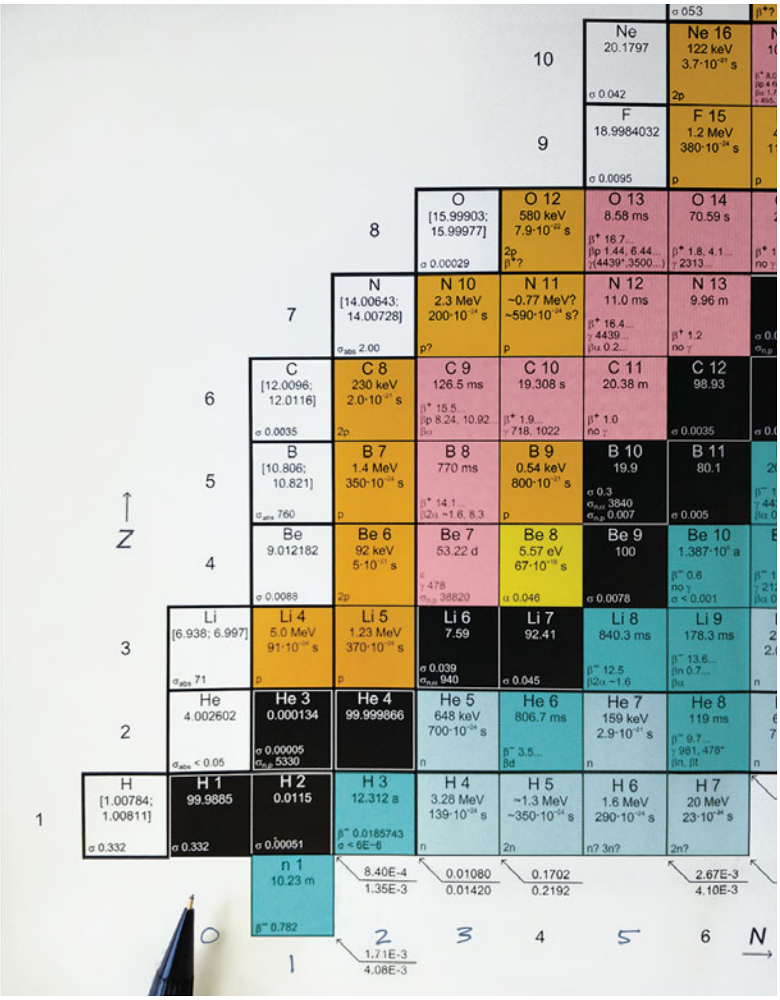


In this Chapter we begin our discussion of nuclear physics. We study the properties of nuclei, the various forms of radioactivity, and how radioactive decay can be used in a variety of fields to determine the age of old objects, from bones and trees to rocks and other mineral substances, and obtain information on the history of the Earth.

Shown is one version of a **Chart of the Nuclides**. Each horizontal row has a square for each known isotope (nuclide) of one element with a particular Z value (= number of electrons in the neutral atom = number of protons in the nucleus). At the far left is a white box with the average atomic weight (or a range if uncertain) of the naturally occurring isotopes of that element. Each vertical column contains nuclides with the same neutron number N. For $N = 1$ (to right of pencil), starting at the bottom, there is a lone neutron, then above it $^2_1$H, then $^3_2$He and $^4_3$Li. Each square is color coded: black means a stable nuclide. Radioactive nuclides are blue green for β^- decay, pink for β^+ decay or electron capture (ε) such as $^7_4$Be, yellow for α decay, and so on. Thus $^1_1$H and $^2_1$H are stable but $^3_1$H (tritium) undergoes β^- decay with half-life = 12.3 years ("a" is for Latin "anno" = year). The squares contain the atomic mass of that isotope, or half-life and energy released if radioactive. Other details may be alternate decay modes and certain cross sections (σ).

CHAPTER 30

Nuclear Physics and Radioactivity

CHAPTER-OPENING QUESTION—Guess now!

If half of an 80-μg sample of $^{60}_{27}$Co decays in 5.3 years, how much $^{60}_{27}$Co is left in 10.6 years?

(a) 10 μg.
(b) 20 μg.
(c) 30 μg.
(d) 40 μg.
(e) 0 μg.

In the early part of the twentieth century, Rutherford's experiments (Section 27–10) led to the idea that at the center of an atom there is a tiny but massive nucleus with a positive charge. At the same time that the quantum theory was being developed and scientists were attempting to understand the structure of the atom and its electrons, investigations into the nucleus itself had also begun. In this Chapter and the next, we take a brief look at *nuclear physics*.

CONTENTS

30–1 Structure and Properties of the Nucleus

An important question for physicists was whether the nucleus had a structure, and what that structure might be. By the early 1930s, a model of the nucleus had been developed that is still useful. According to this model, a nucleus is made up of two types of particles: protons and neutrons. [These "particles" also have wave properties, but for ease of visualization and language, we usually refer to them simply as "particles."] A **proton** is the nucleus of the simplest atom, hydrogen. The proton has a positive charge ($= +e = +1.60 \times 10^{-19}$ C, the same magnitude as for the electron) and its mass is measured to be

$$m_p = 1.67262 \times 10^{-27}\ \text{kg}.$$

The **neutron**, whose existence was ascertained in 1932 by the English physicist James Chadwick (1891–1974), is electrically neutral $(q = 0)$, as its name implies. Its mass is very slightly larger than that of the proton:

$$m_n = 1.67493 \times 10^{-27}\ \text{kg}.$$

These two constituents of a nucleus, neutrons and protons, are referred to collectively as **nucleons**.

Although a normal hydrogen nucleus consists of a single proton alone, the nuclei of all other elements consist of both neutrons and protons. The different nuclei are often referred to as **nuclides**. The number of protons in a nucleus (or nuclide) is called the **atomic number** and is designated by the symbol Z. The total number of nucleons, neutrons plus protons, is designated by the symbol A and is called the **atomic mass number**, or sometimes simply **mass number**. This name is used since the mass of a nucleus is very closely A times the mass of one nucleon. A nuclide with 7 protons and 8 neutrons thus has $Z = 7$ and $A = 15$. The **neutron number** N is $N = A - Z$.

To specify a given nuclide, we need give only A and Z. A special symbol is commonly used which takes the form

$$^{A}_{Z}\text{X},$$

where X is the chemical symbol for the element (see Appendix B, and the Periodic Table inside the back cover), A is the atomic mass number, and Z is the atomic number. For example, $^{15}_{7}\text{N}$ means a nitrogen nucleus containing 7 protons and 8 neutrons for a total of 15 nucleons. In a neutral atom, the number of electrons orbiting the nucleus is equal to the atomic number Z (since the charge on an electron has the same magnitude but opposite sign to that of a proton). The main properties of an atom, and how it interacts with other atoms, are largely determined by the number of electrons. Hence Z determines what kind of atom it is: carbon, oxygen, gold, or whatever. It is redundant to specify both the symbol of a nucleus and its atomic number Z as described above. If the nucleus is nitrogen, for example, we know immediately that $Z = 7$. The subscript Z is thus sometimes dropped and $^{15}_{7}\text{N}$ is then written simply ^{15}N; in words we say "nitrogen fifteen."

For a particular type of atom (say, carbon), nuclei are found to contain different numbers of neutrons, although they all have the same number of protons. For example, carbon nuclei always have 6 protons, but they may have 5, 6, 7, 8, 9, or 10 neutrons. Nuclei that contain the same number of protons but different numbers of neutrons are called **isotopes**. Thus, $^{11}_{6}\text{C}$, $^{12}_{6}\text{C}$, $^{13}_{6}\text{C}$, $^{14}_{6}\text{C}$, $^{15}_{6}\text{C}$, and $^{16}_{6}\text{C}$ are all isotopes of carbon. The isotopes of a given element are not all equally common. For example, 98.9% of naturally occurring carbon (on Earth) is the isotope $^{12}_{6}\text{C}$, and about 1.1% is $^{13}_{6}\text{C}$. These percentages are referred to as the **natural abundances**.† Even hydrogen has isotopes: 99.99% of natural hydrogen is $^{1}_{1}\text{H}$, a simple proton, as the nucleus; there are also $^{2}_{1}\text{H}$, called **deuterium**, and $^{3}_{1}\text{H}$, **tritium**, which besides the proton contain 1 or 2 neutrons. (The bare nucleus in each case is called the **deuteron** and **triton**.)

†The mass value for each element as given in the Periodic Table (inside back cover) is an average weighted according to the natural abundances of its isotopes.

Many isotopes that do not occur naturally can be produced in the laboratory by means of nuclear reactions (more on this later). Indeed, all elements beyond uranium $(Z > 92)$ do not occur naturally on Earth and are only produced artificially (in the laboratory), as are many nuclides with $Z \leq 92$.

The approximate size of nuclei was determined originally by Rutherford from the scattering of charged particles by thin metal foils. We cannot speak about a definite size for nuclei because of the wave–particle duality (Section 27–7): their spatial extent must remain somewhat fuzzy. Nonetheless a rough "size" can be measured by scattering high-speed electrons off nuclei. It is found that nuclei have a roughly spherical shape with a radius that increases with A according to the approximate formula

$$r \approx (1.2 \times 10^{-15}\,\text{m})(A^{\frac{1}{3}}). \tag{30–1}$$

Since the volume of a sphere is $V = \frac{4}{3}\pi r^3$, we see that the volume of a nucleus is approximately proportional to the number of nucleons, $V \propto A$ (because $(A^{\frac{1}{3}})^3 = A$). This is what we would expect if nucleons were like impenetrable billiard balls: if you double the number of balls, you double the total volume. Hence, all nuclei have nearly the same density, and it is enormous (see Example 30–2).

The metric abbreviation for 10^{-15} m is the fermi (after Enrico Fermi, Fig. 30–7) or the femtometer, fm (see Table 1–4 or inside the front cover). Thus $1.2 \times 10^{-15}\,\text{m} = 1.2\,\text{fm}$ or 1.2 fermis.

EXAMPLE 30–1 ESTIMATE **Nuclear sizes.** Estimate the diameter of the smallest and largest naturally occurring nuclei: (*a*) $^1_1$H, (*b*) $^{238}_{92}$U.

APPROACH The radius r of a nucleus is related to its number of nucleons A by Eq. 30–1. The diameter $d = 2r$.

SOLUTION (*a*) For hydrogen, $A = 1$, Eq. 30–1 gives

$$d = \text{diameter} = 2r \approx 2(1.2 \times 10^{-15}\,\text{m})(A^{\frac{1}{3}}) = 2.4 \times 10^{-15}\,\text{m}$$

since $A^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$.

(*b*) For uranium $d \approx (2.4 \times 10^{-15}\,\text{m})(238)^{\frac{1}{3}} = 15 \times 10^{-15}\,\text{m}$.

The range of nuclear diameters is only from 2.4 fm to 15 fm.

NOTE Because nuclear radii vary as $A^{\frac{1}{3}}$, the largest nuclei (such as uranium with $A = 238$) have a radius only about $\sqrt[3]{238} \approx 6$ times that of the smallest, hydrogen $(A = 1)$.

EXAMPLE 30–2 ESTIMATE **Nuclear and atomic densities.** Compare the density of nuclear matter to the density of normal solids.

APPROACH The density of normal liquids and solids is on the order of 10^3 to $10^4\,\text{kg/m}^3$ (see Table 10–1), and because the atoms are close packed, atoms have about this density too. We therefore compare the density (mass per volume) of a nucleus to that of its atom as a whole.

SOLUTION The mass of a proton is greater than the mass of an electron by a factor

$$\frac{1.67 \times 10^{-27}\,\text{kg}}{9.1 \times 10^{-31}\,\text{kg}} \approx 2000.$$

Thus, over 99.9% of the mass of an atom is in the nucleus, and for our estimate we can say the mass of the atom equals the mass of the nucleus, $m_{\text{nucl}}/m_{\text{atom}} = 1$. Atoms have a radius of about 10^{-10} m (Chapter 27) and nuclei on the order of 10^{-15} m (Eq. 30–1). Thus the ratio of nuclear density to atomic density is about

$$\frac{\rho_{\text{nucl}}}{\rho_{\text{atom}}} = \frac{(m_{\text{nucl}}/V_{\text{nucl}})}{(m_{\text{atom}}/V_{\text{atom}})} = \left(\frac{m_{\text{nucl}}}{m_{\text{atom}}}\right)\frac{\frac{4}{3}\pi r^3_{\text{atom}}}{\frac{4}{3}\pi r^3_{\text{nucl}}} \approx (1)\frac{(10^{-10})^3}{(10^{-15})^3} = 10^{15}.$$

The nucleus is 10^{15} times more dense than ordinary matter.

The masses of nuclei can be determined from the radius of curvature of fast-moving nuclei (as ions) in a known magnetic field using a mass spectrometer, as discussed in Section 20–11. Indeed the existence of different isotopes of the same element (different number of neutrons) was discovered using this device.

Nuclear masses can be specified in **unified atomic mass units** (u). On this scale, a neutral $^{12}_{6}C$ atom is given the exact value 12.000000 u. A neutron then has a measured mass of 1.008665 u, a proton 1.007276 u, and a neutral hydrogen atom $^{1}_{1}H$ (proton plus electron) 1.007825 u. The masses of many nuclides are given in Appendix B. It should be noted that the masses in this Table, as is customary, are for the *neutral atom* (including electrons), and not for a bare nucleus.

CAUTION
Masses are for neutral atom (nucleus plus electrons)

Masses may be specified using the electron-volt energy unit, $1\text{ eV} = 1.6022\times10^{-19}\text{ J}$ (Section 17–4). This can be done because mass and energy are related, and the precise relationship is given by Einstein's equation $E = mc^2$ (Chapter 26). Since the mass of a proton is $1.67262 \times 10^{-27}\text{ kg}$, or 1.007276 u, then 1 u is equal to

$$1.0000\text{ u} = (1.0000\text{ u})\left(\frac{1.67262\times10^{-27}\text{ kg}}{1.007276\text{ u}}\right) = 1.66054\times10^{-27}\text{ kg};$$

this is equivalent to an energy (see Table inside front cover) in MeV $(= 10^6\text{ eV})$ of

$$E = mc^2 = \frac{(1.66054\times10^{-27}\text{ kg})(2.9979\times10^8\text{ m/s})^2}{(1.6022\times10^{-19}\text{ J/eV})} = 931.5\text{ MeV}.$$

Thus,

$$1\text{ u} = 1.6605\times10^{-27}\text{ kg} = 931.5\text{ MeV}/c^2.$$

The rest masses of some of the basic particles are given in Table 30–1. As a rule of thumb, to remember, the masses of neutron and proton are about $1\text{ GeV}/c^2$ $(= 1000\text{ MeV}/c^2)$ which is about 2000 times the mass of an electron $(\approx \frac{1}{2}\text{MeV}/c^2)$.

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

	Mass		
Object	**kg**	**u**	**MeV/c^2**
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^{1}_{1}H$ atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Just as an electron has intrinsic spin and angular momentum quantum numbers, so too do nuclei and their constituents, the proton and neutron. Both the proton and the neutron are spin $\frac{1}{2}$ particles, just like the electron. A nucleus, made up of protons and neutrons, has a **nuclear spin** quantum number, I, that can be either integer or half integer, depending on whether it is made up of an even or an odd number of nucleons.

30–2 Binding Energy and Nuclear Forces

Binding Energies

The total mass of a stable nucleus is always less than the sum of the masses of its separate protons and neutrons, as the following Example shows.

EXAMPLE 30–3 **$^{4}_{2}He$ mass compared to its constituents.** Compare the mass of a $^{4}_{2}He$ atom to the total mass of its constituent particles.

