3–6, we treated the football as if it were a particle, ignoring its rotation. We also ignored air resistance. Because air resistance is significant on a football, our results are only estimates (mainly overestimates).

**CONCEPTUAL EXAMPLE 3–7 The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance  $d$  away, Fig. 3–24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

**RESPONSE** Both the water balloon and the boy in the tree start falling at the same instant, and in a time  $t$  they each fall the same vertical distance  $y = \frac{1}{2}gt^2$ , much like Fig. 3–19. In the time it takes the water balloon to travel the horizontal distance  $d$ , the balloon will have the same  $y$  position as the falling boy. Splat. If the boy had stayed in the tree, he would have avoided the humiliation.

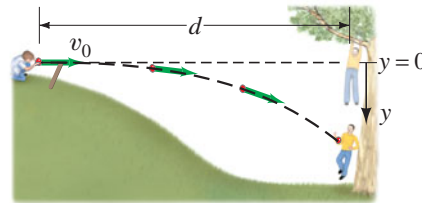


FIGURE 3–24 Example 3–7.

### Level Horizontal Range

The total distance the football traveled in Example 3–6 is called the horizontal **range**  $R$ . We now derive a formula for the range, which applies to a projectile that lands at the same level it started ( $= y_0$ ): that is,  $y$  (final)  $= y_0$  (see Fig. 3–25a). Looking back at Example 3–6 part (c), we see that  $x = R = v_{x0}t$  where (from part b)  $t = 2v_{y0}/g$ . Thus

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g}, \quad [y = y_0]$$

where  $v_{x0} = v_0 \cos \theta_0$  and  $v_{y0} = v_0 \sin \theta_0$ . This can be rewritten, using the trigonometric identity  $2 \sin \theta \cos \theta = \sin 2\theta$  (Appendix A or inside the rear cover):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \quad [\text{only if } y(\text{final}) = y_0]$$

Note that the *maximum* range, for a given initial velocity  $v_0$ , is obtained when  $\sin 2\theta$  takes on its maximum value of 1.0, which occurs for  $2\theta_0 = 90^\circ$ ; so

$$\theta_0 = 45^\circ \text{ for maximum range, and } R_{\max} = v_0^2/g.$$

The maximum range increases by the square of  $v_0$ , so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

When air resistance is important, the range is less for a given  $v_0$ , and the maximum range is obtained at an angle smaller than  $45^\circ$ .

**EXAMPLE 3–8 Range of a cannon ball.** Suppose one of Napoleon's cannons had a muzzle speed,  $v_0$ , of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

**APPROACH** We use the equation just derived for the range,  $R = v_0^2 \sin 2\theta_0/g$ , with  $R = 320$  m.

**SOLUTION** We solve for  $\sin 2\theta_0$  in the range formula:

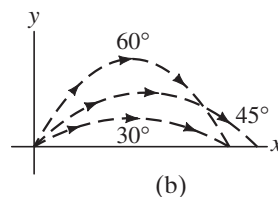
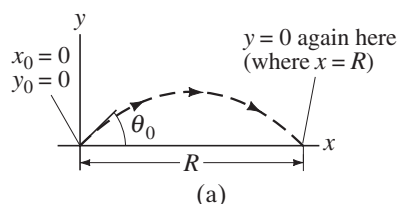
$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320 \text{ m})(9.80 \text{ m/s}^2)}{(60.0 \text{ m/s})^2} = 0.871.$$

We want to solve for an angle  $\theta_0$  that is between  $0^\circ$  and  $90^\circ$ , which means  $2\theta_0$  in this equation can be as large as  $180^\circ$ . Thus,  $2\theta_0 = 60.6^\circ$  is a solution, so  $\theta_0 = 30.3^\circ$ . But  $2\theta_0 = 180^\circ - 60.6^\circ = 119.4^\circ$  is also a solution (see Appendix A–7), so  $\theta_0$  can also be  $\theta_0 = 59.7^\circ$ . In general we have two solutions (see Fig. 3–25b), which in the present case are given by

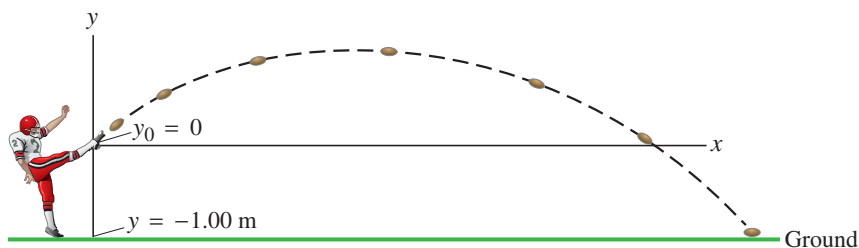
$$\theta_0 = 30.3^\circ \text{ or } 59.7^\circ.$$

Either angle gives the same range. Only when  $\sin 2\theta_0 = 1$  (so  $\theta_0 = 45^\circ$ ) is there a single solution (that is, both solutions are the same).

**FIGURE 3–25** (a) The range  $R$  of a projectile. (b) There are generally two angles  $\theta_0$  that will give the same range. If one angle is  $\theta_{01}$ , the other is  $\theta_{02} = 90^\circ - \theta_{01}$ . Example 3–8.



**FIGURE 3–26** Example 3–9: the football leaves the punter’s foot at  $y = 0$ , and reaches the ground where  $y = -1.00$  m.



**PHYSICS APPLIED**  
*Sports*

**PROBLEM SOLVING**  
*Do not use any formula unless you are sure its range of validity fits the problem; the range formula does not apply here because  $y \neq y_0$*

**EXAMPLE 3–9** **A punt.** Suppose the football in Example 3–6 was punted, and left the punter’s foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set  $x_0 = 0$ ,  $y_0 = 0$ .

**APPROACH** The only difference here from Example 3–6 is that the football hits the ground *below* its starting point of  $y_0 = 0$ . That is, the ball hits the ground at  $y = -1.00$  m. See Fig. 3–26. Thus we cannot use the range formula which is valid only if  $y$  (final) =  $y_0$ . As in Example 3–6,  $v_0 = 20.0$  m/s,  $\theta_0 = 37.0^\circ$ .

**SOLUTION** With  $y = -1.00$  m and  $v_{y0} = 12.0$  m/s (see Example 3–6), we use the  $y$  version of Eq. 2–11b with  $a_y = -g$ ,

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2,$$

and obtain

$$-1.00 \text{ m} = 0 + (12.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

We rearrange this equation into standard form ( $ax^2 + bx + c = 0$ ) so we can use the quadratic formula:

$$(4.90 \text{ m/s}^2)t^2 - (12.0 \text{ m/s})t - (1.00 \text{ m}) = 0.$$

The quadratic formula (Appendix A–4) gives

$$t = \frac{12.0 \text{ m/s} \pm \sqrt{(-12.0 \text{ m/s})^2 - 4(4.90 \text{ m/s}^2)(-1.00 \text{ m})}}{2(4.90 \text{ m/s}^2)} \\ = 2.53 \text{ s} \quad \text{or} \quad -0.081 \text{ s}.$$

The second solution would correspond to a time prior to the kick, so it doesn’t apply. With  $t = 2.53$  s for the time at which the ball touches the ground, the horizontal distance the ball traveled is (using  $v_{x0} = 16.0$  m/s from Example 3–6):

$$x = v_{x0}t = (16.0 \text{ m/s})(2.53 \text{ s}) = 40.5 \text{ m}.$$

Our assumption in Example 3–6 that the ball leaves the foot at ground level would result in an underestimate of about 1.3 m in the distance our punt traveled.



