

A science fantasy book called *Mr Tompkins in Wonderland* (1940), by physicist George Gamow, imagined a world in which the speed of light was only 10 m/s (20 mi/h). Mr Tompkins had studied relativity and when he began “speeding” on a bicycle, he “expected that he would be immediately shortened, and was very happy about it as his increasing figure had lately caused him some anxiety. To his great surprise, however, nothing happened to him or to his cycle. On the other hand, the picture around him completely changed. The streets grew shorter, the windows of the shops began to look like narrow slits, and the policeman on the corner became the thinnest man he had ever seen. ‘By Jove!’ exclaimed Mr Tompkins excitedly, ‘I see the trick now. This is where the word *relativity* comes in.’”

Relativity does indeed predict that objects moving relative to us at high speed, close to the speed of light c , are shortened in length. We don’t notice it as Mr Tompkins did, because $c = 3 \times 10^8$ m/s is incredibly fast. We will study length contraction, time dilation, simultaneity non-agreement, and how energy and mass are equivalent ($E = mc^2$).



CHAPTER 26

The Special Theory of Relativity

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CHAPTER-OPENING QUESTION—Guess now!

A rocket is headed away from Earth at a speed of $0.80c$. The rocket fires a small payload at a speed of $0.70c$ (relative to the rocket) aimed away from Earth. How fast is the payload moving relative to Earth?

- (a) $1.50c$;
- (b) a little less than $1.50c$;
- (c) a little over c ;
- (d) a little under c ;
- (e) $0.75c$.

Physics at the end of the nineteenth century looked back on a period of great progress. The theories developed over the preceding three centuries had been very successful in explaining a wide range of natural phenomena. Newtonian mechanics beautifully explained the motion of objects on Earth and in the heavens. Furthermore, it formed the basis for successful treatments of fluids, wave motion, and sound. Kinetic theory explained the behavior of gases and other materials. Maxwell’s theory of electromagnetism embodied all of electric and magnetic phenomena, and it predicted the existence of electromagnetic waves that would behave just like light—so light came to be thought of as an electromagnetic wave. Indeed, it seemed that the natural world, as seen through the eyes of physicists, was very well explained. A few puzzles remained, but it was felt that these would soon be explained using already known principles.

It did not turn out so simply. Instead, these puzzles were to be solved only by the introduction, in the early part of the twentieth century, of two revolutionary new theories that changed our whole conception of nature: the *theory of relativity* and *quantum theory*.

Physics as it was known at the end of the nineteenth century (what we've covered up to now in this book) is referred to as **classical physics**. The new physics that grew out of the great revolution at the turn of the twentieth century is now called **modern physics**. In this Chapter, we present the special theory of relativity, which was first proposed by Albert Einstein (1879–1955; Fig. 26–1) in 1905. In Chapter 27, we introduce the equally momentous quantum theory.

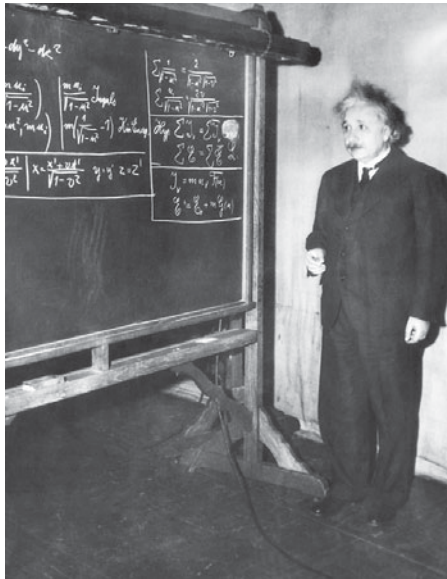


FIGURE 26–1 Albert Einstein (1879–1955), one of the great minds of the twentieth century, was the creator of the special and general theories of relativity.

26–1 Galilean–Newtonian Relativity

Einstein's special theory of relativity deals with how we observe events, particularly how objects and events are observed from different frames of reference.[†] This subject had already been explored by Galileo and Newton.