PROBLEM SOLVING
Keep track of electron masses

APPROACH The $^{4}_{2}He$ nucleus contains 2 protons and 2 neutrons. Tables normally give the masses of neutral atoms—that is, nucleus plus its Z electrons. We must therefore be sure to balance out the electrons when we compare masses. Thus we use the mass of $^{1}_{1}H$ rather than that of a proton alone. We look up the mass of the $^{4}_{2}He$ atom in Appendix B (it includes the mass of 2 electrons), as well as the mass for the 2 neutrons and 2 hydrogen atoms $(= 2\text{ protons} + 2\text{ electrons})$.

SOLUTION The mass of a neutral $^4_2$He atom, from Appendix B, is 4.002603 u. The mass of two neutrons and two H atoms (2 protons including the 2 electrons) is

$$2m_n = 2(1.008665\text{ u}) = 2.017330\text{ u}$$
$$2m(^1_1\text{H}) = 2(1.007825\text{ u}) = \underline{2.015650\text{ u}}$$
$$\text{sum} = 4.032980\text{ u}.$$

Thus the mass of $^4_2$He is measured to be less than the masses of its constituents by an amount $4.032980\text{ u} - 4.002603\text{ u} = 0.030377\text{ u}$.

Where has this lost mass of 0.030377 u disappeared to? It must be $E = mc^2$.

If the four nucleons suddenly came together to form a $^4_2$He nucleus, the mass "loss" would appear as energy of another kind (such as radiation, or kinetic energy). The mass (or energy) difference in the case of $^4_2$He, given in energy units, is $(0.030377\text{ u})(931.5\text{ MeV/u}) = 28.30\text{ MeV}$. This difference is referred to as the **total binding energy** of the nucleus. The total binding energy represents the amount of energy that must be put *into* a nucleus in order to break it apart into its constituents. If the mass of, say, a $^4_2$He nucleus were exactly equal to the mass of two neutrons plus two protons, the nucleus could fall apart without any input of energy. To be stable, the mass of a nucleus *must* be less than that of its constituent nucleons, so that energy input *is* needed to break it apart.

CAUTION
Mass of nucleus must be less *than mass of constituents*

Binding energy is not something a nucleus has—it is energy it "lacks" relative to the total mass of its separate constituents.

[As a comparison, we saw in Chapter 27 that the binding energy of the one electron in the hydrogen atom is 13.6 eV; so the mass of a $^1_1$H atom is less than that of a single proton plus a single electron by $13.6\text{ eV}/c^2$. The binding energies of nuclei are on the order of MeV, so the eV binding energies of electrons can be ignored. Nuclear binding energies, compared to nuclear masses, are on the order of $(28\text{ MeV}/4000\text{ MeV}) \approx 1 \times 10^{-2}$, where we used helium's binding energy of 28.3 MeV (see above) and mass $\approx 4 \times 940\text{ MeV} \approx 4000\text{ MeV}$.]

EXERCISE A Determine how much less the mass of the $^7_3$Li nucleus is compared to that of its constituents. See Appendix B.

The **binding energy per nucleon** is defined as the total binding energy of a nucleus divided by A, the total number of nucleons. We calculated above that the binding energy of $^4_2$He is 28.3 MeV, so its binding energy per nucleon is $28.3\text{ MeV}/4 = 7.1\text{ MeV}$. Figure 30–1 shows the measured binding energy per nucleon as a function of A for stable nuclei. The curve rises as A increases and reaches a plateau at about 8.7 MeV per nucleon above $A \approx 40$. Beyond $A \approx 80$, the curve decreases slowly, indicating that larger nuclei are held together less tightly than those in the middle of the Periodic Table. We will see later that these characteristics allow the release of nuclear energy in the processes of fission and fusion.

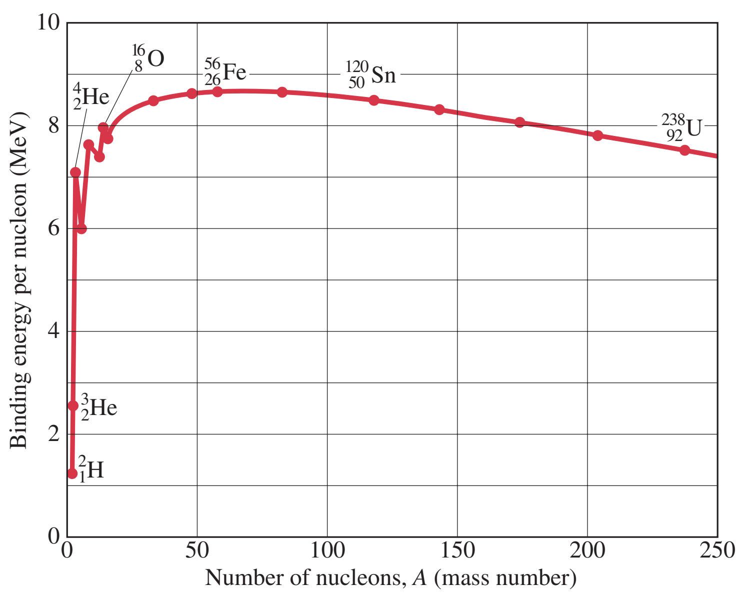


FIGURE 30–1 Binding energy per nucleon for the more stable nuclides as a function of mass number A.

EXAMPLE 30–4 **Binding energy for iron.** Calculate the total binding energy and the binding energy per nucleon for ${}^{56}_{26}\text{Fe}$, the most common stable isotope of iron.

APPROACH We subtract the mass of a ${}^{56}_{26}\text{Fe}$ atom from the total mass of 26 hydrogen atoms and 30 neutrons, all found in Appendix B. Then we convert mass units to energy units; finally we divide by $A = 56$, the total number of nucleons.

SOLUTION ${}^{56}_{26}\text{Fe}$ has 26 protons and 30 neutrons whose separate masses are

$$\begin{aligned} 26m({}^{1}_{1}\text{H}) &= (26)(1.007825\,\text{u}) = 26.20345\,\text{u} \quad \text{(includes 26 electrons)} \\ 30m_\text{n} &= (30)(1.008665\,\text{u}) = \underline{30.25995\,\text{u}} \\ \text{sum} &= 56.46340\,\text{u}. \\ \text{Subtract mass of } {}^{56}_{26}\text{Fe:} &= \underline{-55.93494\,\text{u}} \quad \text{(Appendix B)} \\ \Delta m &= 0.52846\,\text{u}. \end{aligned}$$

The total binding energy is thus

$$(0.52846\,\text{u})(931.5\,\text{MeV/u}) = 492.26\,\text{MeV}$$

and the binding energy per nucleon is

$$\frac{492.26\,\text{MeV}}{56\text{ nucleons}} = 8.79\,\text{MeV}.$$

NOTE The binding energy per nucleon graph (Fig. 30–1) peaks about here, for iron. So the iron nucleus, and its neighbors, are the most stable of nuclei.

EXERCISE B Determine the binding energy per nucleon for ${}^{16}_{8}\text{O}$.

EXAMPLE 30–5 **Binding energy of last neutron.** What is the binding energy of the last neutron in ${}^{13}_{6}\text{C}$?

APPROACH If ${}^{13}_{6}\text{C}$ lost one neutron, it would be ${}^{12}_{6}\text{C}$. We subtract the mass of ${}^{13}_{6}\text{C}$ from the masses of ${}^{12}_{6}\text{C}$ and a free neutron.

SOLUTION Obtaining the masses from Appendix B, we have

$$\begin{aligned} \text{Mass } {}^{12}_{6}\text{C} &= 12.000000\,\text{u} \\ \text{Mass } {}^{1}_{0}\text{n} &= \underline{1.008665\,\text{u}} \\ \text{Total} &= 13.008665\,\text{u}. \\ \text{Subtract mass of } {}^{13}_{6}\text{C:} & \quad \underline{-13.003355\,\text{u}} \\ \Delta m &= 0.005310\,\text{u} \end{aligned}$$

which in energy is $(931.5\,\text{MeV/u})(0.005310\,\text{u}) = 4.95\,\text{MeV}$. That is, it would require 4.95 MeV input of energy to remove one neutron from ${}^{13}_{6}\text{C}$.

Nuclear Forces

We can analyze nuclei not only from the point of view of energy, but also from the point of view of the forces that hold them together. We might not expect a collection of protons and neutrons to come together spontaneously, since protons are all positively charged and thus exert repulsive electric forces on each other. Since stable nuclei *do* stay together, another force must be acting. This new force has to be stronger than the electric force in order to hold the nucleus together, and is called the **strong nuclear force**. The strong nuclear force acts as an attractive force between all nucleons, protons and neutrons alike. Thus protons attract each other via the strong nuclear force at the same time they repel each other via the electric force. Neutrons, because they are electrically neutral, only attract other neutrons or protons via the strong nuclear force.

The strong nuclear force turns out to be far more complicated than the gravitational and electromagnetic forces. One important aspect of the strong nuclear force is that it is a **short-range** force: it acts only over a very short distance.

It is very strong between two nucleons if they are less than about 10^{-15} m apart, but it is essentially zero if they are separated by a distance greater than this. Compare this to electric and gravitational forces, which decrease as $1/r^2$ but continue acting over any distances and are therefore called **long-range** forces.

The strong nuclear force has some strange features. For example, if a nuclide contains too many or too few neutrons relative to the number of protons, the binding of the nucleons is reduced; nuclides that are too unbalanced in this regard are unstable. As shown in Fig. 30–2, stable nuclei tend to have the same number of protons as neutrons $(N = Z)$ up to about $A = 30$. Beyond this, stable nuclei contain more neutrons than protons. This makes sense since, as Z increases, the electrical repulsion increases, so a greater number of neutrons—which exert only the attractive strong nuclear force—are required to maintain stability. For very large Z, no number of neutrons can overcome the greatly increased electric repulsion. Indeed, there are no completely stable nuclides above $Z = 82$.

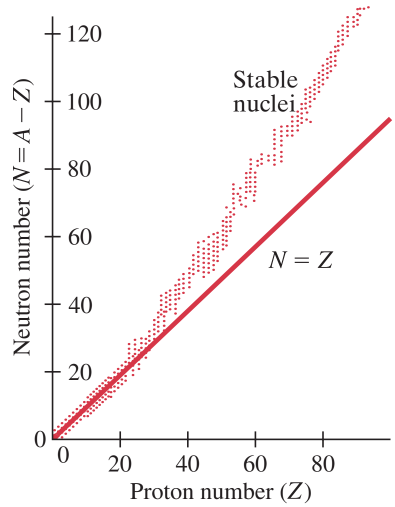


FIGURE 30–2 Number of neutrons versus number of protons for stable nuclides, which are represented by dots. The straight line represents $N = Z$.

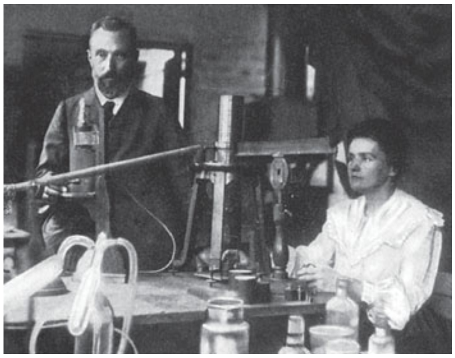
What we mean by a *stable nucleus* is one that stays together indefinitely. What then is an *unstable nucleus*? It is one that comes apart; and this results in radioactive decay. Before we discuss the important subject of radioactivity (next Section), we note that there is a second type of nuclear force that is much weaker than the strong nuclear force. It is called the **weak nuclear force**, and we are aware of its existence only because it shows itself in certain types of radioactive decay. These two nuclear forces, the strong and the weak, together with the gravitational and electromagnetic forces, comprise the four fundamental types of force in nature.

30–3 Radioactivity

Nuclear physics had its beginnings in 1896. In that year, Henri Becquerel (1852–1908) made an important discovery: in his studies of phosphorescence, he found that a certain mineral (which happened to contain uranium) would darken a photographic plate even when the plate was wrapped to exclude light. It was clear that the mineral emitted some new kind of radiation that, unlike X-rays (Section 25–11), occurred without any external stimulus. This new phenomenon eventually came to be called **radioactivity**.

Soon after Becquerel's discovery, Marie Curie (1867–1934) and her husband, Pierre Curie (1859–1906), isolated two previously unknown elements that were very highly radioactive (Fig. 30–3). These were named polonium and radium. Other radioactive elements were soon discovered as well. The radioactivity was found in every case to be unaffected by the strongest physical and chemical treatments, including strong heating or cooling or the action of strong chemicals. It was suspected that the source of radioactivity must be deep within the atom, coming from the nucleus. It became apparent that radioactivity is the result of the **disintegration** or **decay** of an unstable nucleus. Certain isotopes are not stable, and they decay with the emission of some type of radiation or "rays."

FIGURE 30–3 Marie and Pierre Curie in their laboratory (about 1906) where radium was discovered.



Many unstable isotopes occur in nature, and such radioactivity is called "natural radioactivity." Other unstable isotopes can be produced in the laboratory by nuclear reactions (Section 31–1); these are said to be produced "artificially" and to have "artificial radioactivity." Radioactive isotopes are sometimes referred to as **radioisotopes** or **radionuclides**.

Rutherford and others began studying the nature of the rays emitted in radioactivity about 1898. They classified the rays into three distinct types according to their penetrating power. One type of radiation could barely penetrate a piece of paper. The second type could pass through as much as 3 mm of aluminum. The third was extremely penetrating: it could pass through several centimeters of lead and still be detected on the other side. They named these three types of radiation alpha (α), beta (β), and gamma (γ), respectively, after the first three letters of the Greek alphabet.

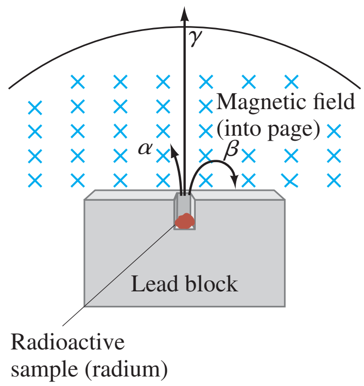


FIGURE 30–4 Alpha and beta rays are bent in opposite directions by a magnetic field, whereas gamma rays are not bent at all.

Each type of ray was found to have a different charge and hence is bent differently in a magnetic field, Fig. 30–4; α rays are positively charged, β rays are negatively charged, and γ rays are neutral. It was soon found that all three types of radiation consisted of familiar kinds of particles. Gamma rays are very high-energy *photons* whose energy is even higher than that of X-rays. Beta rays were found to be identical to *electrons* that orbit the nucleus, but they are created within the nucleus itself. Alpha rays (or α particles) are simply the nuclei of *helium* atoms, ${}^{4}_{2}\text{He}$; that is, an α ray consists of two protons and two neutrons bound together.

We now discuss each of these three types of radioactivity, or decay, in more detail.

30–4 Alpha Decay

Experiments show that when nuclei decay, the number of nucleons (= mass number A) is conserved, as well as electric charge (= Ze). When a nucleus emits an α particle (${}^{4}_{2}\text{He}$), the remaining nucleus will be different from the original: it has lost two protons and two neutrons. Radium 226 (${}^{226}_{88}\text{Ra}$), for example, is an α emitter. It decays to a nucleus with $Z = 88 - 2 = 86$ and $A = 226 - 4 = 222$. The nucleus with $Z = 86$ is radon (Rn)—see Appendix B or the Periodic Table. Thus radium decays to radon with the emission of an α particle. This is written

$$ {}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}. $$

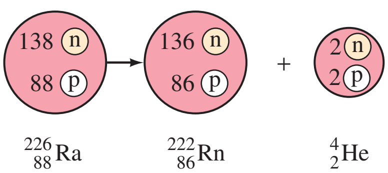


FIGURE 30–5 Radioactive decay of radium to radon with emission of an alpha particle.