**FIGURE 3–27** Examples of projectile motion: a boy jumping, and glowing lava from the volcano Stromboli.

## \*3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a *parabola*, if we can ignore air resistance and can assume that  $\vec{g}$  is constant. To do so, we need to find  $y$  as a function of  $x$  by eliminating  $t$  between the two equations for horizontal and vertical motion (Eq. 2–11b in Table 3–2), and for simplicity we set  $x_0 = y_0 = 0$ :

$$x = v_{x0}t \\ y = v_{y0}t - \frac{1}{2}gt^2.$$

From the first equation, we have  $t = x/v_{x0}$ , and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2. \quad (3-6)$$

We see that  $y$  as a function of  $x$  has the form

$$y = Ax - Bx^2,$$

where  $A$  and  $B$  are constants for any specific projectile motion. This is the standard equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo’s day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

\*Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.



## 3–8 Relative Velocity

We now consider how observations made in different frames of reference are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the train tracks will measure 80 km/h for the speed of each of the trains. Observers on either one of the trains (a different frame of reference) will measure a speed of 160 km/h for the train approaching them.

Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of  $90 \text{ km/h} - 75 \text{ km/h} = 15 \text{ km/h}$ .

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the **relative velocity**. But if they are not along the same line, we must make use of vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process. Each velocity is labeled by *two subscripts*: *the first refers to the object, the second to the reference frame in which it has this velocity*. For example, suppose a boat heads directly across a river, as shown in Fig. 3–28. We let  $\vec{v}_{BW}$  be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly,  $\vec{v}_{BS}$  is the velocity of the **B**oat with respect to the **S**hore, and  $\vec{v}_{WS}$  is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that  $\vec{v}_{BW}$  is what the boat's motor produces (against the water), whereas  $\vec{v}_{BS}$  is equal to  $\vec{v}_{BW}$  plus the effect of the current,  $\vec{v}_{WS}$ . Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 3–28)

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}. \quad (3-7)$$

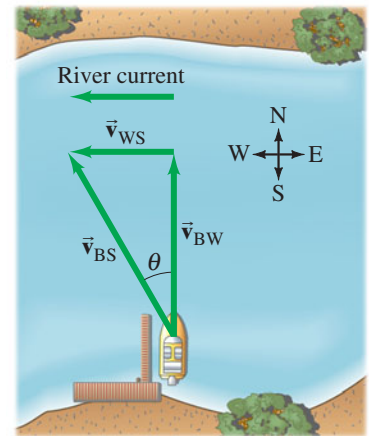
By writing the subscripts using this convention, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–7 are the same; also, the outer subscripts on the right of Eq. 3–7 (the B and the S) are the same as the two subscripts for the sum vector on the left,  $\vec{v}_{BS}$ . By following this convention (first subscript for the object, second for the reference frame), you can write down the correct equation relating velocities in different reference frames.<sup>†</sup>

Equation 3–7 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity  $\vec{v}_{FB}$  relative to the boat, his velocity relative to the shore is  $\vec{v}_{FS} = \vec{v}_{FB} + \vec{v}_{BW} + \vec{v}_{WS}$ . The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\vec{v}_{BA} = -\vec{v}_{AB}. \quad (3-8)$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.



**FIGURE 3–28** A boat heads north directly across a river which flows west. Velocity vectors are shown as green arrows:

- $\vec{v}_{BS}$  = velocity of **B**oat with respect to the **S**hore,
- $\vec{v}_{BW}$  = velocity of **B**oat with respect to the **W**ater,
- $\vec{v}_{WS}$  = velocity of **W**ater with respect to the **S**hore (river current).

As it crosses the river, the boat is dragged downstream by the current.

<sup>†</sup>We thus can see, for example, that the equation  $\vec{v}_{BW} = \vec{v}_{BS} + \vec{v}_{WS}$  is wrong: the inner subscripts are not the same, and the outer ones on the right do not correspond to the subscripts on the left.

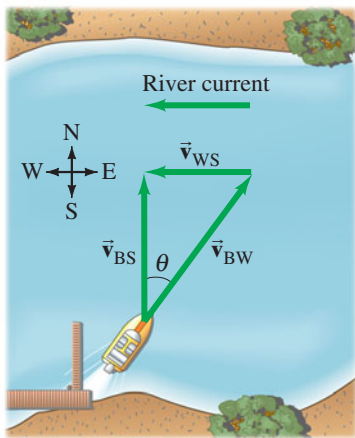


FIGURE 3-29 Example 3-10.

**EXAMPLE 3-10 Heading upstream.** A boat's speed in still water is  $v_{BW} = 1.85$  m/s. If the boat is to travel north directly across a river whose westward current has speed  $v_{WS} = 1.20$  m/s, at what upstream angle must the boat head? (See Fig. 3-29.)

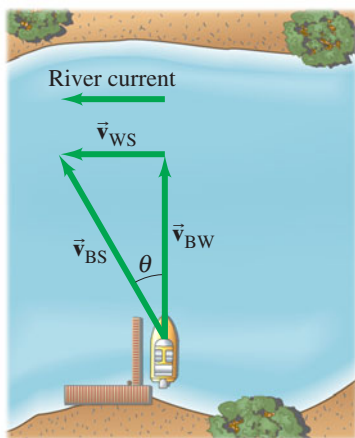
**APPROACH** If the boat heads straight across the river, the current will drag the boat downstream (westward). To overcome the river's current, the boat must have an upstream (eastward) component of velocity as well as a cross-stream (northward) component. Figure 3-29 has been drawn with  $\vec{v}_{BS}$ , the velocity of the **B**oat relative to the **S**hore, pointing directly across the river because this is where the boat is supposed to go. (Note that  $\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$ .)

**SOLUTION** Vector  $\vec{v}_{BW}$  points upstream at angle  $\theta$  as shown. From the diagram,

$$\sin \theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20 \text{ m/s}}{1.85 \text{ m/s}} = 0.6486.$$

Thus  $\theta = 40.4^\circ$ , so the boat must head upstream at a  $40.4^\circ$  angle.

FIGURE 3-30 Example 3-11. A boat heading directly across a river whose current moves at 1.20 m/s.



**EXAMPLE 3-11 Heading across the river.** The same boat ( $v_{BW} = 1.85$  m/s) now heads directly across the river whose current is still 1.20 m/s. (a) What is the velocity (magnitude and direction) of the boat relative to the shore? (b) If the river is 110 m wide, how long will it take to cross and how far downstream will the boat be then?