The special theory of relativity deals with events that are observed and measured from so-called **inertial reference frames** (Section 4–2 and Appendix C), which are reference frames in which Newton's first law is valid: if an object experiences no net force, the object either remains at rest or continues in motion with constant speed in a straight line. It is usually easiest to analyze events when they are observed and measured by observers at rest in an inertial frame. The Earth, though not quite an inertial frame (it rotates), is close enough that for most purposes we can approximate it as an inertial frame. Rotating or otherwise accelerating frames of reference are noninertial frames,[‡] and won't concern us in this Chapter (they are dealt with in Einstein's general theory of relativity, as we will see in Chapter 33).

A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame, since Newton's laws hold in it as well. When we say that we observe or make measurements from a certain reference frame, it means that we are at rest in that reference frame.

[†]A reference frame is a set of coordinate axes fixed to some object such as the Earth, a train, or the Moon. See Section 2–1.

[‡]On a rotating platform (say a merry-go-round), for example, a ball at rest starts moving outward even though no object exerts a force on it. This is therefore not an inertial frame. See Appendix C, Fig. C–1.

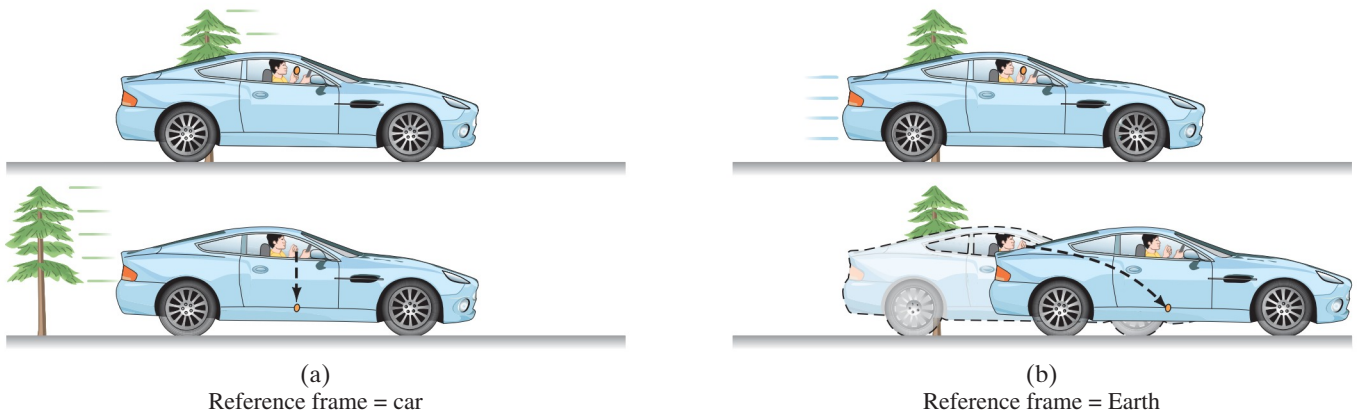


FIGURE 26-2 A coin is dropped by a person in a moving car. The upper views show the moment of the coin's release, the lower views are a short time later. (a) In the reference frame of the car, the coin falls straight down (and the tree moves to the left). (b) In a reference frame fixed on the Earth, the coin has an initial velocity (= to car's) and follows a curved (parabolic) path.

Both Galileo and Newton were aware of what we now call the **relativity principle** applied to mechanics: that *the basic laws of physics are the same in all inertial reference frames*. You may have recognized its validity in everyday life. For example, objects move in the same way in a smoothly moving (constant-velocity) train or airplane as they do on Earth. (This assumes no vibrations or rocking which would make the reference frame noninertial.) When you walk, drink a cup of soup, play pool, or drop a pencil on the floor while traveling in a train, airplane, or ship moving at constant velocity, the objects move just as they do when you are at rest on Earth. Suppose you are in a car traveling rapidly at constant velocity. If you drop a coin from above your head inside the car, how will it fall? It falls straight downward with respect to the car, and hits the floor directly below the point of release, Fig. 26–2a. This is just how objects fall on the Earth—straight down—and thus our experiment in the moving car is in accord with the relativity principle. (If you drop the coin out the car's window, this won't happen because the moving air drags the coin backward relative to the car.)