See Fig. 30–5.

When α decay occurs, a different element is formed. The **daughter** nucleus (${}^{222}_{86}\text{Rn}$ in this case) is different from the **parent** nucleus (${}^{226}_{88}\text{Ra}$ in this case). This changing of one element into another is called **transmutation** of the elements.

Alpha decay can be written in general as

$$ {}^{A}_{Z}\text{N} \rightarrow {}^{A-4}_{Z-2}\text{N}' + {}^{4}_{2}\text{He} \qquad [\alpha \text{ decay}] $$

where N is the parent, N' the daughter, and Z and A are the atomic number and atomic mass number, respectively, of the parent.

EXERCISE C ${}^{154}_{66}\text{Dy}$ decays by α emission to what element? (*a*) Pb, (*b*) Gd, (*c*) Sm, (*d*) Er, (*e*) Yb.

Alpha decay occurs because the strong nuclear force is unable to hold very large nuclei together. The nuclear force is a short-range force: it acts only between neighboring nucleons. But the electric force acts all the way across a large nucleus. For very large nuclei, the large Z means the repulsive electric force becomes so large (Coulomb's law) that the strong nuclear force is unable to hold the nucleus together.

We can express the instability of the parent nucleus in terms of energy (or mass): the mass of the parent nucleus is greater than the mass of the daughter nucleus plus the mass of the α particle. The mass difference appears as kinetic energy, which is carried away by the α particle and the recoiling daughter nucleus. The total energy released is called the **disintegration energy**, Q, or the ***Q*-value** of the decay. From conservation of energy,

$$M_P c^2 = M_D c^2 + m_\alpha c^2 + Q,$$

where Q equals the kinetic energy of the daughter and α particle, and M_P, M_D, and m_α are the masses of the parent, daughter, and α particle, respectively. Thus

$$Q = M_P c^2 - (M_D + m_\alpha)c^2. \qquad (30\text{–}2)$$

If the parent had *less* mass than the daughter plus the α particle (so $Q < 0$), the decay would violate conservation of energy. Such decays have never been observed, another confirmation of this great conservation law.

EXAMPLE 30–6 Uranium decay energy release. Calculate the disintegration energy when $^{232}_{92}\text{U}$ (mass = 232.037156 u) decays to $^{228}_{90}\text{Th}$ (228.028741 u) with the emission of an α particle. (As always, masses given are for neutral atoms.)

APPROACH We use conservation of energy as expressed in Eq. 30–2. $^{232}_{92}\text{U}$ is the parent, $^{228}_{90}\text{Th}$ is the daughter.

SOLUTION Since the mass of the ^4_2He is 4.002603 u (Appendix B), the total mass in the final state $(m_{\text{Th}} + m_{\text{He}})$ is

$$228.028741\text{ u} + 4.002603\text{ u} = 232.031344\text{ u}.$$

The mass lost when the $^{232}_{92}\text{U}$ decays $(m_U - m_{\text{Th}} - m_{\text{He}})$ is

$$232.037156\text{ u} - 232.031344\text{ u} = 0.005812\text{ u}.$$

Because $1\text{ u} = 931.5\text{ MeV}$, the energy Q released is

$$Q = (0.005812\text{ u})(931.5\text{ MeV/u}) = 5.4\text{ MeV}$$

and this energy appears as kinetic energy of the α particle and the daughter nucleus.

Additional Example

EXAMPLE 30–7 Kinetic energy of the α in $^{232}_{92}\text{U}$ decay. For the $^{232}_{92}\text{U}$ decay of Example 30–6, how much of the 5.4-MeV disintegration energy will be carried off by the α particle?

APPROACH In any reaction, momentum must be conserved as well as energy.

SOLUTION Before disintegration, the nucleus can be assumed to be at rest, so the total momentum was zero. After disintegration, the total vector momentum must still be zero so the magnitude of the α particle's momentum must equal the magnitude of the daughter's momentum (Fig. 30–6):

$$m_\alpha v_\alpha = m_D v_D.$$

Thus $v_\alpha = m_D v_D / m_\alpha$ and the α's kinetic energy is

$$\text{KE}_\alpha = \tfrac{1}{2} m_\alpha v_\alpha^2 = \tfrac{1}{2} m_\alpha \left(\frac{m_D v_D}{m_\alpha}\right)^2 = \tfrac{1}{2} m_D v_D^2 \left(\frac{m_D}{m_\alpha}\right) = \left(\frac{m_D}{m_\alpha}\right)\text{KE}_D$$

$$= \left(\frac{228.028741\text{ u}}{4.002603\text{ u}}\right)\text{KE}_D = 57\,\text{KE}_D.$$

The total disintegration energy is $Q = \text{KE}_\alpha + \text{KE}_D = 57\,\text{KE}_D + \text{KE}_D = 58\,\text{KE}_D$. Hence

$$\text{KE}_\alpha = 57\,\text{KE}_D = \frac{57}{58}Q = 5.3\text{ MeV}.$$

The lighter α particle carries off (57/58) or 98% of the total kinetic energy. The total energy released is 5.4 MeV, so the daughter nucleus, which recoils in the opposite direction, carries off only 0.1 MeV.

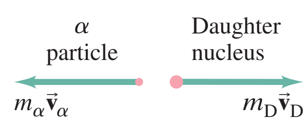


FIGURE 30–6 Momentum conservation in Example 30–7.

Why α Particles?

Why, you may wonder, do nuclei emit this combination of four nucleons called an α particle? Why not just four separate nucleons, or even one? The answer is that the α particle is very strongly bound, so that its mass is significantly less than that of four separate nucleons. That helps the final state in α decay to have less total mass, thus allowing certain nuclides to decay which could not decay to, say, 2 protons plus 2 neutrons. For example, ${}^{232}_{92}\mathrm{U}$ could not decay to $2\mathrm{p} + 2\mathrm{n}$ because the masses of the daughter ${}^{228}_{90}\mathrm{Th}$ plus four separate nucleons is $228.028741\,\mathrm{u} + 2(1.007825\,\mathrm{u}) + 2(1.008665\,\mathrm{u}) = 232.061721\,\mathrm{u}$, which is greater than the mass of the ${}^{232}_{92}\mathrm{U}$ parent (232.037156 u). Such a decay would violate the conservation of energy. Indeed, we have never seen ${}^{232}_{92}\mathrm{U} \rightarrow {}^{228}_{90}\mathrm{Th} + 2\mathrm{p} + 2\mathrm{n}$. Similarly, it is almost always true that the emission of a single nucleon is energetically not possible; see Example 30–5.

Smoke Detectors—An Application

PHYSICS APPLIED
Smoke detector

One widespread application of nuclear physics is present in nearly every home in the form of an ordinary **smoke detector**. One type of smoke detector contains about 0.2 mg of the radioactive americium isotope, ${}^{241}_{95}\mathrm{Am}$, in the form of $\mathrm{AmO_2}$. The radiation continually ionizes the nitrogen and oxygen molecules in the air space between two oppositely charged plates. The resulting conductivity allows a small steady electric current. If smoke enters, the radiation is absorbed by the smoke particles rather than by the air molecules, thus reducing the current. The current drop is detected by the device's electronics and sets off the alarm. The radiation dose that escapes from an intact americium smoke detector is much less than the natural radioactive background, and so can be considered relatively harmless. There is no question that smoke detectors save lives and reduce property damage.

30–5 Beta Decay

β^- Decay

Transmutation of elements also occurs when a nucleus decays by β decay—that is, with the emission of an electron or β^- particle. The nucleus ${}^{14}_{6}\mathrm{C}$, for example, emits an electron when it decays:

$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^- + \text{neutrino},$$

where e^- is the symbol for the electron. The particle known as the neutrino has charge $q = 0$ and a very small mass, long thought to be zero. It was not initially detected and was only later hypothesized to exist, as we shall discuss later in this Section. No nucleons are lost when an electron is emitted, and the total number of nucleons, A, is the same in the daughter nucleus as in the parent. But because an electron has been emitted from the nucleus itself, the charge on the daughter nucleus is $+1e$ greater than that on the parent. The parent nucleus in the decay written above had $Z = +6$, so from charge conservation the nucleus remaining behind must have a charge of $+7e$. So the daughter nucleus has $Z = 7$, which is nitrogen.

CAUTION
β-decay e^- comes from nucleus (it is not an orbital electron)

It must be carefully noted that the electron emitted in β decay is *not* an orbital electron. Instead, the electron is created *within the nucleus itself*. What happens is that one of the neutrons changes to a proton and in the process (to conserve charge) emits an electron. Indeed, free neutrons actually do decay in this fashion:

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \text{neutrino}.$$

To remind us of their origin in the nucleus, the electrons emitted in β decay are often referred to as "β particles." They are, nonetheless, indistinguishable from orbital electrons.

EXAMPLE 30–8 **Energy release in $^{14}_{6}C$ decay.** How much energy is released when $^{14}_{6}C$ decays to $^{14}_{7}N$ by β emission?

APPROACH We find the mass difference before and after decay, Δm. The energy released is $E = (\Delta m)c^2$. The masses given in Appendix B are those of the neutral atom, and we have to keep track of the electrons involved. Assume the parent nucleus has six orbiting electrons so it is neutral; its mass is 14.003242 u. The daughter in this decay, $^{14}_{7}N$, is not neutral because it has the same six orbital electrons circling it but the nucleus has a charge of $+7e$. However, the mass of this daughter with its six electrons, plus the mass of the emitted electron (which makes a total of seven electrons), is just the mass of a neutral nitrogen atom.

SOLUTION The total mass in the final state is

$$(\text{mass of } ^{14}_{7}\text{N nucleus} + 6 \text{ electrons}) + (\text{mass of 1 electron}),$$

and this is equal to

$$\text{mass of neutral } ^{14}_{7}\text{N (includes 7 electrons)},$$

which from Appendix B is a mass of 14.003074 u. So the mass difference is $14.003242\,\text{u} - 14.003074\,\text{u} = 0.000168\,\text{u}$, which is equivalent to an energy change $\Delta m\, c^2 = (0.000168\,\text{u})(931.5\,\text{MeV/u}) = 0.156\,\text{MeV}$ or 156 keV.

NOTE The neutrino doesn't contribute to either the mass or charge balance because it has $q = 0$ and $m \approx 0$.

CAUTION

Be careful with atomic and electron masses in β decay

According to Example 30–8, we would expect the emitted electron to have a kinetic energy of 156 keV. (The daughter nucleus, because its mass is very much larger than that of the electron, recoils with very low velocity and hence gets very little of the kinetic energy—see Example 30–7.) Indeed, very careful measurements indicate that a few emitted β particles do have kinetic energy close to this calculated value. But the vast majority of emitted electrons have somewhat less energy. In fact, the energy of the emitted electron can be anywhere from zero up to the maximum value as calculated above. This range of electron kinetic energy was found for any β decay. It was as if the law of conservation of energy was being violated, and Bohr actually considered this possibility. Careful experiments indicated that linear momentum and angular momentum also did not seem to be conserved. Physicists were troubled at the prospect of giving up these laws, which had worked so well in all previous situations.

In 1930, Wolfgang Pauli proposed an alternate solution: perhaps a new particle that was very difficult to detect was emitted during β decay in addition to the electron. This hypothesized particle could be carrying off the energy, momentum, and angular momentum required to maintain the conservation laws. This new particle was named the **neutrino**—meaning "little neutral one"—by the great Italian physicist Enrico Fermi (1901–1954; Fig. 30–7), who in 1934 worked out a detailed theory of β decay. (It was Fermi who, in this theory, postulated the existence of the fourth force in nature which we call the *weak nuclear force*.) The neutrino has zero charge, spin of $\frac{1}{2}\hbar$, and was long thought to have zero mass, although today we are quite sure that it has a very tiny mass $(< 0.14\,\text{eV}/c^2)$. If its mass were zero, it would be much like a photon in that it is neutral and would travel at the speed of light. But the neutrino is very difficult to detect. In 1956, complex experiments produced further evidence for the existence of the neutrino; but by then, most physicists had already accepted its existence.

The symbol for the neutrino is the Greek letter nu (ν). The correct way of writing the decay of $^{14}_{6}C$ is then

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}.$$

The bar $(^{-})$ over the neutrino symbol is to indicate that it is an "antineutrino." (Why this is called an antineutrino rather than simply a neutrino is discussed in Chapter 32.)

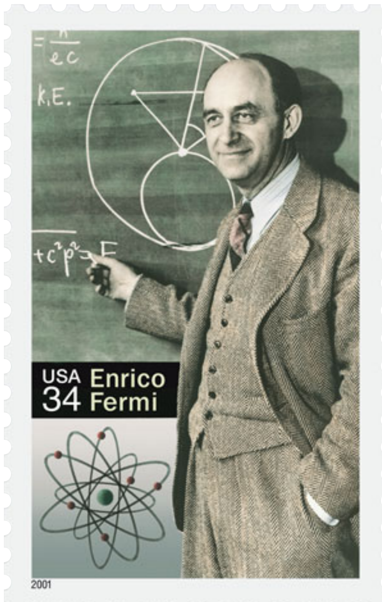


FIGURE 30–7 Enrico Fermi, as portrayed on a US postage stamp. Fermi contributed significantly to both theoretical and experimental physics, a feat almost unique in modern times: statistical theory of identical particles that obey the exclusion principle (= fermions); theory of the weak interaction and β decay; neutron physics; induced radioactivity and new elements; first nuclear reactor; first resonance of particle physics; led and inspired a vast amount of other nuclear research.

In β decay, it is the weak nuclear force that plays the crucial role. The neutrino is unique in that it interacts with matter only via the weak force, which is why it is so hard to detect.

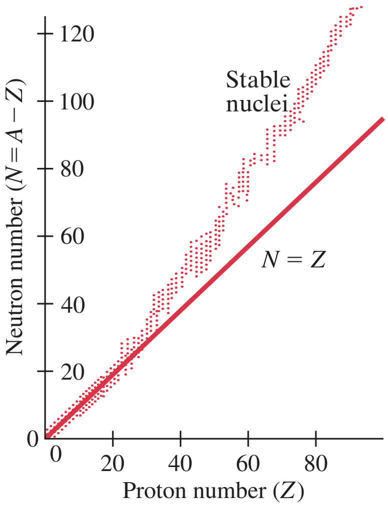


FIGURE 30–2 (Repeated.) Number of neutrons versus number of protons for stable nuclides, which are represented by dots. The straight line represents $N = Z$.

β^+ Decay

Many isotopes decay by electron emission. They are always isotopes that have too many neutrons compared to the number of protons. That is, they are isotopes that lie above the stable isotopes plotted in Fig. 30–2. But what about unstable isotopes that have too few neutrons compared to their number of protons—those that fall below the stable isotopes of Fig. 30–2? These, it turns out, decay by emitting a **positron** instead of an electron. A positron (sometimes called an e^+ or β^+ particle) has the same mass as the electron, but it has a positive charge of $+1e$. Because it is so like an electron, except for its charge, the positron is called the **antiparticle**† to the electron. An example of a β^+ decay is that of ${}^{19}_{10}\mathrm{Ne}$:

$$ {}^{19}_{10}\mathrm{Ne} \rightarrow {}^{19}_{9}\mathrm{F} + e^+ + \nu, $$

where e^+ stands for a positron. Note that the ν emitted here is a neutrino, whereas that emitted in β^- decay is called an antineutrino. Thus an antielectron (= positron) is emitted with a neutrino, whereas an antineutrino is emitted with an electron; this gives a certain balance as discussed in Chapter 32.