**APPROACH** The boat now heads directly across the river and is pulled downstream by the current, as shown in Fig. 3-30. The boat's velocity with respect to the shore,  $\vec{v}_{BS}$ , is the sum of its velocity with respect to the water,  $\vec{v}_{BW}$ , plus the velocity of the water with respect to the shore,  $\vec{v}_{WS}$ : just as before,

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

**SOLUTION** (a) Since  $\vec{v}_{BW}$  is perpendicular to  $\vec{v}_{WS}$ , we can get  $v_{BS}$  using the theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85 \text{ m/s})^2 + (1.20 \text{ m/s})^2} = 2.21 \text{ m/s}.$$

We can obtain the angle (note how  $\theta$  is defined in Fig. 3-30) from:

$$\tan \theta = v_{WS}/v_{BW} = (1.20 \text{ m/s})/(1.85 \text{ m/s}) = 0.6486.$$

A calculator with a key INV TAN or ARC TAN or  $\tan^{-1}$  gives  $\theta = \tan^{-1}(0.6486) = 33.0^\circ$ . Note that this angle is not equal to the angle calculated in Example 3-10.

(b) The travel time for the boat is determined by the time it takes to cross the river. Given the river's width  $D = 110$  m, we can use the velocity component in the direction of  $D$ ,  $v_{BW} = D/t$ . Solving for  $t$ , we get  $t = 110 \text{ m}/1.85 \text{ m/s} = 59.5$  s. The boat will have been carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20 \text{ m/s})(59.5 \text{ s}) = 71.4 \text{ m} \approx 71 \text{ m}.$$

**NOTE** There is no acceleration in this Example, so the motion involves only constant velocities (of the boat or of the river).

## Summary

A quantity such as velocity, that has both a magnitude and a direction, is called a **vector**. A quantity such as mass, that has only a magnitude, is called a **scalar**. On diagrams, vectors are represented by arrows.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first vector to the tip of the last vector. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude  $V$  making an angle  $\theta$  with the  $+x$  axis has components

$$V_x = V \cos \theta, \quad V_y = V \sin \theta. \quad (3-3)$$

Given the components, we can find a vector's magnitude and direction from

$$V = \sqrt{V_x^2 + V_y^2}, \quad \tan \theta = \frac{V_y}{V_x}. \quad (3-4)$$

**Projectile motion** is the motion of an object in the air near the Earth's surface under the effect of gravity alone. It can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration,  $\vec{g}$ , just as for an object falling vertically under the action of gravity.


The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the **relative velocity** of the two reference frames, are known.


## Questions


- One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are their velocities equal? Explain.
- Can you conclude that a car is not accelerating if its speedometer indicates a steady 60 km/h? Explain.
- Give several examples of an object's motion in which a great distance is traveled but the displacement is zero.
- Can the displacement vector for a particle moving in two dimensions be longer than the length of path traveled by the particle over the same time interval? Can it be less? Discuss.
- During baseball practice, a player hits a very high fly ball and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball? Explain.
- If  $\vec{V} = \vec{V}_1 + \vec{V}_2$ , is  $V$  necessarily greater than  $V_1$  and/or  $V_2$ ? Discuss.
- Two vectors have length  $V_1 = 3.5$  km and  $V_2 = 4.0$  km. What are the maximum and minimum magnitudes of their vector sum?
- Can two vectors, of unequal magnitude, add up to give the zero vector? Can *three* unequal vectors? Under what conditions?
- Can the magnitude of a vector ever (a) equal, or (b) be less than, one of its components?
- Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?
- How could you determine the speed a slingshot imparts to a rock, using only a meter stick, a rock, and the slingshot?
- In archery, should the arrow be aimed directly at the target? How should your angle of aim depend on the distance to the target?
- It was reported in World War I that a pilot flying at an altitude of 2 km caught in his bare hands a bullet fired at the plane! Using the fact that a bullet slows down considerably due to air resistance, explain how this incident occurred.
- You are on the street trying to hit a friend in his dorm window with a water balloon. He has a similar idea and is aiming at you with *his* water balloon. You aim straight at each other and throw at the same instant. Do the water balloons hit each other? Explain why or why not.
- A projectile is launched at an upward angle of  $30^\circ$  to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch, ignoring air resistance? Explain.
- A projectile has the least speed at what point in its path?
- Two cannonballs, A and B, are fired from the ground with identical initial speeds, but with  $\theta_A$  larger than  $\theta_B$ . (a) Which cannonball reaches a higher elevation? (b) Which stays longer in the air? (c) Which travels farther? Explain.
- A person sitting in an enclosed train car, moving at constant velocity, throws a ball straight up into the air in her reference frame. (a) Where does the ball land? What is your answer if the car (b) accelerates, (c) decelerates, (d) rounds a curve, (e) moves with constant velocity but is open to the air?
- If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?
- Two rowers, who can row at the same speed in still water, set off across a river at the same time. One heads straight across and is pulled downstream somewhat by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which rower reaches the opposite side first? Explain.
- If you stand motionless under an umbrella in a rainstorm where the drops fall vertically, you remain relatively dry. However, if you start running, the rain begins to hit your legs even if they remain under the umbrella. Why?

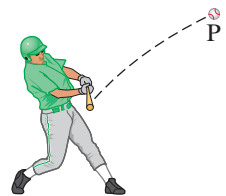
## MisConceptual Questions

- You are adding vectors of length 20 and 40 units. Which of the following choices is a possible resultant magnitude?
  - 0.
  - 18.
  - 37.
  - 64.
  - 100.
- The magnitude of a component of a vector must be
  - less than or equal to the magnitude of the vector.
  - equal to the magnitude of the vector.
  - greater than or equal to the magnitude of the vector.
  - less than, equal to, or greater than the magnitude of the vector.
- You are in the middle of a large field. You walk in a straight line for 100 m, then turn left and walk 100 m more in a straight line before stopping. When you stop, you are 100 m from your starting point. By how many degrees did you turn?
  - $90^\circ$ .
  - $120^\circ$ .
  - $30^\circ$ .
  - $180^\circ$ .
  - This is impossible. You cannot walk 200 m and be only 100 m away from where you started.
- A bullet fired from a rifle begins to fall
  - as soon as it leaves the barrel.
  - after air friction reduces its speed.
  - not at all if air resistance is ignored.
- A baseball player hits a ball that soars high into the air. After the ball has left the bat, and while it is traveling upward (at point P in Fig. 3–31), what is the direction of acceleration? Ignore air resistance.
 

(a) 

(b) 

(c) 
- One ball is dropped vertically from a window. At the same instant, a second ball is thrown horizontally from the same window. Which ball has the greater speed at ground level?
  - The dropped ball.
  - The thrown ball.
  - Neither—they both have the same speed on impact.
  - It depends on how hard the ball was thrown.



**FIGURE 3–31**  
MisConceptual  
Question 5.

7. You are riding in an enclosed train car moving at 90 km/h. If you throw a baseball straight up, where will the baseball land?  
 (a) In front of you.  
 (b) Behind you.  
 (c) In your hand.  
 (d) Can't decide from the given information.
8. Which of the three kicks in Fig. 3–32 is in the air for the longest time? They all reach the same maximum height  $h$ . Ignore air resistance.  
 (a), (b), (c), or (d) all the same time.