Note in this example, however, that to an observer on the Earth, the coin follows a curved path, Fig. 26–2b. The actual path followed by the coin is different as viewed from different frames of reference. This does not violate the relativity principle because this principle states that the *laws* of physics are the same in all inertial frames. The same law of gravity, and the same laws of motion, apply in both reference frames. The acceleration of the coin is the same in both reference frames. The difference in Figs. 26–2a and b is that in the Earth's frame of reference, the coin has an initial velocity (equal to that of the car). The laws of physics therefore predict it will follow a parabolic path like any projectile (Chapter 3). In the car's reference frame, there is no initial velocity, and the laws of physics predict that the coin will fall straight down. The laws are the same in both reference frames, although the specific paths are different.

Galilean–Newtonian relativity involves certain unprovable assumptions that make sense from everyday experience. It is assumed that the lengths of objects are the same in one reference frame as in another, and that time passes at the same rate in different reference frames. In classical mechanics, then, space and time intervals are considered to be **absolute**: their measurement does not change from one reference frame to another. The mass of an object, as well as all forces, are assumed to be unchanged by a change in inertial reference frame.

The position of an object, however, is different when specified in different reference frames, and so is velocity. For example, a person may walk inside a bus toward the front with a speed of 2 m/s. But if the bus moves 10 m/s with respect to the Earth, the person is then moving with a speed of 12 m/s with respect to the Earth. The acceleration of an object, however, is the same in any inertial reference frame according to classical mechanics. This is because the change in velocity, and the time interval, will be the same. For example, the person in the bus may accelerate from 0 to 2 m/s in 1.0 seconds, so $a = 2 \text{ m/s}^2$ in the reference frame of the bus. With respect to the Earth, the acceleration is

$$(12 \text{ m/s} - 10 \text{ m/s}) / (1.0 \text{ s}) = 2 \text{ m/s}^2,$$

which is the same.

CAUTION

Laws are the same, but paths may be different in different reference frames

CAUTION

Length and time intervals are absolute (pre-relativity)

CAUTION

Position and velocity are different in different reference frames, but length is the same (classical)

Since neither F , m , nor a changes from one inertial frame to another, Newton's second law, $F = ma$, does not change. Thus Newton's second law satisfies the relativity principle. The other laws of mechanics also satisfy the relativity principle.

That the laws of mechanics are the same in all inertial reference frames implies that no one inertial frame is special in any sense. We express this important conclusion by saying that **all inertial reference frames are equivalent** for the description of mechanical phenomena. No one inertial reference frame is any better than another. A reference frame fixed to a car or an aircraft traveling at constant velocity is as good as one fixed on the Earth. When you travel smoothly at constant velocity in a car or airplane, it is just as valid to say you are at rest and the Earth is moving as it is to say the reverse.[†] There is no experiment you can do to tell which frame is “really” at rest and which is moving. Thus, there is no way to single out one particular reference frame as being at absolute rest.

A complication arose, however, in the last half of the nineteenth century. Maxwell's comprehensive and successful theory of electromagnetism (Chapter 22) predicted that light is an electromagnetic wave. Maxwell's equations gave the velocity of light c as 3.00×10^8 m/s; and this is just what is measured. The question then arose: in what reference frame does light have precisely the value predicted by Maxwell's theory? It was assumed that light would have a different speed in different frames of reference. For example, if observers could travel on a rocket ship at a speed of 1.0×10^8 m/s away from a source of light, we might expect them to measure the speed of the light reaching them to be $(3.0 \times 10^8 \text{ m/s}) - (1.0 \times 10^8 \text{ m/s}) = 2.0 \times 10^8$ m/s. But Maxwell's equations have no provision for relative velocity. They predicted the speed of light to be $c = 3.0 \times 10^8$ m/s, which seemed to imply that there must be some preferred reference frame where c would have this value.