We can write β^- and β^+ decay, in general, as follows:

$$ {}^{A}_{Z}\mathrm{N} \rightarrow {}^{A}_{Z+1}\mathrm{N}' + e^- + \bar{\nu} \qquad [\beta^- \text{ decay}] $$

$$ {}^{A}_{Z}\mathrm{N} \rightarrow {}^{A}_{Z-1}\mathrm{N}' + e^+ + \nu, \qquad [\beta^+ \text{ decay}] $$

where N is the parent nucleus and N′ is the daughter.

Electron Capture

Besides β^- and β^+ emission, there is a third related process. This is **electron capture** (abbreviated EC in Appendix B) and occurs when a nucleus absorbs one of its orbiting electrons. An example is ${}^{7}_{4}\mathrm{Be}$, which as a result becomes ${}^{7}_{3}\mathrm{Li}$. The process is written

$$ {}^{7}_{4}\mathrm{Be} + e^- \rightarrow {}^{7}_{3}\mathrm{Li} + \nu, $$

or, in general,

$$ {}^{A}_{Z}\mathrm{N} + e^- \rightarrow {}^{A}_{Z-1}\mathrm{N}' + \nu. \qquad [\text{electron capture}] $$

Usually it is an electron in the innermost (K) shell that is captured, in which case the process is called **K-capture**. The electron disappears in the process, and a proton in the nucleus becomes a neutron; a neutrino is emitted as a result. This process is inferred experimentally by detection of emitted X-rays (due to other electrons jumping down to fill the state of the captured e^-).

30–6 Gamma Decay

Gamma rays are photons having very high energy. They have their origin in the decay of a nucleus, much like emission of photons by excited atoms. Like an atom, a nucleus itself can be in an excited state. When it jumps down to a lower energy state, or to the ground state, it emits a photon which we call a γ ray. The possible states of a nucleus are much farther apart in energy than those of an atom: on the order of keV or MeV, as compared to a few eV for electrons in an atom. Hence, the emitted photons have energies that can range from a few keV to several MeV. For a given decay, the γ ray always has the same energy. Since a γ ray carries no charge, there is no change in the element as a result of a γ decay.

How does a nucleus get into an excited state? It may occur because of a violent collision with another particle. More commonly, the nucleus remaining after a previous radioactive decay may be in an excited state. A typical example is shown

†Discussed in Chapter 32. Briefly, an antiparticle has the same mass as its corresponding particle, but opposite charge. A particle and its antiparticle can quickly annihilate each other, releasing energy in the form of two γ rays: $e^+ + e^- \rightarrow 2\gamma$.

in the energy-level diagram of Fig. 30–8. ${}^{12}_{5}B$ can decay by β decay directly to the ground state of ${}^{12}_{6}C$; or it can go by β decay to an excited state of ${}^{12}_{6}C$, written ${}^{12}_{6}C^*$, which itself decays by emission of a 4.4-MeV γ ray to the ground state of ${}^{12}_{6}C$.

We can write γ decay as

$$ {}^{A}_{Z}N^* \rightarrow {}^{A}_{Z}N + \gamma, \qquad [\gamma \text{ decay}] $$

where the asterisk means "excited state" of that nucleus.

What, you may wonder, is the difference between a γ ray and an X-ray? They both are electromagnetic radiation (photons) and, though γ rays usually have higher energy than X-rays, their range of energies overlap to some extent. The difference is not intrinsic. We use the term X-ray if the photon is produced by an electron–atom interaction, and γ ray if the photon is produced in a nuclear process.

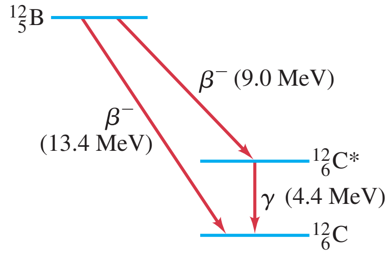


FIGURE 30–8 Energy-level diagram showing how ${}^{12}_{5}B$ can decay to the ground state of ${}^{12}_{6}C$ by β decay (total energy released = 13.4 MeV), or can instead β decay to an excited state of ${}^{12}_{6}C$ (indicated by *), which subsequently decays to its ground state by emitting a 4.4-MeV γ ray.

* Isomers; Internal Conversion

In some cases, a nucleus may remain in an excited state for some time before it emits a γ ray. The nucleus is then said to be in a **metastable state** and is called an **isomer**.

An excited nucleus can sometimes return to the ground state by another process known as **internal conversion** with no γ ray emitted. In this process, the excited nucleus interacts with one of the orbital electrons and ejects this electron from the atom with the same kinetic energy (minus the binding energy of the electron) that an emitted γ ray would have had.

30–7 Conservation of Nucleon Number and Other Conservation Laws

In all three types of radioactive decay, the classical conservation laws hold. Energy, linear momentum, angular momentum, and electric charge are all conserved. These quantities are the same before the decay as after. But a new conservation law is also revealed, the **law of conservation of nucleon number**. According to this law, the total number of nucleons (A) remains constant in any process, although one type can change into the other type (protons into neutrons or vice versa). This law holds in all three types of decay. [In Chapter 32 we will generalize this and call it conservation of baryon number.]

Table 30–2 gives a summary of α, β, and γ decay.

TABLE 30–2 The Three Types of Radioactive Decay

α decay:

$$ {}^{A}_{Z}N \rightarrow {}^{A-4}_{Z-2}N' + {}^{4}_{2}He $$

β decay:

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z+1}N' + e^- + \bar{\nu} $$

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z-1}N' + e^+ + \nu $$

$$ {}^{A}_{Z}N + e^- \rightarrow {}^{A}_{Z-1}N' + \nu \ [\text{EC}]^{\dagger} $$

γ decay:

$$ {}^{A}_{Z}N^* \rightarrow {}^{A}_{Z}N + \gamma $$

†Electron capture.

*Indicates the excited state of a nucleus.

30–8 Half-Life and Rate of Decay

A macroscopic sample of any radioactive isotope consists of a vast number of radioactive nuclei. These nuclei do not all decay at one time. Rather, they decay one by one over a period of time. This is a random process: we can not predict exactly when a given nucleus will decay. But we can determine, on a probabilistic basis, approximately how many nuclei in a sample will decay over a given time period, by assuming that each nucleus has the same probability of decaying in each second that it exists.

The number of decays ΔN that occur in a very short time interval Δt is then proportional to Δt and to the total number N of radioactive (parent) nuclei present:

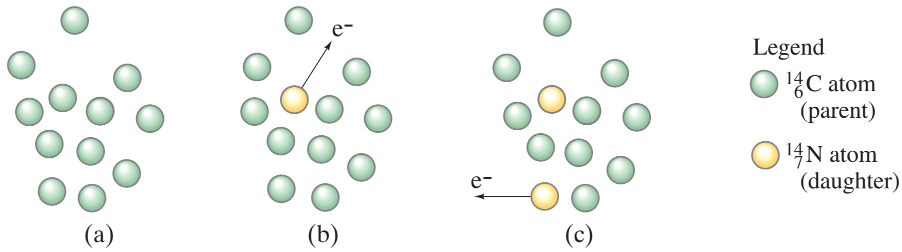
$$ \Delta N = -\lambda N \, \Delta t \qquad \textbf{(30–3a)} $$

where the minus sign means N is decreasing. We rewrite this to get the **rate of decay** (number of decays per second):

$$ \frac{\Delta N}{\Delta t} = -\lambda N. \qquad \textbf{(30–3b)} $$

In these equations, λ is a measurable constant called the **decay constant**, which is different for different isotopes. The greater λ is, the greater the rate of decay, $\Delta N/\Delta t$, and the more "radioactive" that isotope is said to be.

FIGURE 30–9 Radioactive nuclei decay one by one. Hence, the number of parent nuclei in a sample is continually decreasing. When a ${}^{14}_{6}C$ nucleus emits an electron (b), the nucleus becomes a ${}^{14}_{7}N$ nucleus. Another decays in (c).



The number of decays that occur in the short time interval Δt is designated ΔN because each decay that occurs corresponds to a decrease by one in the number N of parent nuclei present. That is, radioactive decay is a "one-shot" process, Fig. 30–9. Once a particular parent nucleus decays into its daughter, it cannot do it again.

Exponential Decay

Equation 30–3a or b can be solved for N (using calculus) and the result is

$$N = N_0 e^{-\lambda t}, \qquad \textbf{(30–4)}$$

where N_0 is the number of parent nuclei present at any chosen time $t = 0$, and N is the number remaining after a time t. The symbol e is the natural exponential (encountered earlier in Sections 19–6 and 21–12) whose value is $e = 2.718\cdots$. Thus the number of parent nuclei in a sample decreases exponentially in time. This is shown in Fig. 30–10a for the decay of ${}^{14}_{6}C$. Equation 30–4 is called the **radioactive decay law**.

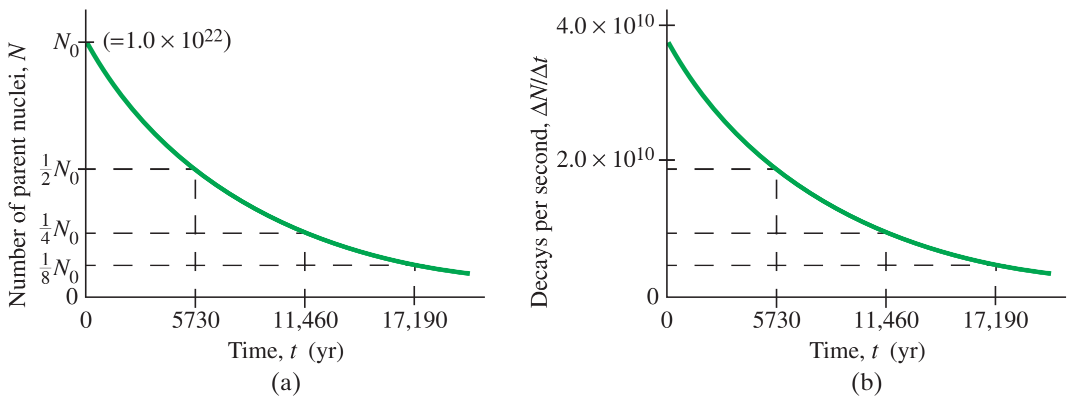


FIGURE 30–10 (a) The number N of parent nuclei in a given sample of ${}^{14}_{6}C$ decreases exponentially. We assume a sample that has $N_0 = 1.00 \times 10^{22}$ nuclei. (b) The number of decays per second also decreases exponentially. The half-life of ${}^{14}_{6}C$ is 5730 yr, which means that the number of parent nuclei, N, and the rate of decay, $\Delta N/\Delta t$, decrease by half every 5730 yr.

The number of decays per second, or decay rate R, is the magnitude of $\Delta N/\Delta t$, and is also called the **activity** of the sample. The magnitude (always positive) of a quantity is often indicated using vertical lines. The magnitude of $\Delta N/\Delta t$ is written $|\Delta N/\Delta t|$ and it is proportional to N (see Eq. 30–3b). So it too decreases exponentially in time at the same rate (Fig. 30–10b). The activity of a pure sample at time t is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = R_0 e^{-\lambda t}, \qquad \textbf{(30–5)}$$

where $R_0 = |\Delta N/\Delta t|_0$ is the activity at $t = 0$.

Equation 30–5 is also referred to as the **radioactive decay law** (as is Eq. 30–4).

Half-Life

The rate of decay of any isotope is often specified by giving its "half-life" rather than the decay constant λ. The **half-life** of an isotope is defined as the time it takes for half the original amount of parent isotope in a given sample to decay.

For example, the half-life of $^{14}_{6}C$ is about 5730 years. If at some time a piece of petrified wood contains, say, 1.00×10^{22} nuclei of $^{14}_{6}C$, then 5730 years later it will contain half as many, 0.50×10^{22} nuclei. After another 5730 years it will contain 0.25×10^{22} nuclei, and so on. This is shown in Fig. 30–10a. Since the rate of decay $\Delta N/\Delta t$ is proportional to N, it, too, decreases by a factor of 2 every half-life (Fig. 30–10b).

The half-lives of known radioactive isotopes vary from very short ($\approx 10^{-22}$ s) to more than 10^{23} yr $(> 10^{30}\,\mathrm{s})$. The half-lives of many isotopes are given in Appendix B. It should be clear that the half-life (which we designate $T_{\frac{1}{2}}$) bears an inverse relationship to the decay constant. The longer the half-life of an isotope, the more slowly it decays, and hence λ is smaller. Conversely, very active isotopes (large λ) have very short half-lives. The precise relationship between half-life and decay constant is

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \qquad \textbf{(30–6)}$$

We derive this in the next (optional) subsection.

EXERCISE D The half-life of $^{22}_{11}Na$ is 2.6 years. How much $^{22}_{11}Na$ will be left of a pure 1.0-μg sample after 7.8 yr? (*a*) None. (*b*) $\frac{1}{8}$ μg. (*c*) $\frac{1}{4}$ μg. (*d*) $\frac{1}{2}$ μg. (*e*) 0.693 μg.

EXERCISE E Return to the Chapter-Opening Question, page 857, and answer it again now. Try to explain why you may have answered differently the first time.

* Deriving the Half-Life Formula

We can derive Eq. 30–6 starting from Eq. 30–4 by setting $N = N_0/2$ at $t = T_{\frac{1}{2}}$:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

so

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

and

$$e^{\lambda T_{\frac{1}{2}}} = 2.$$

We take natural logs of both sides ("ln" and "e" are inverse operations, meaning $\ln\left(e^x\right) = x$) and find

$$\ln\left(e^{\lambda T_{\frac{1}{2}}}\right) = \ln 2,$$

so

$$\lambda T_{\frac{1}{2}} = \ln 2 = 0.693$$

and

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda},$$

which is Eq. 30–6.

* Mean Life

Sometimes the **mean life** τ of an isotope is quoted, which is defined as $\tau = 1/\lambda$. Then Eq. 30–4 can be written $N = N_0 e^{-t/\tau}$, just as for RC and LR circuits (Chapters 19 and 21 where τ was called the time constant). The mean life of an isotope is then given by (see also Eq. 30–6)

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693}. \qquad \text{[mean life]} \quad \textbf{(30–7)}$$

The mean life and half-life differ by a factor of 0.693, so confusing them can cause serious error (and has). The radioactive decay law, Eq. 30–5, can then be written as $R = R_0 e^{-t/\tau}$.

CAUTION
Do not confuse half-life and mean life

30–9 Calculations Involving Decay Rates and Half-Life

Let us now consider Examples of what we can determine about a sample of radioactive material if we know the half-life.

EXAMPLE 30–9 **Sample activity.** The isotope ${}^{14}_{6}C$ has a half-life of 5730 yr. If a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

APPROACH We first use the half-life to find the decay constant (Eq. 30–6), and use that to find the activity, Eq. 30–3b. The number of seconds in a year is $(60)(60)(24)(365\frac{1}{4}) = 3.156 \times 10^7$ s.

SOLUTION The decay constant λ from Eq. 30–6 is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730\,\text{yr})(3.156 \times 10^7\,\text{s/yr})} = 3.83 \times 10^{-12}\,\text{s}^{-1}.$$

From Eqs. 30–3b and 30–5, the activity or rate of decay is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N = (3.83 \times 10^{-12}\,\text{s}^{-1})(1.00 \times 10^{22}) = 3.83 \times 10^{10}\,\text{decays/s}.$$

Notice that the graph of Fig. 30–10b starts at this value, corresponding to the original value of $N = 1.0 \times 10^{22}$ nuclei in Fig. 30–10a.