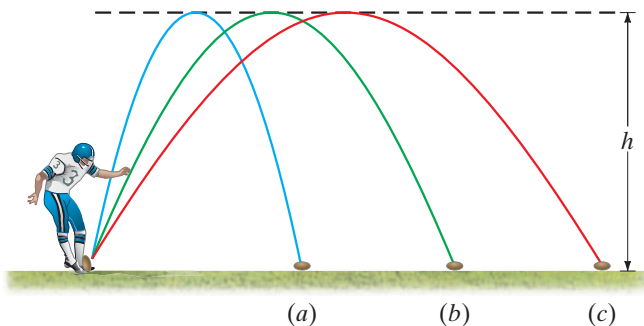


FIGURE 3–32 MisConceptual Question 8.

9. A baseball is hit high and far. Which of the following statements is true? At the highest point,  
 (a) the magnitude of the acceleration is zero.  
 (b) the magnitude of the velocity is zero.  
 (c) the magnitude of the velocity is the slowest.  
 (d) more than one of the above is true.  
 (e) none of the above are true.

10. A hunter is aiming horizontally at a monkey who is sitting in a tree. The monkey is so terrified when it sees the gun that it falls off the tree. At that very instant, the hunter pulls the trigger. What will happen?  
 (a) The bullet will miss the monkey because the monkey falls down while the bullet speeds straight forward.  
 (b) The bullet will hit the monkey because both the monkey and the bullet are falling downward at the same rate due to gravity.  
 (c) The bullet will miss the monkey because although both the monkey and the bullet are falling downward due to gravity, the monkey is falling faster.  
 (d) It depends on how far the hunter is from the monkey.
11. Which statements are *not* valid for a projectile? Take up as positive.  
 (a) The projectile has the same  $x$  velocity at any point on its path.  
 (b) The acceleration of the projectile is positive and decreasing when the projectile is moving upwards, zero at the top, and increasingly negative as the projectile descends.  
 (c) The acceleration of the projectile is a constant negative value.  
 (d) The  $y$  component of the velocity of the projectile is zero at the highest point of the projectile's path.  
 (e) The velocity at the highest point is zero.
12. A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is  
 (a) less than 20 m/s.  
 (b) exactly 20 m/s.  
 (c) more than 20 m/s.

For assigned homework and other learning materials, go to the MasteringPhysics website.



## Problems

### 3–2 to 3–4 Vector Addition

- (I) A car is driven 225 km west and then 98 km southwest ( $45^\circ$ ). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.
- (I) A delivery truck travels 21 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.
- (I) If  $V_x = 9.80$  units and  $V_y = -6.40$  units, determine the magnitude and direction of  $\vec{V}$ .
- (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m,  $36^\circ$  north of east; (2) 18 m,  $37^\circ$  east of north; and (3) 26 m,  $33^\circ$  west of south.
- (II)  $\vec{V}$  is a vector 24.8 units in magnitude and points at an angle of  $23.4^\circ$  above the negative  $x$  axis. (a) Sketch this vector. (b) Calculate  $V_x$  and  $V_y$ . (c) Use  $V_x$  and  $V_y$  to obtain (again) the magnitude and direction of  $\vec{V}$ . [Note: Part (c) is a good way to check if you've resolved your vector correctly.]
- (II) Vector  $\vec{V}_1$  is 6.6 units long and points along the negative  $x$  axis. Vector  $\vec{V}_2$  is 8.5 units long and points at  $+55^\circ$  to the positive  $x$  axis. (a) What are the  $x$  and  $y$  components of each vector? (b) Determine the sum  $\vec{V}_1 + \vec{V}_2$  (magnitude and angle).

7. (II) Figure 3–33 shows two vectors,  $\vec{A}$  and  $\vec{B}$ , whose magnitudes are  $A = 6.8$  units and  $B = 5.5$  units. Determine  $\vec{C}$  if (a)  $\vec{C} = \vec{A} + \vec{B}$ , (b)  $\vec{C} = \vec{A} - \vec{B}$ , (c)  $\vec{C} = \vec{B} - \vec{A}$ . Give the magnitude and direction for each.

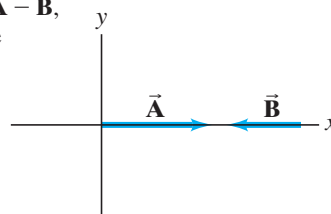


FIGURE 3–33 Problem 7.

8. (II) An airplane is traveling 835 km/h in a direction  $41.5^\circ$  west of north (Fig. 3–34).  
 (a) Find the components of the velocity vector in the northerly and westerly directions. (b) How far north and how far west has the plane traveled after 1.75 h?

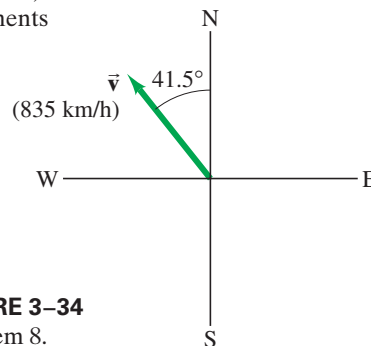
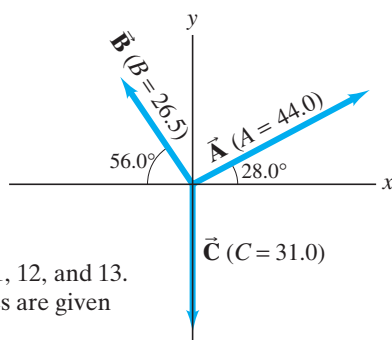


FIGURE 3–34 Problem 8.

9. (II) Three vectors are shown in Fig. 3–35. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (a) components, (b) magnitude and angle with the  $+x$  axis.



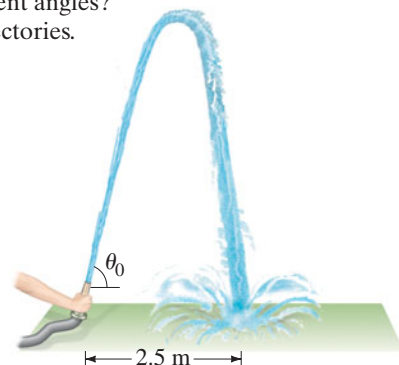
**FIGURE 3–35**  
Problems 9, 10, 11, 12, and 13.  
Vector magnitudes are given in arbitrary units.