We discussed in Chapters 11 and 12 that waves can travel on water and along ropes or strings, and sound waves travel in air and other materials. Nineteenth-century physicists viewed the material world in terms of the laws of mechanics, so it was natural for them to assume that light too must travel in some *medium*. They called this transparent medium the **ether** and assumed it permeated all space.[‡] It was therefore assumed that the velocity of light given by Maxwell's equations must be with respect to the ether.[§]

Scientists soon set out to determine the speed of the Earth relative to this absolute frame, whatever it might be. A number of clever experiments were designed. The most direct were performed by A. A. Michelson and E. W. Morley in the 1880s. They measured the difference in the speed of light in different directions using Michelson's interferometer (Section 24–9). They expected to find a difference depending on the orientation of their apparatus with respect to the ether. For just as a boat has different speeds relative to the land when it moves upstream, downstream, or across the stream, so too light would be expected to have different speeds depending on the velocity of the ether past the Earth.

Strange as it may seem, they detected no difference at all. This was a great puzzle. A number of explanations were put forth over a period of years, but they led to contradictions or were otherwise not generally accepted. This **null result** was one of the great puzzles at the end of the nineteenth century.

Then in 1905, Albert Einstein proposed a radical new theory that reconciled these many problems in a simple way. But at the same time, as we shall see, it completely changed our ideas of space and time.

[†]We use the reasonable approximation that Earth is an inertial reference frame.

[‡]The medium for light waves could not be air, since light travels from the Sun to Earth through nearly empty space. Therefore, another medium was postulated, the ether. The ether was not only transparent but, because of difficulty in detecting it, was assumed to have zero density.

[§]Also, it appeared that Maxwell's equations did *not* satisfy the relativity principle: They were simplest in the frame where $c = 3.00 \times 10^8$ m/s, in a reference frame at rest in the ether. In any other reference frame, extra terms were needed to account for relative velocity. Although other laws of physics obeyed the relativity principle, the laws of electricity and magnetism apparently did not. Einstein's second postulate (next Section) resolved this problem: Maxwell's equations do satisfy relativity.

26–2 Postulates of the Special Theory of Relativity

The problems that existed at the start of the twentieth century with regard to electromagnetic theory and Newtonian mechanics were beautifully resolved by Einstein's introduction of the special theory of relativity in 1905. Unaware of the Michelson–Morley null result, Einstein was motivated by certain questions regarding electromagnetic theory and light waves. For example, he asked himself: “What would I see if I rode a light beam?” The answer was that instead of a traveling electromagnetic wave, he would see alternating electric and magnetic fields at rest whose magnitude changed in space, but did not change in time. Such fields, he realized, had never been detected and indeed were not consistent with Maxwell's electromagnetic theory. He argued, therefore, that it was unreasonable to think that the speed of light relative to any observer could be reduced to zero, or in fact reduced at all. This idea became the second postulate of his theory of relativity.

In his famous 1905 paper, Einstein proposed doing away with the idea of the ether and the accompanying assumption of a preferred or absolute reference frame at rest. This proposal was embodied in two postulates. The first was an extension of the Galilean–Newtonian relativity principle to include not only the laws of mechanics but also those of the rest of physics, including electricity and magnetism:

RELATIVITY PRINCIPLE

First postulate (the relativity principle): The laws of physics have the same form in all inertial reference frames.

The first postulate can also be stated as: *there is no experiment you can do in an inertial reference frame to determine if you are at rest or moving uniformly at constant velocity.*

The second postulate is consistent with the first:

SPEED OF LIGHT PRINCIPLE

Second postulate (constancy of the speed of light): Light propagates through empty space with a definite speed c independent of the speed of the source or observer.

These two postulates form the foundation of Einstein's **special theory of relativity**. It is called “special” to distinguish it from his later “general theory of relativity,” which deals with noninertial (accelerating) reference frames (Chapter 33). The special theory, which is what we discuss here, deals only with inertial frames.