NOTE The unit "decays/s" is often written simply as s^{-1} since "decays" is not a unit but refers only to the number. This simple unit of activity is called the becquerel: $1\,\text{Bq} = 1\,\text{decay/s}$, as discussed in Chapter 31.

CONCEPTUAL EXAMPLE 30–10 **Safety: Activity versus half-life.** One might think that a short half-life material is safer than a long half-life material because it will not last as long. Is that true?

RESPONSE No. A shorter half-life means the activity is higher and thus more "radioactive" and can cause more biological damage. In contrast, a longer half-life for the same sample size N means a lower activity but we have to worry about it for longer and find safe storage until it reaches a safe (low) level of activity.

EXAMPLE 30–11 **A sample of radioactive ${}^{13}_{7}N$.** A laboratory has 1.49 μg of pure ${}^{13}_{7}N$, which has a half-life of 10.0 min (600 s). (*a*) How many nuclei are present initially? (*b*) What is the rate of decay (activity) initially? (*c*) What is the activity after 1.00 h? (*d*) After approximately how long will the activity drop to less than one per second $(= 1\,\text{s}^{-1})$?

APPROACH We use the definition of the mole and Avogadro's number (Sections 13–6 and 13–8) to find (*a*) the number of nuclei. For (*b*) we get λ from the given half-life and use Eq. 30–3b for the rate of decay. For (*c*) and (*d*) we use Eq. 30–5.

SOLUTION (*a*) The atomic mass is 13.0, so 13.0 g will contain 6.02×10^{23} nuclei (Avogadro's number). We have only 1.49×10^{-6} g, so the number of nuclei N_0 that we have initially is given by the ratio

$$\frac{N_0}{6.02 \times 10^{23}} = \frac{1.49 \times 10^{-6}\,\text{g}}{13.0\,\text{g}}.$$

Solving for N_0, we find $N_0 = 6.90 \times 10^{16}$ nuclei.

(*b*) From Eq. 30–6,

$$\lambda = 0.693/T_{\frac{1}{2}} = (0.693)/(600\,\text{s}) = 1.155 \times 10^{-3}\,\text{s}^{-1}.$$

Then, at $t = 0$ (see Eqs. 30–3b and 30–5)

$$R_0 = \left|\frac{\Delta N}{\Delta t}\right|_0 = \lambda N_0 = (1.155 \times 10^{-3}\,\text{s}^{-1})(6.90 \times 10^{16}) = 7.97 \times 10^{13}\,\text{decays/s}.$$

(*c*) After 1.00 h = 3600 s, the magnitude of the activity will be (Eq. 30–5)

$$R = R_0 e^{-\lambda t} = \left(7.97 \times 10^{13}\,\mathrm{s}^{-1}\right)e^{-(1.155\times10^{-3}\,\mathrm{s}^{-1})(3600\,\mathrm{s})} = 1.25 \times 10^{12}\,\mathrm{s}^{-1}.$$

(*d*) We want to determine the time t when $R = 1.00\,\mathrm{s}^{-1}$. From Eq. 30–5, we have

$$e^{-\lambda t} = \frac{R}{R_0} = \frac{1.00\,\mathrm{s}^{-1}}{7.97 \times 10^{13}\,\mathrm{s}^{-1}} = 1.25 \times 10^{-14}.$$

We take the natural log (ln) of both sides $\left(\ln e^{-\lambda t} = -\lambda t\right)$ and divide by λ to find

$$t = -\frac{\ln\left(1.25 \times 10^{-14}\right)}{\lambda} = 2.77 \times 10^4\,\mathrm{s} = 7.70\,\mathrm{h}.$$

Easy Alternate Solution to (c) 1.00 h = 60.0 minutes is 6 half-lives, so the activity will decrease to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$ of its original value, or $\left(7.97 \times 10^{13}\right)/(64) = 1.25 \times 10^{12}$ per second.

30–10 Decay Series

It is often the case that one radioactive isotope decays to another isotope that is also radioactive. Sometimes this daughter decays to yet a third isotope which also is radioactive. Such successive decays are said to form a **decay series**. An important example is illustrated in Fig. 30–11. As can be seen, ${}^{238}_{92}\mathrm{U}$ decays by α emission to ${}^{234}_{90}\mathrm{Th}$, which in turn decays by β decay to ${}^{234}_{91}\mathrm{Pa}$. The series continues as shown, with several possible branches near the bottom, ending at the stable lead isotope, ${}^{206}_{82}\mathrm{Pb}$. The two last decays can be

$${}^{206}_{81}\mathrm{Tl} \rightarrow {}^{206}_{82}\mathrm{Pb} + e^- + \bar{\nu}, \qquad \left(T_{\frac{1}{2}} = 4.2\,\mathrm{min}\right)$$

or

$${}^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + \alpha. \qquad \left(T_{\frac{1}{2}} = 138\,\mathrm{days}\right)$$

Other radioactive series also exist.

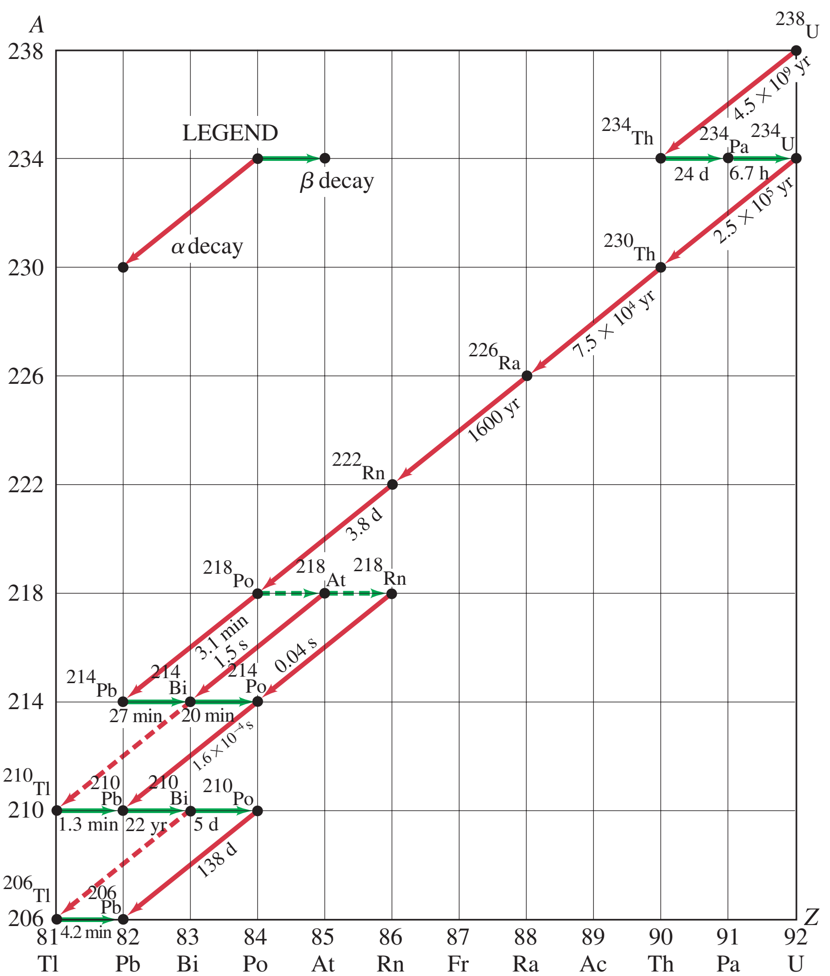


FIGURE 30–11 Decay series beginning with ${}^{238}_{92}\mathrm{U}$. Nuclei in the series are specified by a dot representing A and Z values. Half-lives are given in seconds (s), minutes (min), hours (h), days (d), or years (yr). Note that a horizontal arrow represents β decay (A does not change), whereas a diagonal line represents α decay (A changes by 4, Z changes by 2). For the four nuclides shown that can decay by both α and β decay, the more prominent decay (in these four cases, >99.9%) is shown as a solid arrow and the less common decay (<0.1%) as a dashed arrow.

Because of such decay series, certain radioactive elements are found in nature that otherwise would not be. When the solar system (including Earth) was formed about 5 billion years ago, it is believed that nearly all nuclides were present, having been formed (by fusion and neutron capture, Sections 31–3 and 33–2) in a nearby supernova explosion (Section 33–2). Many isotopes with short half-lives decayed quickly and no longer are detected in nature today. But long-lived isotopes, such as $^{238}_{92}\mathrm{U}$ with a half-life of 4.5×10^9 yr, still do exist in nature today. Indeed, about half of the original $^{238}_{92}\mathrm{U}$ still remains. We might expect, however, that radium $\left(^{226}_{88}\mathrm{Ra}\right)$, with a half-life of 1600 yr, would have disappeared from the Earth long ago. Indeed, the original $^{226}_{88}\mathrm{Ra}$ nuclei must by now have all decayed. However, because $^{238}_{92}\mathrm{U}$ decays (in several steps, Fig. 30–11) to $^{226}_{88}\mathrm{Ra}$, the supply of $^{226}_{88}\mathrm{Ra}$ is continually replenished, which is why it is still found on Earth today. The same can be said for many other radioactive nuclides.

CONCEPTUAL EXAMPLE 30–12 **Decay chain.** In the decay chain of Fig. 30–11, if we look at the decay of $^{234}_{92}\mathrm{U}$, we see four successive nuclides with half-lives of 250,000 yr, 75,000 yr, 1600 yr, and a little under 4 days. Each decay in the chain has an alpha particle of a characteristic energy, and so we can monitor the radioactive decay rate of each nuclide. Given a sample that was pure $^{234}_{92}\mathrm{U}$ a million years ago, which alpha decay would you expect to have the highest activity rate in the sample?

RESPONSE The first instinct is to say that the process with the shortest half-life would show the highest activity. Surprisingly, perhaps, the activities of the four nuclides in this sample are all the same. The reason is that in each case the decay of the parent acts as a bottleneck to the decay of the daughter. Compared to the 1600-yr half-life of $^{226}_{88}\mathrm{Ra}$, for example, its daughter $^{222}_{86}\mathrm{Rn}$ decays almost immediately, but it cannot decay until it is made. (This is like an automobile assembly line: if worker A takes 20 minutes to do a task and then worker B takes only 1 minute to do the next task, worker B still does only one car every 20 minutes.)

30–11 Radioactive Dating

Radioactive decay has many interesting applications. One is the technique of *radioactive dating* by which the age of ancient materials can be determined.

PHYSICS APPLIED
Carbon-14 dating

The age of any object made from once-living matter, such as wood, can be determined using the natural radioactivity of $^{14}_{6}\mathrm{C}$. All living plants absorb carbon dioxide (CO_2) from the air and use it to synthesize organic molecules. The vast majority of these carbon atoms are $^{12}_{6}\mathrm{C}$, but a small fraction, about 1.3×10^{-12}, is the radioactive isotope $^{14}_{6}\mathrm{C}$. The ratio of $^{14}_{6}\mathrm{C}$ to $^{12}_{6}\mathrm{C}$ in the atmosphere has remained roughly constant over many thousands of years, in spite of the fact that $^{14}_{6}\mathrm{C}$ decays with a half-life of about 5730 yr. This is because energetic nuclei in the cosmic radiation, which impinges on the Earth from outer space, strike nuclei of atoms in the atmosphere and break those nuclei into pieces, releasing free neutrons. Those neutrons can collide with nitrogen nuclei in the atmosphere to produce the nuclear transformation $\mathrm{n} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{14}_{6}\mathrm{C} + \mathrm{p}$. That is, a neutron strikes and is absorbed by a $^{14}_{7}\mathrm{N}$ nucleus, and a proton is knocked out in the process. The remaining nucleus is $^{14}_{6}\mathrm{C}$. This continual production of $^{14}_{6}\mathrm{C}$ in the atmosphere roughly balances the loss of $^{14}_{6}\mathrm{C}$ by radioactive decay.

As long as a plant or tree is alive, it continually uses the carbon from carbon dioxide in the air to build new tissue and to replace old. Animals eat plants, so they too are continually receiving a fresh supply of carbon for their tissues.

Organisms cannot distinguish† $^{14}_{6}C$ from $^{12}_{6}C$, and because the ratio of $^{14}_{6}C$ to $^{12}_{6}C$ in the atmosphere remains nearly constant, the ratio of the two isotopes within the living organism remains nearly constant as well. When an organism dies, carbon dioxide is no longer taken in and utilized. Because the $^{14}_{6}C$ decays radioactively, the ratio of $^{14}_{6}C$ to $^{12}_{6}C$ in a dead organism decreases over time. The half-life of $^{14}_{6}C$ is about 5730 yr, so the $^{14}_{6}C/^{12}_{6}C$ ratio decreases by half every 5730 yr. If, for example, the $^{14}_{6}C/^{12}_{6}C$ ratio of an ancient wooden tool is half of what it is in living trees, then the object must have been made from a tree that was felled about 5730 years ago.

Actually, corrections must be made for the fact that the $^{14}_{6}C/^{12}_{6}C$ ratio in the atmosphere has not remained precisely constant over time. The determination of what this ratio has been over the centuries has required techniques such as comparing the expected ratio to the actual ratio for objects whose age is known, such as very old trees whose annual rings can be counted reasonably accurately.

EXAMPLE 30–13 **An ancient animal.** The mass of carbon in an animal bone fragment found in an archeological site is 200 g. If the bone registers an activity of 16 decays/s, what is its age?

PHYSICS APPLIED
Archeological dating

APPROACH First we determine how many $^{14}_{6}C$ atoms there were in our 200-g sample when the animal was alive, given the known fraction of $^{14}_{6}C$ to $^{12}_{6}C$, 1.3×10^{-12}. Then we use Eq. 30–3b to find the activity back then, and Eq. 30–5 to find out how long ago that was by solving for the time t.

SOLUTION The 200 g of carbon is nearly all $^{12}_{6}C$; 12.0 g of $^{12}_{6}C$ contains 6.02×10^{23} atoms, so 200 g contains

$$\left(\frac{6.02 \times 10^{23}\ \text{atoms/mol}}{12.0\ \text{g/mol}}\right)(200\ \text{g}) = 1.00 \times 10^{25}\ \text{atoms.}$$

When the animal was alive, the ratio of $^{14}_{6}C$ to $^{12}_{6}C$ in the bone was 1.3×10^{-12}. The number of $^{14}_{6}C$ nuclei at that time was

$$N_0 = \left(1.00 \times 10^{25}\ \text{atoms}\right)\left(1.3 \times 10^{-12}\right) = 1.3 \times 10^{13}\ \text{atoms.}$$

From Eq. 30–3b with $\lambda = 3.83 \times 10^{-12}\ \text{s}^{-1}$ (Example 30–9) the magnitude of the activity when the animal was still alive $(t = 0)$ was

$$R_0 = \left|\frac{\Delta N}{\Delta t}\right|_0 = \lambda N_0 = \left(3.83 \times 10^{-12}\ \text{s}^{-1}\right)\left(1.3 \times 10^{13}\right) = 50\ \text{s}^{-1}.$$

From Eq. 30–5

$$R = R_0 e^{-\lambda t}$$

where R, its activity now, is given as $16\ \text{s}^{-1}$. Then

$$16\ \text{s}^{-1} = \left(50\ \text{s}^{-1}\right)e^{-\lambda t}$$

or

$$e^{\lambda t} = \frac{50}{16}.$$

We take the natural logs of both sides (and divide by λ) to get

$$t = \frac{1}{\lambda}\ln\left(\frac{50}{16}\right) = \frac{1}{3.83 \times 10^{-12}\ \text{s}^{-1}}\ln\left(\frac{50}{16}\right)$$

$$= 2.98 \times 10^{11}\ \text{s} = 9400\ \text{yr},$$

which is the time elapsed since the death of the animal.