10. (II) (a) Given the vectors  $\vec{A}$  and  $\vec{B}$  shown in Fig. 3–35, determine  $\vec{B} - \vec{A}$ . (b) Determine  $\vec{A} - \vec{B}$  without using your answer in (a). Then compare your results and see if they are opposite.
11. (II) Determine the vector  $\vec{A} - \vec{C}$ , given the vectors  $\vec{A}$  and  $\vec{C}$  in Fig. 3–35.
12. (II) For the vectors shown in Fig. 3–35, determine (a)  $\vec{B} - 3\vec{A}$ , (b)  $2\vec{A} - 3\vec{B} + 2\vec{C}$ .
13. (II) For the vectors given in Fig. 3–35, determine (a)  $\vec{A} - \vec{B} + \vec{C}$ , (b)  $\vec{A} + \vec{B} - \vec{C}$ , and (c)  $\vec{C} - \vec{A} - \vec{B}$ .
14. (II) Suppose a vector  $\vec{V}$  makes an angle  $\phi$  with respect to the  $y$  axis. What could be the  $x$  and  $y$  components of the vector  $\vec{V}$ ?
15. (II) The summit of a mountain, 2450 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction  $38.4^\circ$  west of north. What are the components of the displacement vector from camp to summit? What is its magnitude? Choose the  $x$  axis east,  $y$  axis north, and  $z$  axis up.
16. (III) You are given a vector in the  $xy$  plane that has a magnitude of 90.0 units and a  $y$  component of  $-65.0$  units. (a) What are the two possibilities for its  $x$  component? (b) Assuming the  $x$  component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the  $-x$  direction.

### 3–5 and 3–6 Projectile Motion (neglect air resistance)

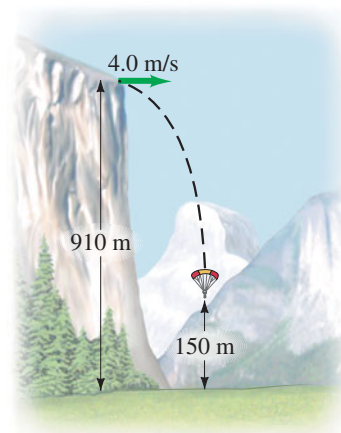
17. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 3.0 m/s. How far from the base of the rock will she land?
18. (I) A diver running 2.5 m/s dives out horizontally from the edge of a vertical cliff and 3.0 s later reaches the water below. How high was the cliff and how far from its base did the diver hit the water?
19. (II) Estimate by what factor a person can jump farther on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.
20. (II) A ball is thrown horizontally from the roof of a building 7.5 m tall and lands 9.5 m from the base. What was the ball's initial speed?
21. (II) A ball thrown horizontally at 12.2 m/s from the roof of a building lands 21.0 m from the base of the building. How high is the building?

22. (II) A football is kicked at ground level with a speed of 18.0 m/s at an angle of  $31.0^\circ$  to the horizontal. How much later does it hit the ground?
23. (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.5 m away (Fig. 3–36)? Why are there two different angles? Sketch the two trajectories.



**FIGURE 3–36**  
Problem 23.

24. (II) You buy a plastic dart gun, and being a clever physics student you decide to do a quick calculation to find its maximum horizontal range. You shoot the gun straight up, and it takes 4.0 s for the dart to land back at the barrel. What is the maximum horizontal range of your gun?
25. (II) A grasshopper hops along a level road. On each hop, the grasshopper launches itself at angle  $\theta_0 = 45^\circ$  and achieves a range  $R = 0.80$  m. What is the average horizontal speed of the grasshopper as it hops along the road? Assume that the time spent on the ground between hops is negligible.
26. (II) Extreme-sports enthusiasts have been known to jump off the top of El Capitan, a sheer granite cliff of height 910 m in Yosemite National Park. Assume a jumper runs horizontally off the top of El Capitan with speed 4.0 m/s and enjoys a free fall until she is 150 m above the valley floor, at which time she opens her parachute (Fig. 3–37). (a) How long is the jumper in free fall? Ignore air resistance. (b) It is important to be as far away from the cliff as possible before opening the parachute. How far from the cliff is this jumper when she opens her chute?



**FIGURE 3–37**  
Problem 26.

27. (II) A projectile is fired with an initial speed of 36.6 m/s at an angle of  $42.2^\circ$  above the horizontal on a long flat firing range. Determine (a) the maximum height reached by the projectile, (b) the total time in the air, (c) the total horizontal distance covered (that is, the range), and (d) the speed of the projectile 1.50 s after firing.

28. (II) An athlete performing a long jump leaves the ground at a  $27.0^\circ$  angle and lands 7.80 m away. (a) What was the takeoff speed? (b) If this speed were increased by just 5.0%, how much longer would the jump be?
29. (II) A shot-putter throws the “shot” (mass = 7.3 kg) with an initial speed of 14.4 m/s at a  $34.0^\circ$  angle to the horizontal. Calculate the horizontal distance traveled by the shot if it leaves the athlete’s hand at a height of 2.10 m above the ground.
30. (II) A baseball is hit with a speed of 27.0 m/s at an angle of  $45.0^\circ$ . It lands on the flat roof of a 13.0-m-tall nearby building. If the ball was hit when it was 1.0 m above the ground, what horizontal distance does it travel before it lands on the building?
31. (II) A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 m below. If the plane is traveling horizontally with a speed of 250 km/h (69.4 m/s), how far in advance of the recipients (horizontal distance) must the goods be dropped (Fig. 3–38)?

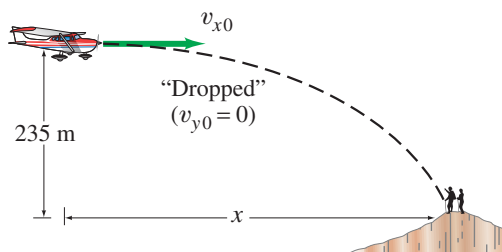


FIGURE 3–38 Problem 31.

32. (III) Suppose the rescue plane of Problem 31 releases the supplies a horizontal distance of 425 m in advance of the mountain climbers. What vertical velocity (up or down) should the supplies be given so that they arrive precisely at the climbers’ position (Fig. 3–39)? With what speed do the supplies land?

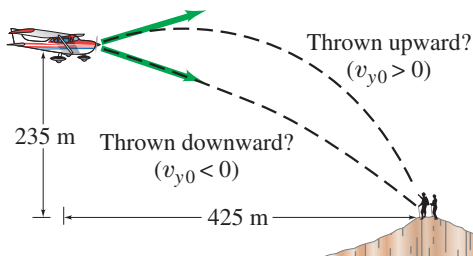


FIGURE 3–39 Problem 32.

33. (III) A diver leaves the end of a 4.0-m-high diving board and strikes the water 1.3 s later, 3.0 m beyond the end of the board. Considering the diver as a particle, determine: (a) her initial velocity,  $\vec{v}_0$ ; (b) the maximum height reached; and (c) the velocity  $\vec{v}_f$  with which she enters the water.
34. (III) Show that the time required for a projectile to reach its highest point is equal to the time for it to return to its original height if air resistance is negligible.
35. (III) Suppose the kick in Example 3–6 is attempted 36.0 m from the goalposts, whose crossbar is 3.05 m above the ground. If the football is directed perfectly between the goalposts, will it pass over the bar and be a field goal? Show why or why not. If not, from what horizontal distance must this kick be made if it is to score?

36. (III) Revisit Example 3–7, and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 3–40) and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

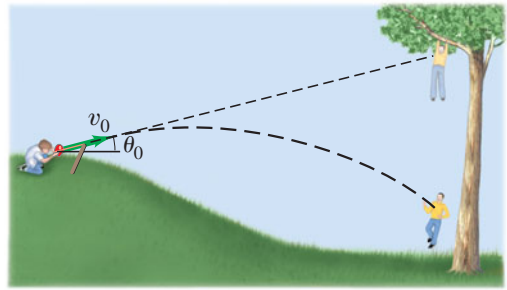


FIGURE 3–40 Problem 36.