The second postulate may seem hard to accept, for it seems to violate common sense. First of all, we have to think of light traveling through empty space. Giving up the ether is not too hard, however, since it had never been detected. But the second postulate also tells us that the speed of light in vacuum is always the same, 3.00×10^8 m/s, no matter what the speed of the observer or the source. Thus, a person traveling toward or away from a source of light will measure the same speed for that light as someone at rest with respect to the source. This conflicts with our everyday experience: we would expect to have to add in the velocity of the observer. On the other hand, perhaps we can't expect our everyday experience to be helpful when dealing with the high velocity of light. Furthermore, the null result of the Michelson–Morley experiment is fully consistent with the second postulate.[†]

Einstein's proposal has a certain beauty. By doing away with the idea of an absolute reference frame, it was possible to reconcile classical mechanics with Maxwell's electromagnetic theory. The speed of light predicted by Maxwell's equations *is* the speed of light in vacuum in *any* reference frame.

Einstein's theory required us to give up common sense notions of space and time, and in the following Sections we will examine some strange but interesting consequences of special relativity. Our arguments for the most part will be simple ones.

[†]The Michelson–Morley experiment can also be considered as evidence for the first postulate, since it was intended to measure the motion of the Earth relative to an absolute reference frame. Its failure to do so implies the absence of any such preferred frame.

We will use a technique that Einstein himself did: we will imagine very simple experimental situations in which little mathematics is needed. In this way, we can see many of the consequences of relativity theory without getting involved in detailed calculations. Einstein called these **thought experiments**.

26–3 Simultaneity

An important consequence of the theory of relativity is that we can no longer regard time as an absolute quantity. No one doubts that time flows onward and never turns back. But according to relativity, the time interval between two events, and even whether or not two events are simultaneous, depends on the observer's reference frame. By an **event**, which we use a lot here, we mean something that happens at a particular place and at a particular time.

Two events are said to occur simultaneously if they occur at exactly the same time. But how do we know if two events occur precisely at the same time? If they occur at the same point in space—such as two apples falling on your head at the same time—it is easy. But if the two events occur at widely separated places, it is more difficult to know whether the events are simultaneous since we have to take into account the time it takes for the light from them to reach us. Because light travels at finite speed, a person who sees two events must calculate back to find out when they actually occurred. For example, if two events are *observed* to occur at the same time, but one actually took place farther from the observer than the other, then the more distant one must have occurred earlier, and the two events were not simultaneous.

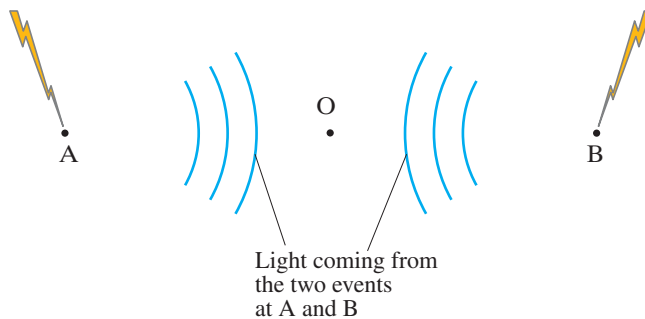
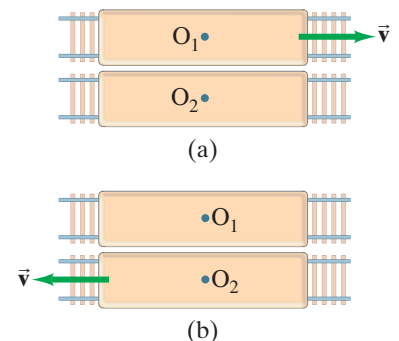


FIGURE 26–3 A moment after lightning strikes at points A and B, the pulses of light (shown as blue waves) are traveling toward the observer O, but O “sees” the lightning only when the light reaches O.