†Organisms operate almost exclusively via chemical reactions—which involve only the outer orbital electrons of the atom; extra neutrons in the nucleus have essentially no effect.

Geological Time Scale Dating

Carbon dating is useful only for determining the age of objects less than about 60,000 years old. The amount of ${}^{14}_{6}C$ remaining in objects older than that is usually too small to measure accurately, although new techniques are allowing detection of even smaller amounts of ${}^{14}_{6}C$, pushing the time frame further back. On the other hand, radioactive isotopes with longer half-lives can be used in certain circumstances to obtain the age of older objects. For example, the decay of ${}^{238}_{92}U$, because of its long half-life of 4.5×10^9 years, is useful in determining the ages of rocks on a geologic time scale. When molten material on Earth long ago solidified into rock as the temperature dropped, different compounds solidified according to the melting points, and thus different compounds separated to some extent. Uranium present in a material became fixed in position and the daughter nuclei that result from the decay of uranium were also fixed in that position. Thus, by measuring the amount of ${}^{238}_{92}U$ remaining in the material relative to the amount of daughter nuclei, the time when the rock solidified can be determined.

PHYSICS APPLIED
Geological dating

PHYSICS APPLIED
Oldest Earth rocks and earliest life

Radioactive dating methods using ${}^{238}_{92}U$ and other isotopes have shown the age of the oldest Earth rocks to be about 4×10^9 yr. The age of rocks in which the oldest fossilized organisms are embedded indicates that life appeared more than $3\frac{1}{2}$ billion years ago. The earliest fossilized remains of mammals are found in rocks 200 million years old, and humanlike creatures seem to have appeared more than 2 million years ago. Radioactive dating has been indispensable for the reconstruction of Earth's history.

*30–12 Stability and Tunneling

Radioactive decay occurs only if the mass of the parent nucleus is greater than the sum of the masses of the daughter nucleus and all particles emitted. For example, ${}^{238}_{92}U$ can decay to ${}^{234}_{90}Th$ because the mass of ${}^{238}_{92}U$ is greater than the mass of the ${}^{234}_{90}Th$ plus the mass of the α particle. Because systems tend to go in the direction that reduces their internal or potential energy (a ball rolls downhill, a positive charge moves toward a negative charge), you may wonder why an unstable nucleus doesn't fall apart immediately. In other words, why do ${}^{238}_{92}U$ nuclei $\left(T_{\frac{1}{2}} = 4.5 \times 10^9\,\text{yr}\right)$ and other isotopes have such long half-lives? Why don't parent nuclei all decay at once?

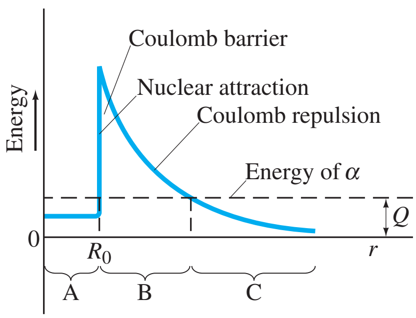


FIGURE 30–12 Potential energy for alpha particle and nucleus, showing the "Coulomb barrier" through which the α particle must tunnel to escape. The Q-value of the reaction is also indicated.

The answer has to do with quantum theory and the nature of the forces involved. One way to view the situation is with the aid of a potential-energy diagram, as in Fig. 30–12. Let us consider the particular case of the decay ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$. The blue line represents the potential energy, including rest mass, where we imagine the α particle as a separate entity within the ${}^{238}_{92}U$ nucleus. The region labeled A in Fig. 30–12 represents the PE of the α particle when it is held within the uranium nucleus by the strong nuclear force (R_0 is the nuclear radius). Region C represents the PE when the α particle is free of the nucleus. The downward-curving PE (proportional to $1/r$) represents the electrical repulsion (Coulomb's law) between the positively charged α and the ${}^{234}_{90}Th$ nucleus. To get to region C, the α particle has to get through the "**Coulomb barrier**" shown. Since the PE just beyond $r = R_0$ (region B) is greater than the energy of the alpha particle (dashed line), the α particle could not escape the nucleus if it were governed by classical physics. It could escape only if there were an input of energy equal to the height of the barrier. Nuclei decay spontaneously, however, without any input of energy. How, then, does the α particle get from region A to region C? It actually passes through the barrier in a process known as quantum-mechanical **tunneling**. Classically, this could not happen, because an α particle in region B (within the barrier) would be violating the conservation-of-energy principle.†

†The total energy E (dashed line in Fig. 30–12) would be less than the PE; because $\text{KE} = \frac{1}{2}mv^2 > 0$, then classically, $E = \text{KE} + \text{PE}$ could not be less than the PE.

The uncertainty principle, however, tells us that energy conservation can be violated by an amount ΔE for a length of time Δt given by

$$(\Delta E)(\Delta t) \approx \frac{h}{2\pi}.$$

We saw in Section 28–3 that this is a result of the wave–particle duality. Thus quantum mechanics allows conservation of energy to be violated for brief periods that may be long enough for an α particle to "tunnel" through the barrier. ΔE would represent the energy difference between the average barrier height and the particle's energy, and Δt the time to pass through the barrier. The higher and wider the barrier, the less time the α particle has to escape and the less likely it is to do so. It is therefore the height and width of this barrier that control the rate of decay and half-life of an isotope.

30–13 Detection of Particles

Individual particles such as electrons, protons, α particles, neutrons, and γ rays are not detected directly by our senses. Consequently, a variety of instruments have been developed to detect them.

Counters

One of the most common detectors is the **Geiger counter**. As shown in Fig. 30–13, it consists of a cylindrical metal tube filled with a certain type of gas. A long wire runs down the center and is kept at a high positive voltage $(\approx 10^3\ \mathrm{V})$ with respect to the outer cylinder. The voltage is just slightly less than that required to ionize the gas atoms. When a charged particle enters through the thin "window" at one end of the tube, it ionizes a few atoms of the gas. The freed electrons are attracted toward the positive wire, and as they are accelerated they strike and ionize additional atoms. An "avalanche" of electrons is quickly produced, and when it reaches the wire anode, it produces a voltage pulse. The pulse, after being amplified, can be sent to an electronic counter, which counts how many particles have been detected. Or the pulses can be sent to a loudspeaker and each detection of a particle is heard as a "click." Only a fraction of the radiation emitted by a sample is detected by any detector.

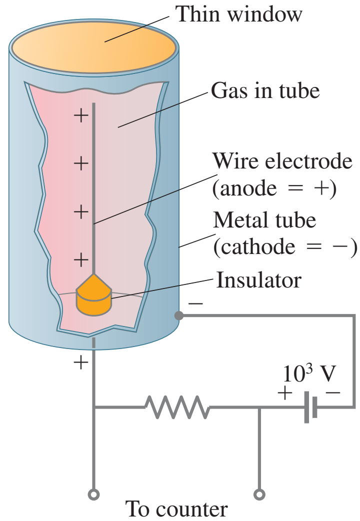
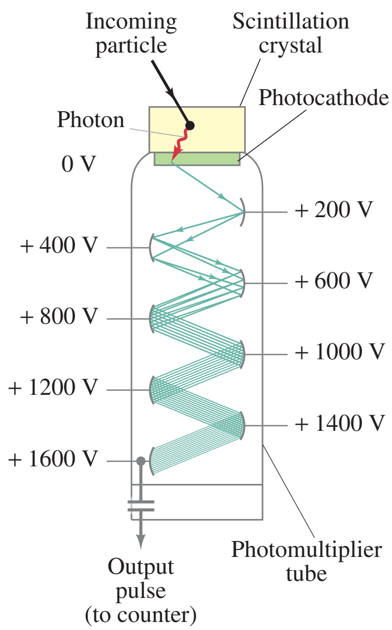


FIGURE 30–13 Diagram of a Geiger counter.

A **scintillation counter** makes use of a solid, liquid, or gas known as a **scintillator** or **phosphor**. The atoms of a scintillator are easily excited when struck by an incoming particle and emit visible light when they return to their ground states. Typical scintillators are crystals of NaI and certain plastics. One face of a solid scintillator is cemented to a photomultiplier tube, and the whole is wrapped with opaque material to keep it light-tight (in the dark) or is placed within a light-tight container. The **photomultiplier** (PM) **tube** converts the energy of the scintillator-emitted photon(s) into an electric signal. A PM tube is a vacuum tube containing several electrodes (typically 8 to 14), called *dynodes,* which are maintained at successively higher voltages as shown in Fig. 30–14. At its top surface is a photoelectric surface, called the *photocathode*, whose work function (Section 27–3) is low enough that an electron is easily released when struck by a photon from the scintillator. Such an electron is accelerated toward the positive voltage of the first dynode. When it strikes the first dynode, the electron has acquired sufficient kinetic energy so that it can eject two to five more electrons. These, in turn, are accelerated toward the higher voltage second dynode, and a multiplication process begins. The number of electrons striking the last dynode may be 10^6 or more. Thus the passage of a particle through the scintillator results in an electric signal at the output of the PM tube that can be sent to an electronic counter just as for a Geiger tube. Solid scintillators are much more dense than the gas of a Geiger counter, and so are much more efficient detectors—especially for γ rays, which interact less with matter than do α or β particles. Scintillators that can measure the total energy deposited are much used today and are called **calorimeters**.

FIGURE 30–14 Scintillation counter with a photomultiplier tube.



In tracer work (Section 31–7), **liquid scintillators** are often used. Radioactive samples taken at different times or from different parts of an organism are placed directly in small bottles containing the liquid scintillator. This is particularly convenient for detection of β rays from ${}^{3}_{1}H$ and ${}^{14}_{6}C$, which have very low energies and have difficulty passing through the outer covering of a crystal scintillator or Geiger tube. A PM tube is still used to produce the electric signal from the liquid scintillator.

A **semiconductor detector** consists of a reverse-biased *pn* junction diode (Section 29–9). A charged particle passing through the junction can excite electrons into the conduction band, leaving holes in the valence band. The freed charges produce a short electric current pulse that can be counted as for Geiger and scintillation counters.

Hospital workers and others who work around radiation may carry *film badges* which detect the accumulation of radiation exposure. The film inside is periodically replaced and developed, the darkness of the developed film being related to total exposure (see Section 31–5).

Visualization

The devices discussed so far are used for counting the number of particles (or decays of a radioactive isotope). There are also devices that allow the track of charged particles to be *seen*. Very important are semiconductor detectors. **Silicon wafer semiconductors** have their surface etched into separate tiny pixels, each providing particle position information. They are much used in elementary particle physics (Chapter 32) to track the positions of particles produced and to determine their point of origin and/or their momentum (with the help of a magnetic field). The pixel arrangement can be CCD or CMOS (Section 25–1), the latter able to incorporate electronics inside, allowing fast readout.

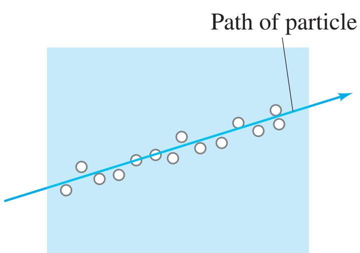


FIGURE 30–15 In a cloud chamber or bubble chamber, droplets or bubbles are formed around ions produced by the passage of a charged particle.

One of the oldest tracking devices is the **photographic emulsion**, which can be small and portable, used now particularly for cosmic-ray studies from balloons. A charged particle passing through an emulsion ionizes the atoms along its path. These points undergo a chemical change, and when the emulsion is developed (like film) the particle's path is revealed.

In a **cloud chamber**, used in the early days of nuclear physics, a gas is cooled to a temperature slightly below its usual condensation point ("supercooled"). Tiny droplets form around ions produced when a charged particle passes through (Fig. 30–15). Light scattering from these droplets reveals the track of the particle.

The **bubble chamber**, invented in 1952 by Donald A. Glaser (1926–2013), makes use of a superheated liquid kept close to its normal boiling point. Bubbles characteristic of boiling form around ions produced by the passage of a charged particle, revealing paths of particles that recently passed through. Because a bubble chamber uses a liquid, often liquid hydrogen, many more interactions can occur than in a cloud chamber. A magnetic field applied across the chamber makes charged particle paths curve (Chapter 20) and allows the momentum of charged particles to be determined from the radius of curvature of their paths.

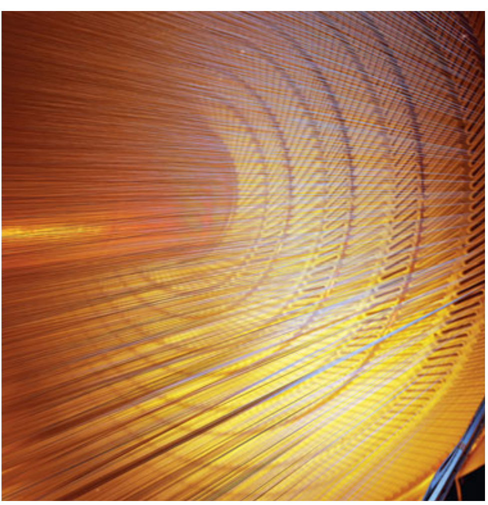


FIGURE 30–16 Multiwire chamber inside the Collider Detector at Fermilab (CDF). Figure 32–15 in Chapter 32 was made with this detector.

A **multiwire**† **chamber** consists of a set of closely spaced fine wires immersed in a gas (Fig. 30–16). Many wires are grounded, and the others between are kept at very high voltage. A charged particle passing through produces ions in the gas. Freed electrons drift toward the nearest high voltage wire, creating an "avalanche" of many more ions, and producing an electric pulse or signal at that wire. The positions of the particles are determined electronically by the position of the wire and by the time it takes the pulses to reach "readout" electronics at the ends of the wires. The paths of the particles are reconstructed electronically by computers which can "draw" a picture of the tracks, as shown in Fig. 32–15, Chapter 32. An external magnetic field curves the paths, allowing the momentum of the particles to be measured.

Many detectors are also **calorimeters** which measure the energy of the particles.

† Also called *wire drift chamber* or *wire proportional chamber*.

Summary

Nuclear physics is the study of atomic nuclei. Nuclei contain **protons** and **neutrons**, which are collectively known as **nucleons**. The total number of nucleons, A, is the nucleus's **atomic mass number**. The number of protons, Z, is the **atomic number**. The number of neutrons equals $A - Z$. **Isotopes** are nuclei with the same Z, but with different numbers of neutrons. For an element X, an isotope of given Z and A is represented by

$$^A_Z\text{X}.$$

The nuclear radius is approximately proportional to $A^{\frac{1}{3}}$, indicating that all nuclei have about the same density. Nuclear masses are specified in **unified atomic mass units** (u), where the mass of $^{12}_{6}\text{C}$ (including its 6 electrons) is defined as exactly 12.000000 u. In terms of the energy equivalent (because $E = mc^2$),

$$1\text{ u} = 931.5\text{ MeV}/c^2 = 1.66 \times 10^{-27}\text{ kg}.$$

The mass of a stable nucleus is less than the sum of the masses of its constituent nucleons. The difference in mass (times c^2) is the **total binding energy**. It represents the energy needed to break the nucleus into its constituent nucleons. The **binding energy per nucleon** averages about 8 MeV per nucleon, and is lowest for low mass and high mass nuclei.

Unstable nuclei undergo **radioactive decay**; they change into other nuclei with the emission of an α, β, or γ particle. An α particle is a ^4_2He nucleus; a β particle is an electron or positron; and a γ ray is a high-energy photon. In β decay, a **neutrino** is also emitted. The transformation of **parent** nuclei into **daughter** nuclei is called **transmutation** of the elements. Radioactive decay occurs spontaneously only when the mass of the products is less than the mass of the parent nucleus. The loss in mass appears as kinetic energy of the products.