37. (III) A stunt driver wants to make his car jump over 8 cars parked side by side below a horizontal ramp (Fig. 3–41). (a) With what minimum speed must he drive off the horizontal ramp? The vertical height of the ramp is 1.5 m above the cars and the horizontal distance he must clear is 22 m. (b) If the ramp is now tilted upward, so that “takeoff angle” is  $7.0^\circ$  above the horizontal, what is the new minimum speed?

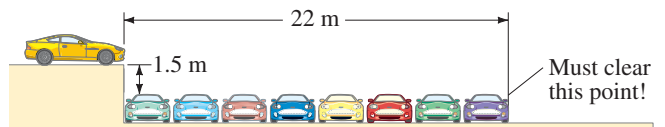


FIGURE 3–41 Problem 37.

### 3–8 Relative Velocity

38. (I) A person going for a morning jog on the deck of a cruise ship is running toward the bow (front) of the ship at 2.0 m/s while the ship is moving ahead at 8.5 m/s. What is the velocity of the jogger relative to the water? Later, the jogger is moving toward the stern (rear) of the ship. What is the jogger’s velocity relative to the water now?
39. (I) Huck Finn walks at a speed of 0.70 m/s across his raft (that is, he walks perpendicular to the raft’s motion relative to the shore). The heavy raft is traveling down the Mississippi River at a speed of 1.50 m/s relative to the river bank (Fig. 3–42). What is Huck’s velocity (speed and direction) relative to the river bank?

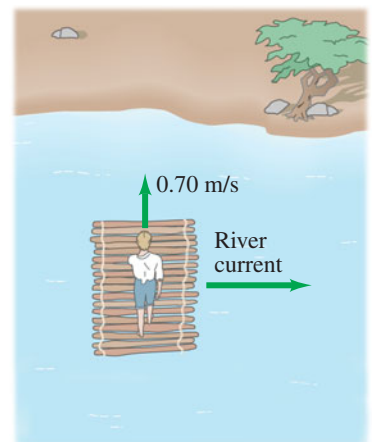


FIGURE 3–42 Problem 39.

40. (II) Determine the speed of the boat with respect to the shore in Example 3–10.

41. (II) Two planes approach each other head-on. Each has a speed of 780 km/h, and they spot each other when they are initially 10.0 km apart. How much time do the pilots have to take evasive action?
42. (II) A passenger on a boat moving at 1.70 m/s on a still lake walks up a flight of stairs at a speed of 0.60 m/s, Fig. 3–43. The stairs are angled at  $45^\circ$  pointing in the direction of motion as shown. What is the velocity of the passenger relative to the water?

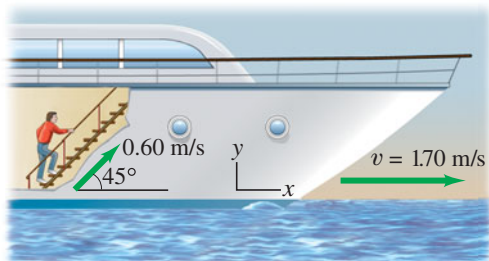


FIGURE 3–43 Problem 42.

43. (II) A person in the passenger basket of a hot-air balloon throws a ball horizontally outward from the basket with speed 10.0 m/s (Fig. 3–44). What initial velocity (magnitude and direction) does the ball have relative to a person standing on the ground (a) if the hot-air balloon is rising at 3.0 m/s relative to the ground during this throw, (b) if the hot-air balloon is descending at 3.0 m/s relative to the ground?



FIGURE 3–44 Problem 43.

44. (II) An airplane is heading due south at a speed of 688 km/h. If a wind begins blowing from the southwest at a speed of 90.0 km/h (average), calculate (a) the velocity (magnitude and direction) of the plane, relative to the ground, and (b) how far from its intended position it will be after 11.0 min if the pilot takes no corrective action. [Hint: First draw a diagram.]
45. (II) In what direction should the pilot aim the plane in Problem 44 so that it will fly due south?
46. (II) A swimmer is capable of swimming 0.60 m/s in still water. (a) If she aims her body directly across a 45-m-wide river whose current is 0.50 m/s, how far downstream (from a point opposite her starting point) will she land? (b) How long will it take her to reach the other side?
47. (II) (a) At what upstream angle must the swimmer in Problem 46 aim, if she is to arrive at a point directly across the stream? (b) How long will it take her?

48. (II) A boat, whose speed in still water is 2.50 m/s, must cross a 285-m-wide river and arrive at a point 118 m upstream from where it starts (Fig. 3–45). To do so, the pilot must head the boat at a  $45.0^\circ$  upstream angle. What is the speed of the river's current?

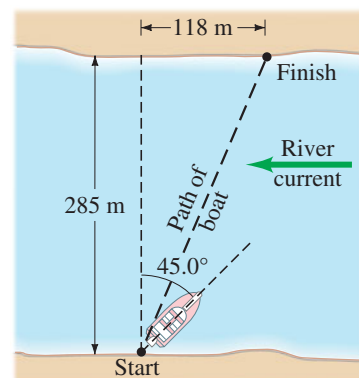


FIGURE 3–45 Problem 48.

49. (II) A child, who is 45 m from the bank of a river, is being carried helplessly downstream by the river's swift current of 1.0 m/s. As the child passes a lifeguard on the river's bank, the lifeguard starts swimming in a straight line (Fig. 3–46) until she reaches the child at a point downstream. If the lifeguard can swim at a speed of 2.0 m/s relative to the water, how long does it take her to reach the child? How far downstream does the lifeguard intercept the child?

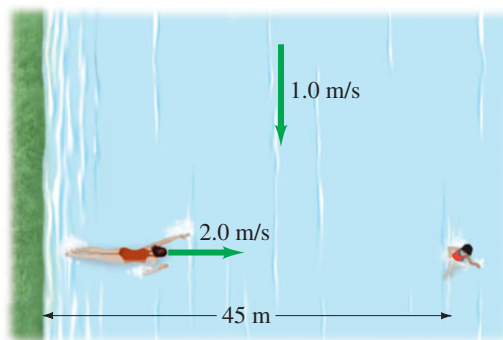


FIGURE 3–46 Problem 49.

50. (III) An airplane, whose air speed is 580 km/h, is supposed to fly in a straight path  $38.0^\circ$  N of E. But a steady 82 km/h wind is blowing from the north. In what direction should the plane head? [Hint: Use the law of sines, Appendix A–7.]
51. (III) Two cars approach a street corner at right angles to each other (Fig. 3–47). Car 1 travels at a speed relative to Earth  $v_{1E} = 35$  km/h, and car 2 at  $v_{2E} = 55$  km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

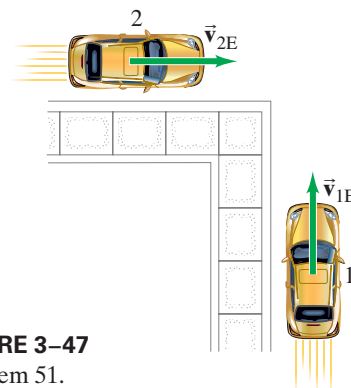


FIGURE 3–47 Problem 51.