Nuclei are held together by the **strong nuclear force**. The **weak nuclear force** makes itself apparent in β decay. These two forces, plus the gravitational and electromagnetic forces, are the four known types of force.

Electric charge, linear and angular momentum, mass–energy, and **nucleon number** are **conserved** in all decays.

Radioactive decay is a statistical process. For a given type of radioactive nucleus, the number of nuclei that decay (ΔN) in a time Δt is proportional to the number N of parent nuclei present:

$$\Delta N = -\lambda N\,\Delta t; \qquad \textbf{(30–3a)}$$

the minus sign means N *decreases* in time.

The proportionality constant λ is called the **decay constant** and is characteristic of the given nucleus. The number N of nuclei remaining after a time t decreases exponentially,

$$N = N_0 e^{-\lambda t}, \qquad \textbf{(30–4)}$$

as does the **activity**, R = magnitude of $\Delta N/\Delta t$:

$$R = \left|\frac{\Delta N}{\Delta t}\right|_0 e^{-\lambda t}. \qquad \textbf{(30–5)}$$

The **half-life**, $T_{\frac{1}{2}}$, is the time required for half the nuclei of a radioactive sample to decay. It is related to the decay constant by

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda}\cdot \qquad \textbf{(30–6)}$$

Radioactive dating is the use of radioactive decay to determine the age of certain objects, such as carbon dating.

[*Alpha decay occurs via a purely quantum-mechanical process called **tunneling** through a barrier.]

Particle **detectors** include **Geiger counters**, **scintillators** with attached **photomultiplier tubes**, and **semiconductor detectors**. Detectors that can image particle tracks include **semiconductors**, photographic **emulsions, bubble chambers**, and **multiwire chambers**.

Questions

1. What do different isotopes of a given element have in common? How are they different?

2. What are the elements represented by the X in the following: (*a*) $^{232}_{92}\text{X}$; (*b*) $^{18}_{7}\text{X}$; (*c*) $^{1}_{1}\text{X}$; (*d*) $^{86}_{38}\text{X}$; (*e*) $^{252}_{100}\text{X}$?

3. How many protons and how many neutrons do each of the isotopes in Question 2 have?

4. Identify the element that has 87 nucleons and 50 neutrons.

5. Why are the atomic masses of many elements (see the Periodic Table) not close to whole numbers?

6. Why are atoms much more likely to emit an alpha particle than to emit separate neutrons and protons?

7. What are the similarities and the differences between the strong nuclear force and the electric force?

8. What is the experimental evidence in favor of radioactivity being a nuclear process?

9. The isotope $^{64}_{29}\text{Cu}$ is unusual in that it can decay by γ, β^-, and β^+ emission. What is the resulting nuclide for each case?

10. A $^{238}_{92}\text{U}$ nucleus decays via α decay to a nucleus containing how many neutrons?

11. Describe, in as many ways as you can, the difference between α, β, and γ rays.

12. Fill in the missing particle or nucleus:
(*a*) $^{45}_{20}\text{Ca} \rightarrow ? + e^- + \bar{\nu}$
(*b*) $^{58}_{29}\text{Cu}^* \rightarrow ? + \gamma$
(*c*) $^{46}_{24}\text{Cr} \rightarrow {}^{46}_{23}\text{V} + ?$
(*d*) $^{234}_{94}\text{Pu} \rightarrow ? + \alpha$
(*e*) $^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + ?$

13. Immediately after a $^{238}_{92}\text{U}$ nucleus decays to $^{234}_{90}\text{Th} + {}^4_2\text{He}$, the daughter thorium nucleus may still have 92 electrons circling it. Since thorium normally holds only 90 electrons, what do you suppose happens to the two extra ones?

14. When a nucleus undergoes either β^- or β^+ decay, what happens to the energy levels of the atomic electrons? What is likely to happen to these electrons following the decay?

15. The alpha particles from a given alpha-emitting nuclide are generally monoenergetic; that is, they all have the same kinetic energy. But the beta particles from a beta-emitting nuclide have a spectrum of energies. Explain the difference between these two cases.

16. Do isotopes that undergo electron capture generally lie above or below the stable nuclides in Fig. 30–2?

17. Can hydrogen or deuterium emit an α particle? Explain.

18. Why are many artificially produced radioactive isotopes rare in nature?

19. An isotope has a half-life of one month. After two months, will a given sample of this isotope have completely decayed? If not, how much remains?

20. Why are none of the elements with $Z > 92$ stable?

21. A proton strikes a $^{6}_{3}\text{Li}$ nucleus. As a result, an α particle and another particle are released. What is the other particle?

22. Can $^{14}_{6}\text{C}$ dating be used to measure the age of stone walls and tablets of ancient civilizations? Explain.

23. Explain the absence of β^{+} emitters in the radioactive decay series of Fig. 30–11.

24. As $^{222}_{86}\text{Rn}$ decays into $^{206}_{82}\text{Pb}$, how many alpha and beta particles are emitted? Does it matter which path in the decay series is chosen? Why or why not?

25. A ^{238}U nucleus (initially at rest) decays into a ^{234}Th nucleus and an alpha particle. Which has the greater (i) momentum, (ii) velocity, (iii) kinetic energy? Explain.
(*a*) The ^{234}Th nucleus.
(*b*) The alpha particle.
(*c*) Both the same.

MisConceptual Questions

1. Elements of the Periodic Table are distinguished by
(*a*) the number of protons in the nucleus.
(*b*) the number of neutrons in the nucleus.
(*c*) the number of electrons in the atom.
(*d*) Both (*a*) and (*b*).
(*e*) (*a*), (*b*), and (*c*).

2. A nucleus has
(*a*) more energy than its component neutrons and protons have.
(*b*) less energy than its component neutrons and protons have.
(*c*) the same energy as its component neutrons and protons have.
(*d*) more energy than its component neutrons and protons have when the nucleus is at rest but less energy than when it is moving.

3. Which of the following will generally create a more stable nucleus?
(*a*) Having more nucleons.
(*b*) Having more protons than neutrons.
(*c*) Having a larger binding energy per nucleon.
(*d*) Having the same number of electrons as protons.
(*e*) Having a larger total binding energy.

4. There are 82 protons in a lead nucleus. Why doesn't the lead nucleus burst apart?
(*a*) Coulomb repulsive force doesn't act inside the nucleus.
(*b*) Gravity overpowers the Coulomb repulsive force inside the nucleus.
(*c*) The negatively charged neutrons balance the positively charged protons.
(*d*) Protons lose their positive charge inside the nucleus.
(*e*) The strong nuclear force holds the nucleus together.

5. The half-life of a radioactive nucleus is
(*a*) half the time it takes for the entire substance to decay.
(*b*) the time it takes for half of the substance to decay.
(*c*) the same as the decay constant.
(*d*) Both (*a*) and (*b*) (they are the same).
(*e*) All of the above.

6. As a radioactive sample decays,
(*a*) the half-life increases.
(*b*) the half-life decreases.
(*c*) the activity remains the same.
(*d*) the number of radioactive nuclei increases.
(*e*) None of the above.

7. If the half-life of a radioactive sample is 10 years, then it should take _____ years for the sample to decay completely.
(*a*) 10. (*b*) 20. (*c*) 40.
(*d*) Cannot be determined.

8. A sample's half-life is 1 day. What fraction of the original sample will have decayed after 3 days?
(*a*) $\frac{1}{8}$. (*b*) $\frac{1}{4}$. (*c*) $\frac{1}{2}$. (*d*) $\frac{3}{4}$. (*e*) $\frac{7}{8}$. (*f*) All of it.

9. After three half-lives, what fraction of the original radioactive material is left?
(*a*) None. (*b*) $\frac{1}{16}$. (*c*) $\frac{1}{8}$. (*d*) $\frac{1}{4}$. (*e*) $\frac{3}{4}$. (*f*) $\frac{7}{8}$.

10. Technicium $^{98}_{43}\text{Tc}$ has a half-life of 4.2×10^{6} yr. Strontium $^{90}_{38}\text{Sr}$ has a half-life of 28.79 yr. Which statements are true?
(*a*) The decay constant of Sr is greater than the decay constant of Tc.
(*b*) The activity of 100 g of Sr is less than the activity of 100 g of Tc.
(*c*) The long half-life of Tc means that it decays by alpha decay.
(*d*) A Tc atom has a higher probability of decaying in 1 yr than a Sr atom.
(*e*) 28.79 g of Sr has the same activity as 4.2×10^{6} g of Tc.

11. A material having which decay constant would have the shortest half-life?
(*a*) 100/second.
(*b*) 5/year.
(*c*) 8/century.
(*d*) 10^{9}/day.

12. Uranium-238 decays to lead-206 through a series of
(*a*) alpha decays.
(*b*) beta decays.
(*c*) gamma decays.
(*d*) some combination of alpha, beta, and gamma decays.

13. Carbon dating is useful only for determining the age of objects less than about _____ years old.
(*a*) 4.5 million.
(*b*) 1.2 million.
(*c*) 600,000.
(*d*) 60,000.
(*e*) 6000.

14. Radon has a half-life of about 1600 years. The Earth is several billion years old, so why do we still find radon on this planet?
(*a*) Ice-age temperatures preserved some of it.
(*b*) Heavier unstable isotopes decay into it.
(*c*) It is created in lightning strikes.
(*d*) It is replenished by cosmic rays.
(*e*) Its half-life has increased over time.
(*f*) Its half-life has decreased over time.

15. How does an atom's nucleus stay together and remain stable?
(*a*) The attractive gravitational force between the protons and neutrons overcomes the repulsive electrostatic force between the protons.
(*b*) Having just the right number of neutrons overcomes the electrostatic force between the protons.
(*c*) A strong covalent bond develops between the neutrons and protons, because they are so close to each other.
(*d*) None of the above.

16. What has greater mass?
(*a*) A neutron and a proton that are far from each other (unbound).
(*b*) A neutron and a proton that are bound together in a hydrogen (deuterium) nucleus.
(*c*) Both the same.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

[See Appendix B for masses]

30–1 Nuclear Properties

1. (I) A pi meson has a mass of 139 MeV/c^2. What is this in atomic mass units?

2. (I) What is the approximate radius of an α particle $\left({}^4_2\text{He}\right)$?

3. (I) By what % is the radius of ${}^{238}_{92}\text{U}$ greater than the radius of ${}^{232}_{92}\text{U}$?

4. (II) (*a*) What is the approximate radius of a ${}^{112}_{48}\text{Cd}$ nucleus? (*b*) Approximately what is the value of A for a nucleus whose radius is 3.7×10^{-15} m?

5. (II) What is the mass of a bare α particle (without electrons) in MeV/c^2?

6. (II) Suppose two alpha particles were held together so they were just "touching" (use Eq. 30–1). Estimate the electrostatic repulsive force each would exert on the other. What would be the acceleration of an alpha particle subjected to this force?

7. (II) (*a*) What would be the radius of the Earth if it had its actual mass but had the density of nuclei? (*b*) By what factor would the radius of a ${}^{238}_{92}\text{U}$ nucleus increase if it had the Earth's density?

8. (II) What stable nucleus has approximately half the radius of a uranium nucleus? [*Hint*: Find A and use Appendix B to get Z.]

9. (II) If an alpha particle were released from rest near the surface of a ${}^{257}_{100}\text{Fm}$ nucleus, what would its kinetic energy be when far away?

10. (II) (*a*) What is the fraction of the hydrogen atom's mass $\left({}^1_1\text{H}\right)$ that is in the nucleus? (*b*) What is the fraction of the hydrogen atom's volume that is occupied by the nucleus?

11. (II) Approximately how many nucleons are there in a 1.0-kg object? Does it matter what the object is made of? Why or why not?

12. (III) How much kinetic energy, in MeV, must an α particle have to just "touch" the surface of a ${}^{232}_{92}\text{U}$ nucleus?

30–2 Binding Energy

13. (I) Estimate the total binding energy for ${}^{63}_{29}\text{Cu}$, using Fig. 30–1.

14. (I) Use Fig. 30–1 to estimate the total binding energy of (*a*) ${}^{238}_{92}\text{U}$, and (*b*) ${}^{84}_{36}\text{Kr}$.

15. (II) Calculate the binding energy per nucleon for a ${}^{15}_{7}\text{N}$ nucleus, using Appendix B.

16. (II) Use Appendix B to calculate the binding energy of ${}^2_1\text{H}$ (deuterium).

17. (II) Determine the binding energy of the last neutron in a ${}^{23}_{11}\text{Na}$ nucleus.

18. (II) Calculate the total binding energy, and the binding energy per nucleon, for (*a*) ${}^7_3\text{Li}$, (*b*) ${}^{195}_{78}\text{Pt}$. Use Appendix B.

19. (II) Compare the average binding energy of a nucleon in ${}^{23}_{11}\text{Na}$ to that in ${}^{24}_{11}\text{Na}$, using Appendix B.

20. (III) How much energy is required to remove (*a*) a proton, (*b*) a neutron, from ${}^{15}_{7}\text{N}$? Explain the difference in your answers.

21. (III) (*a*) Show that the nucleus ${}^8_4\text{Be}$ (mass = 8.005305 u) is unstable and will decay into two α particles. (*b*) Is ${}^{12}_{6}\text{C}$ stable against decay into three α particles? Show why or why not.

30–3 to 30–7 Radioactive Decay

22. (I) The ${}^7_3\text{Li}$ nucleus has an excited state 0.48 MeV above the ground state. What wavelength gamma photon is emitted when the nucleus decays from the excited state to the ground state?

23. (II) Show that the decay ${}^{11}_{6}\text{C} \rightarrow {}^{10}_{5}\text{B} + \text{p}$ is not possible because energy would not be conserved.

24. (II) Calculate the energy released when tritium, ${}^3_1\text{H}$, decays by β^- emission.

25. (II) What is the maximum kinetic energy of an electron emitted in the β decay of a free neutron?

26. (II) Give the result of a calculation that shows whether or not the following decays are possible:
(*a*) ${}^{233}_{92}\text{U} \rightarrow {}^{232}_{92}\text{U} + \text{n}$;
(*b*) ${}^{14}_{7}\text{N} \rightarrow {}^{13}_{7}\text{N} + \text{n}$;
(*c*) ${}^{40}_{19}\text{K} \rightarrow {}^{39}_{19}\text{K} + \text{n}$.

27. (II) ${}^{24}_{11}\text{Na}$ is radioactive. (*a*) Is it a β^- or β^+ emitter? (*b*) Write down the decay reaction, and estimate the maximum kinetic energy of the emitted β.

28. (II) A $^{238}_{92}U$ nucleus emits an α particle with kinetic energy = 4.20 MeV. (*a*) What is the daughter nucleus, and (*b*) what is the approximate atomic mass (in u) of the daughter atom? Ignore recoil of the daughter nucleus.

29. (II) Calculate the maximum kinetic energy of the β particle emitted during the decay of $^{60}_{27}Co$.

30. (II) How much energy is released in electron capture by beryllium: $^{7}_{4}Be + e^- \rightarrow {}^{7}_{3}Li + \nu$?

31. (II) The isotope $^{218}_{84}Po$ can decay by either α or β^- emission. What is the energy release in each case? The mass of $^{218}_{84}Po$ is 218.008973 u.

32. (II) The nuclide $^{32}_{15}P$ decays by emitting an electron whose maximum kinetic energy can be 1.71 MeV. (*a*) What is the daughter nucleus? (*b*) Calculate the daughter's atomic mass (in u).

33. (II) A photon with a wavelength of 1.15×10^{-13} m is ejected from an atom. Calculate its energy and explain why it is a γ ray from the nucleus or a photon from the atom.

34. (II) How much recoil energy does a $^{40}_{19}K$ nucleus get when it emits a 1.46-MeV gamma ray?

35. (II) Determine the maximum kinetic energy of β^+ particles released when $^{11}_{6}C$ decays to $^{11}_{5}B$. What is the maximum energy the neutrino can have? What is the minimum energy of each?

36. (III) Show that when a nucleus decays by β^+ decay, the total energy released is equal to

$$(M_P - M_D - 2m_e)c^2,$$

where M_P and M_D are the masses of the parent and daughter atoms (neutral), and m_e is the mass of an electron or positron.

37. (III) When $^{238}_{92}U$ decays, the α particle emitted has 4.20 MeV of kinetic energy. Calculate the recoil kinetic energy of the daughter nucleus and the Q-value of the decay.

30–8 to 30–11 Half-Life, Decay Rates, Decay Series, Dating

38. (I) (*a*) What is the decay constant of $^{238}_{92}U$ whose half-life is 4.5×10^9 yr? (*b*) The decay constant of a given nucleus is $3.2 \times 10^{-5}\ s^{-1}$. What is its half-life?

39. (I) A radioactive material produces 1120 decays per minute at one time, and 3.6 h later produces 140 decays per minute. What is its half-life?

40. (I) What fraction of a sample of $^{68}_{32}Ge$, whose half-life is about 9 months, will remain after 2.5 yr?

41. (I) What is the activity of a sample of $^{14}_{6}C$ that contains 6.5×10^{20} nuclei?

42. (I) What fraction of a radioactive sample is left after exactly 5 half-lives?

43. (II) The iodine isotope $^{131}_{53}I$ is used in hospitals for diagnosis of thyroid function. If 782 μg are ingested by a patient, determine the activity (*a*) immediately, (*b*) 1.50 h later when the thyroid is being tested, and (*c*) 3.0 months later. Use Appendix B.

44. (II) How many nuclei of $^{238}_{92}U$ remain in a rock if the activity registers 420 decays per second?

45. (II) In a series of decays, the nuclide $^{235}_{92}U$ becomes $^{207}_{82}Pb$. How many α and β^- particles are emitted in this series?

46. (II) $^{124}_{55}Cs$ has a half-life of 30.8 s. (*a*) If we have 8.7 μg initially, how many Cs nuclei are present? (*b*) How many are present 2.6 min later? (*c*) What is the activity at this time? (*d*) After how much time will the activity drop to less than about 1 per second?

47. (II) Calculate the mass of a sample of pure $^{40}_{19}K$ with an initial decay rate of $2.4 \times 10^5\ s^{-1}$. The half-life of $^{40}_{19}K$ is 1.248×10^9 yr.

48. (II) Calculate the activity of a pure 6.7-μg sample of $^{32}_{15}P$ $(T_{\frac{1}{2}} = 1.23 \times 10^6\ s)$.

49. (II) A sample of $^{233}_{92}U$ $(T_{\frac{1}{2}} = 1.59 \times 10^5\ yr)$ contains 4.50×10^{18} nuclei. (*a*) What is the decay constant? (*b*) Approximately how many disintegrations will occur per minute?

50. (II) The activity of a sample drops by a factor of 6.0 in 9.4 minutes. What is its half-life?

51. (II) A 345-g sample of pure carbon contains 1.3 parts in 10^{12} (atoms) of $^{14}_{6}C$. How many disintegrations occur per second?

52. (II) A sample of $^{238}_{92}U$ is decaying at a rate of 4.20×10^2 decays/s. What is the mass of the sample?

53. (II) **Rubidium–strontium dating**. The rubidium isotope $^{87}_{37}Rb$, a β emitter with a half-life of 4.75×10^{10} yr, is used to determine the age of rocks and fossils. Rocks containing fossils of ancient animals contain a ratio of $^{87}_{38}Sr$ to $^{87}_{37}Rb$ of 0.0260. Assuming that there was no $^{87}_{38}Sr$ present when the rocks were formed, estimate the age of these fossils.

54. (II) Two of the naturally occurring radioactive decay sequences start with $^{232}_{90}Th$ and with $^{235}_{92}U$. The first five decays of these two sequences are:

$$\alpha,\ \beta,\ \beta,\ \alpha,\ \alpha$$

and

$$\alpha,\ \beta,\ \alpha,\ \beta,\ \alpha.$$

Determine the resulting intermediate daughter nuclei in each case.

55. (II) An ancient wooden club is found that contains 73 g of carbon and has an activity of 7.0 decays per second. Determine its age assuming that in living trees the ratio of $^{14}C/^{12}C$ atoms is about 1.3×10^{-12}.

56. (II) Use Fig. 30–11 and calculate the relative decay rates for α decay of $^{218}_{84}Po$ and $^{214}_{84}Po$.

57. (III) The activity of a radioactive source decreases by 5.5% in 31.0 hours. What is the half-life of this source?

58. (III) $^{7}_{4}Be$ decays with a half-life of about 53 d. It is produced in the upper atmosphere, and filters down onto the Earth's surface. If a plant leaf is detected to have 350 decays/s of $^{7}_{4}Be$, (*a*) how long do we have to wait for the decay rate to drop to 25 per second? (*b*) Estimate the initial mass of $^{7}_{4}Be$ on the leaf.

59. (III) At $t = 0$, a pure sample of radioactive nuclei contains N_0 nuclei whose decay constant is λ. Determine a formula for the number of daughter nuclei, N_D, as a function of time; assume the daughter is stable and that $N_D = 0$ at $t = 0$.

General Problems

60. Which radioactive isotope of lead is being produced if the measured activity of a sample drops to 1.050% of its original activity in 4.00 h?

61. An old wooden tool is found to contain only 4.5% of the $^{14}_{6}C$ that an equal mass of fresh wood would. How old is the tool?

62. A neutron star consists of neutrons at approximately nuclear density. Estimate, for a 10-km-diameter neutron star, (*a*) its mass number, (*b*) its mass (kg), and (*c*) the acceleration of gravity at its surface.

63. **Tritium dating**. The $^{3}_{1}H$ isotope of hydrogen, which is called *tritium* (because it contains three nucleons), has a half-life of 12.3 yr. It can be used to measure the age of objects up to about 100 yr. It is produced in the upper atmosphere by cosmic rays and brought to Earth by rain. As an application, determine approximately the age of a bottle of wine whose $^{3}_{1}H$ radiation is about $\frac{1}{10}$ that present in new wine.

64. Some elementary particle theories (Section 32–11) suggest that the proton may be unstable, with a half-life $\geq 10^{33}$ yr. (*a*) How long would you expect to wait for one proton in your body to decay (approximate your body as all water)? (*b*) Of the roughly 7 billion people on Earth, about how many would have a proton in their body decay in a 70 yr lifetime?

65. The original experiments which established that an atom has a heavy, positive nucleus were done by shooting alpha particles through gold foil. The alpha particles had a kinetic energy of 7.7 MeV. What is the closest they could get to the center of a gold nucleus? How does this compare with the size of the nucleus?

66. How long must you wait (in half-lives) for a radioactive sample to drop to 2.00% of its original activity?

67. If the potassium isotope $^{40}_{19}K$ gives 42 decays/s in a liter of milk, estimate how much $^{40}_{19}K$ and regular $^{39}_{19}K$ are in a liter of milk. Use Appendix B.

68. Strontium-90 is produced as a nuclear fission product of uranium in both reactors and atomic bombs. Look at its location in the Periodic Table to see what other elements it might be similar to chemically, and tell why you think it might be dangerous to ingest. It has too many neutrons to be stable, and it decays with a half-life of about 29 yr. How long will we have to wait for the amount of $^{90}_{38}Sr$ on the Earth's surface to reach 1% of its current level, assuming no new material is scattered about? Write down the decay reaction, including the daughter nucleus. The daughter is radioactive: write down its decay.

69. The activity of a sample of $^{35}_{16}S$ $\left(T_{\frac{1}{2}} = 87.37 \text{ days}\right)$ is 4.28×10^{4} decays per second. What is the mass of the sample?

70. The nuclide $^{191}_{76}Os$ decays with β^- energy of 0.14 MeV accompanied by γ rays of energy 0.042 MeV and 0.129 MeV. (*a*) What is the daughter nucleus? (*b*) Draw an energy-level diagram showing the ground states of the parent and daughter and excited states of the daughter. (*c*) To which of the daughter states does β^- decay of $^{191}_{76}Os$ occur?

71. Determine the activities of (*a*) 1.0 g of $^{131}_{53}I$ $\left(T_{\frac{1}{2}} = 8.02 \text{ days}\right)$ and (*b*) 1.0 g of $^{238}_{92}U$ $\left(T_{\frac{1}{2}} = 4.47 \times 10^{9} \text{ yr}\right)$.

72. Use Fig. 30–1 to estimate the total binding energy for copper and then estimate the energy, in joules, needed to break a 3.0-g copper penny into its constituent nucleons.

73. Instead of giving atomic masses for nuclides as in Appendix B, some Tables give the **mass excess**, Δ, defined as $\Delta = M - A$, where A is the atomic mass number and M is the mass in u. Determine the mass excess, in u and in MeV/c^2, for: (*a*) $^{4}_{2}He$; (*b*) $^{12}_{6}C$; (*c*) $^{86}_{38}Sr$; (*d*) $^{235}_{92}U$. (*e*) From a glance at Appendix B, can you make a generalization about the sign of Δ as a function of Z or A?

74. When water is placed near an intense neutron source, the neutrons can be slowed down to almost zero speed by collisions with the water molecules, and are eventually captured by a hydrogen nucleus to form the stable isotope called **deuterium**, $^{2}_{1}H$, giving off a gamma ray. What is the energy of the gamma ray?

75. The practical limit for carbon-14 dating is about 60,000 years. If a bone contains 1.0 kg of carbon, and the animal died 60,000 years ago, what is the activity today?

76. Using Section 30–2 and Appendix B, determine the energy required to remove one neutron from $^{4}_{2}He$. How many times greater is this energy than the binding energy of the last neutron in $^{13}_{6}C$?

77. (*a*) If all of the atoms of the Earth were to collapse and simply become nuclei, what would be the Earth's new radius? (*b*) If all of the atoms of the Sun were to collapse and simply become nuclei, what would be the Sun's new radius?

78. (*a*) A 72-gram sample of natural carbon contains the usual fraction of $^{14}_{6}C$. Estimate roughly how long it will take before there is only one $^{14}_{6}C$ nucleus left. (*b*) How does the answer in (*a*) change if the sample is 340 grams? What does this tell you about the limits of carbon dating?

79. If the mass of the proton were just a little closer to the mass of the neutron, the following reaction would be possible even at low collision energies:

$$e^- + p \rightarrow n + \nu.$$

(*a*) Why would this situation be catastrophic? (See last paragraph of Chapter 33.) (*b*) By what percentage would the proton's mass have to be increased to make this reaction possible?

80. What is the ratio of the kinetic energies for an alpha particle and a beta particle if both make tracks with the same radius of curvature in a magnetic field, oriented perpendicular to the paths of the particles?

81. A 1.00-g sample of natural samarium emits α particles at a rate of $120\ \text{s}^{-1}$ due to the presence of $^{147}_{62}Sm$. The natural abundance of $^{147}_{62}Sm$ is 15%. Calculate the half-life for this decay process.

82. Almost all of naturally occurring uranium is $^{238}_{92}U$ with a half-life of 4.468×10^{9} yr. Most of the rest of natural uranium is $^{235}_{92}U$ with a half-life of 7.04×10^{8} yr. Today a sample contains 0.720% $^{235}_{92}U$. (*a*) What was this percentage 1.0 billion years ago? (*b*) What percentage of uranium will be $^{235}_{92}U$ 100 million years from now?

83. A banana contains about 420 mg of potassium, of which a small fraction is the radioactive isotope ${}^{40}_{19}\mathrm{K}$ (Appendix B). Estimate the activity of an average banana due to ${}^{40}_{19}\mathrm{K}$.

84. When ${}^{23}_{10}\mathrm{Ne}$ (mass = 22.9947 u) decays to ${}^{23}_{11}\mathrm{Na}$ (mass = 22.9898 u), what is the maximum kinetic energy of the emitted electron? What is its minimum energy? What is the energy of the neutrino in each case? Ignore recoil of the daughter nucleus.

85. (*a*) In α decay of, say, a ${}^{226}_{88}\mathrm{Ra}$ nucleus, show that the nucleus carries away a fraction $1/\left(1 + \frac{1}{4}A_{\mathrm{D}}\right)$ of the total energy available, where A_{D} is the mass number of the daughter nucleus. [*Hint*: Use conservation of momentum as well as conservation of energy.] (*b*) Approximately what percentage of the energy available is thus carried off by the α particle when ${}^{226}_{88}\mathrm{Ra}$ decays?

86. Decay series, such as that shown in Fig. 30–11, can be classified into four families, depending on whether the mass numbers have the form $4n$, $4n + 1$, $4n + 2$, or $4n + 3$, where n is an integer. Justify this statement and show that for a nuclide in any family, all its daughters will be in the same family.

Search and Learn

1. Describe in detail why we think there is a strong nuclear force.

2. (*a*) Under what circumstances could a fermium nucleus decay into an einsteinium nucleus? (*b*) What about the reverse, an Es nucleus decaying into Fm?

3. Using the uncertainty principle and the radius of a nucleus, estimate the minimum possible kinetic energy of a nucleon in, say, iron. Ignore relativistic corrections. [*Hint*: A particle can have a momentum at least as large as its momentum uncertainty.]

4. In Fig. 30–17, a nucleus decays and emits a particle that enters a region with a uniform magnetic field of 0.012 T directed into the page. The path of the detected particle is shown. (*a*) What type of radioactive decay is this? (*b*) If the radius of the circular arc is 4.7 mm, what is the velocity of the particle?

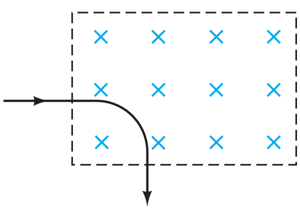


FIGURE 30–17 Search and Learn 4.

5. In both internal conversion and β decay, an electron is emitted. How could you determine which decay process occurred?

6. Suppose we discovered that several thousand years ago cosmic rays had bombarded the Earth's atmosphere a lot more than we had thought. Compared to previous calculations of the carbon-dated age of organic matter, we would now calculate it to be older, younger, or the same age as previously calculated? Explain.

7. In 1991, the frozen remains of a Neolithic-age man, nicknamed Otzi, were found in the Italian Alps by hikers. The body was well preserved, as were his bow, arrows, knife, axe, other tools, and clothing. The date of his death can be determined using carbon-14 dating. (*a*) What is the decay constant for ${}^{14}_{6}\mathrm{C}$? (*b*) How many ${}^{14}_{6}\mathrm{C}$ atoms per gram of ${}^{12}_{6}\mathrm{C}$ are there in a living organism? (*c*) What is the activity per gram in naturally occurring carbon for a living organism? (*d*) For Otzi, the activity per gram of carbon was measured to be 0.121. How long ago did he live?

8. Some radioactive isotopes have half-lives that are greater than the age of the universe (like gadolinium or samarium). The only way to determine these half-lives is to monitor the decay rate of a sample that contains these isotopes. For example, suppose we find an asteroid that currently contains about 15,000 kg of ${}^{152}_{64}\mathrm{Gd}$ (gadolinium) and we detect an activity of 1 decay/s. Estimate the half-life of gadolinium (in years).

ANSWERS TO EXERCISES

A: 0.042130 u.
B: 7.98 MeV/nucleon.
C: (*b*) Gd.
D: (*b*) $\frac{1}{8}\,\mu$g.
E: (*b*) 20 μg.