## General Problems

52. Two vectors,  $\vec{V}_1$  and  $\vec{V}_2$ , add to a resultant  $\vec{V}_R = \vec{V}_1 + \vec{V}_2$ . Describe  $\vec{V}_1$  and  $\vec{V}_2$  if (a)  $V_R = V_1 + V_2$ , (b)  $V_R^2 = V_1^2 + V_2^2$ , (c)  $V_1 + V_2 = V_1 - V_2$ .
53. On mountainous downhill roads, escape routes are sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of  $26^\circ$ , calculate the horizontal and vertical components of the acceleration of a truck that slowed from 110 km/h to rest in 7.0 s. See Fig. 3–48.



FIGURE 3–48 Problem 53.

54. A light plane is headed due south with a speed relative to still air of 185 km/h. After 1.00 h, the pilot notices that they have covered only 135 km and their direction is not south but  $15.0^\circ$  east of south. What is the wind velocity?
55. An Olympic long jumper is capable of jumping 8.0 m. Assuming his horizontal speed is 9.1 m/s as he leaves the ground, how long is he in the air and how high does he go? Assume that he lands standing upright—that is, the same way he left the ground.
56. Romeo is throwing pebbles gently up to Juliet’s window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 8.5 m from the base of the wall (Fig. 3–49). How fast are the pebbles going when they hit her window?

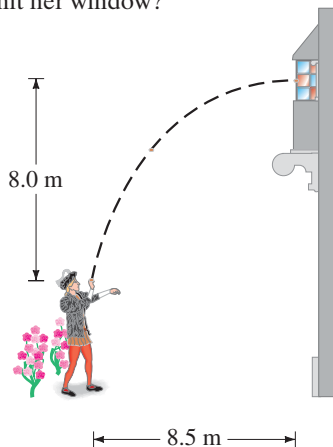


FIGURE 3–49 Problem 56.

57. *Apollo* astronauts took a “nine iron” to the Moon and hit a golf ball about 180 m. Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 32 m, estimate the acceleration due to gravity on the surface of the Moon. (We neglect air resistance in both cases, but on the Moon there is none.)

58. (a) A long jumper leaves the ground at  $45^\circ$  above the horizontal and lands 8.0 m away. What is her “takeoff” speed  $v_0$ ? (b) Now she is out on a hike and comes to the left bank of a river. There is no bridge and the right bank is 10.0 m away horizontally and 2.5 m vertically below. If she long jumps from the edge of the left bank at  $45^\circ$  with the speed calculated in (a), how long, or short, of the opposite bank will she land (Fig. 3–50)?

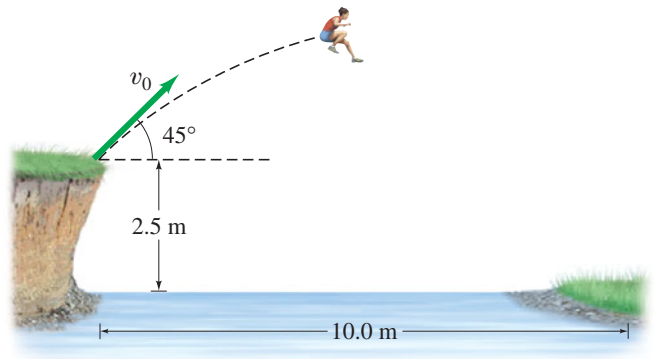


FIGURE 3–50 Problem 58.

59. A projectile is shot from the edge of a cliff 115 m above ground level with an initial speed of 65.0 m/s at an angle of  $35.0^\circ$  with the horizontal, as shown in Fig. 3–51. (a) Determine the time taken by the projectile to hit point P at ground level. (b) Determine the distance  $X$  of point P from the base of the vertical cliff. At the instant just before the projectile hits point P, find (c) the horizontal and the vertical components of its velocity, (d) the magnitude of the velocity, and (e) the angle made by the velocity vector with the horizontal. (f) Find the maximum height above the cliff top reached by the projectile.

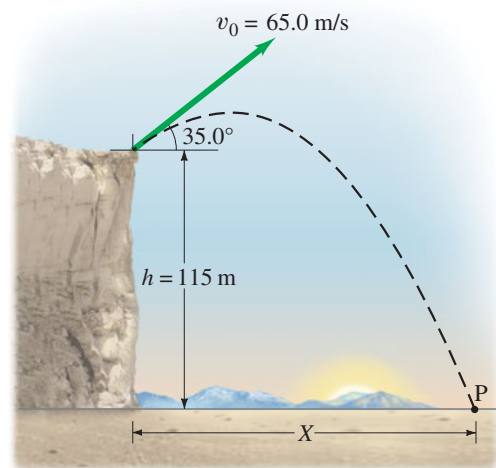


FIGURE 3–51 Problem 59.

60. William Tell must split the apple on top of his son’s head from a distance of 27 m. When William aims directly at the apple, the arrow is horizontal. At what angle should he aim the arrow to hit the apple if the arrow travels at a speed of 35 m/s?



61. Raindrops make an angle  $\theta$  with the vertical when viewed through a moving train window (Fig. 3–52). If the speed of the train is  $v_T$ , what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?



FIGURE 3–52  
Problem 61.

62. A car moving at 95 km/h passes a 1.00-km-long train traveling in the same direction on a track that is parallel to the road. If the speed of the train is 75 km/h, how long does it take the car to pass the train, and how far will the car have traveled in this time? What are the results if the car and train are instead traveling in opposite directions?
63. A hunter aims directly at a target (on the same level) 38.0 m away. (a) If the arrow leaves the bow at a speed of 23.1 m/s, by how much will it miss the target? (b) At what angle should the bow be aimed so the target will be hit?
64. The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 3–53. What minimum pushoff speed is necessary to clear the rocks? How long are they in the air?

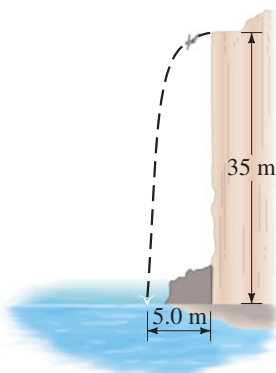


FIGURE 3–53  
Problem 64.

65. When Babe Ruth hit a homer over the 8.0-m-high right-field fence 98 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a  $36^\circ$  angle with the ground.
66. At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is “launched” from a height of 2.50 m? Where will the ball land if it just clears the net (and will it be “good” in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 3–54.

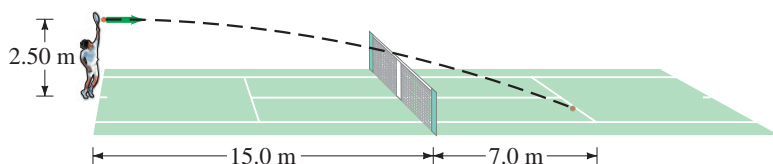


FIGURE 3–54 Problem 66.

67. Spymaster Chris, flying a constant 208 km/h horizontally in a low-flying helicopter, wants to drop secret documents into her contact’s open car which is traveling 156 km/h on a level highway 78.0 m below. At what angle (with the horizontal) should the car be in her sights when the packet is released (Fig. 3–55)?

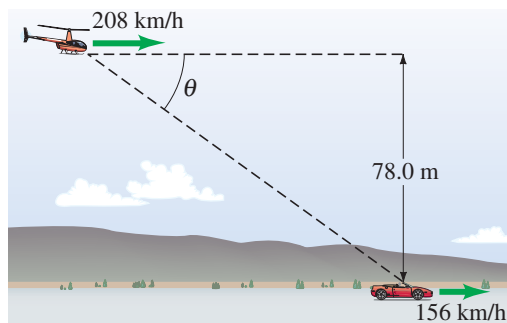


FIGURE 3–55 Problem 67.

68. A basketball leaves a player’s hands at a height of 2.10 m above the floor. The basket is 3.05 m above the floor. The player likes to shoot the ball at a  $38.0^\circ$  angle. If the shot is made from a horizontal distance of 11.00 m and must be accurate to  $\pm 0.22$  m (horizontally), what is the range of initial speeds allowed to make the basket?
69. A boat can travel 2.20 m/s in still water. (a) If the boat points directly across a stream whose current is 1.20 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (b) What will be the position of the boat, relative to its point of origin, after 3.00 s?
70. A projectile is launched from ground level to the top of a cliff which is 195 m away and 135 m high (see Fig. 3–56). If the projectile lands on top of the cliff 6.6 s after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

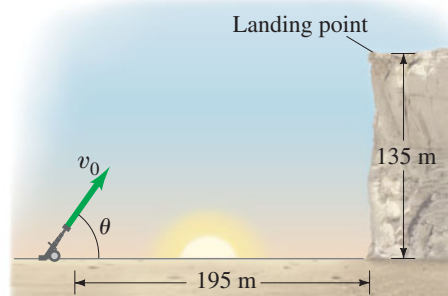
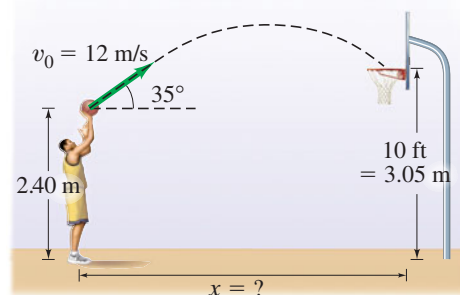


FIGURE 3–56  
Problem 70.

71. A basketball is shot from an initial height of 2.40 m (Fig. 3–57) with an initial speed  $v_0 = 12$  m/s directed at an angle  $\theta_0 = 35^\circ$  above the horizontal. (a) How far from the basket was the player if he made a basket? (b) At what angle to the horizontal did the ball enter the basket?

FIGURE 3–57  
Problem 71.



72. A rock is kicked horizontally at 15 m/s from a hill with a  $45^\circ$  slope (Fig. 3–58). How long does it take for the rock to hit the ground?

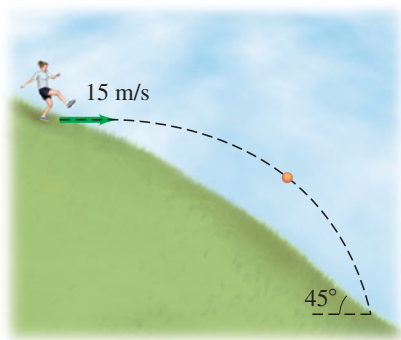


FIGURE 3–58 Problem 72.

73. A batter hits a fly ball which leaves the bat 0.90 m above the ground at an angle of  $61^\circ$  with an initial speed of 28 m/s heading toward centerfield. Ignore air resistance. (a) How far from home plate would the ball land if not caught? (b) The ball is caught by the centerfielder who, starting at a distance of 105 m from home plate just as the ball was hit, runs straight toward home plate at a constant speed and makes the catch at ground level. Find his speed.
74. A ball is shot from the top of a building with an initial velocity of 18 m/s at an angle  $\theta = 42^\circ$  above the horizontal. (a) What are the horizontal and vertical components of the initial velocity? (b) If a nearby building is the same height and 55 m away, how far below the top of the building will the ball strike the nearby building?
75. If a baseball pitch leaves the pitcher's hand horizontally at a velocity of 150 km/h, by what % will the pull of gravity change the magnitude of the velocity when the ball reaches the batter, 18 m away? For this estimate, ignore air resistance and spin on the ball.

## Search and Learn

1. Here is something to try at a sporting event. Show that the maximum height  $h$  attained by an object projected into the air, such as a baseball, football, or soccer ball, is approximately given by

$$h \approx 1.2t^2 \text{ m,}$$

where  $t$  is the total time of flight for the object in seconds. Assume that the object returns to the same level as that from which it was launched, as in Fig. 3–59. For example, if you count to find that a baseball was in the air for  $t = 5.0$  s, the maximum height attained was  $h = 1.2 \times (5.0)^2 = 30$  m. The fun of this relation is that  $h$  can be determined without knowledge of the launch speed  $v_0$  or launch angle  $\theta_0$ . Why is that exactly? See Section 3–6.

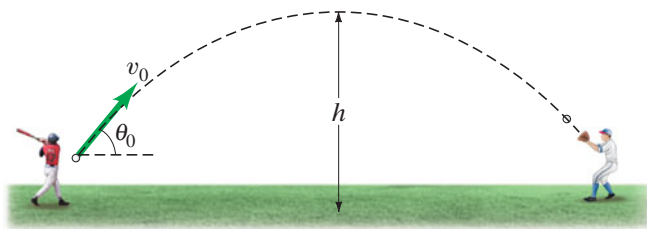


FIGURE 3–59 Search and Learn 1.

2. Two balls are thrown in the air at different angles, but each reaches the same height. Which ball remains in the air longer? Explain, using equations.
3. Show that the speed with which a projectile leaves the ground is equal to its speed just before it strikes the ground at the end of its journey, assuming the firing level equals the landing level.
4. The initial angle of projectile A is  $30^\circ$ , while that of projectile B is  $60^\circ$ . Both have the same level horizontal range. How do the initial velocities and flight times (elapsed time from launch until landing) compare for A and B?
5. You are driving south on a highway at 12 m/s (approximately 25 mi/h) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of  $7.0^\circ$  to the horizontal. Estimate the speed of the vertically falling snowflakes relative to the ground. [Hint: Construct a relative velocity diagram similar to Fig. 3–29 or 3–30. Be careful about which angle is the angle given.]

## ANSWERS TO EXERCISES

**A:**  $3.0\sqrt{2} \approx 4.2$  units.

**B:** (a).

**C:** They hit at the same time.

**D:** (i) Nowhere; (ii) at the highest point; (iii) nowhere.

**E:** (d). It provides the initial velocity of the box.

**F:** (a)  $v = v_{x0} = 16.0$  m/s, horizontal; (b)  $9.80$  m/s<sup>2</sup> down.