We now imagine a simple thought experiment. Assume an observer, called O, is located exactly halfway between points A and B where two events occur, Fig. 26–3. Suppose the two events are lightning that strikes the points A and B, as shown. For brief events like lightning, only short pulses of light (blue in Fig. 26–3) will travel outward from A and B and reach O. Observer O “sees” the events when the pulses of light reach point O. If the two pulses reach O at the same time, then the two events had to be simultaneous. This is because (i) the two light pulses travel at the same speed (postulate 2), and (ii) the distance OA equals OB, so the time for the light to travel from A to O and from B to O must be the same. Observer O can then definitely state that the two events occurred simultaneously. On the other hand, if O sees the light from one event before that from the other, then the former event occurred first.

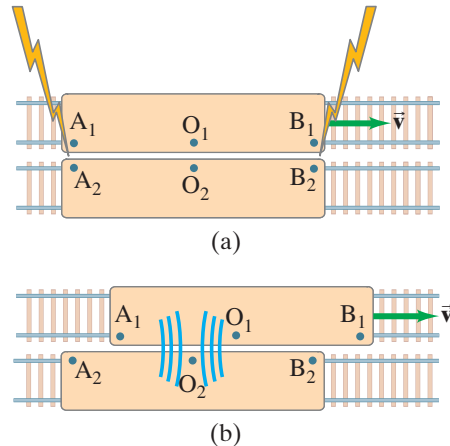
The question we really want to examine is this: if two events are simultaneous to an observer in one reference frame, are they also simultaneous to another observer moving with respect to the first? Let us call the observers O_1 and O_2 and assume they are fixed in reference frames 1 and 2 that move with speed v relative to one another. These two reference frames can be thought of as two rockets or two trains (Fig. 26–4). O_2 says that O_1 is moving to the right with speed v , as in Fig. 26–4a; and O_1 says O_2 is moving to the left with speed v , as in Fig. 26–4b. Both viewpoints are legitimate according to the relativity principle. [There is no third point of view that will tell us which one is “really” moving.]

FIGURE 26–4 Observers O_1 and O_2 , on two different trains (two different reference frames), are moving with relative speed v . (a) O_2 says that O_1 is moving to the right. (b) O_1 says that O_2 is moving to the left. Both viewpoints are legitimate: it all depends on your reference frame.



Now suppose that observers O_1 and O_2 observe and measure two lightning strikes. The lightning bolts mark both trains where they strike: at A_1 and B_1 on O_1 's train, and at A_2 and B_2 on O_2 's train, Fig. 26–5a. For simplicity, we assume that O_1 is exactly halfway between A_1 and B_1 , and O_2 is halfway between A_2 and B_2 . Let us first put ourselves in O_2 's reference frame, so we observe O_1 moving to the right with speed v . Let us also assume that the two events occur *simultaneously* in O_2 's frame, and just at the instant when O_1 and O_2 are opposite each other, Fig. 26–5a. A short time later, Fig. 26–5b, light from A_2 and from B_2 reach O_2 at the same time (we assumed this). Since O_2 knows (or measures) the distances O_2A_2 and O_2B_2 as equal, O_2 knows the two events are simultaneous in the O_2 reference frame.

FIGURE 26–5 Thought experiment on simultaneity. In both (a) and (b) we are in the reference frame of observer O_2 , who sees the reference frame of O_1 moving to the right. In (a), one lightning bolt strikes the two reference frames at A_1 and A_2 , and a second lightning bolt strikes at B_1 and B_2 . (b) A moment later, the light (shown in blue) from the two events reaches O_2 at the same time. So according to observer O_2 , the two bolts of lightning struck simultaneously. But in O_1 's reference frame, the light from B_1 has already reached O_1 , whereas the light from A_1 has not yet reached O_1 . So in O_1 's reference frame, the event at B_1 must have preceded the event at A_1 . Simultaneity in time is not absolute.



But what does observer O_1 observe and measure? From our (O_2) reference frame, we can predict what O_1 will observe. We see that O_1 moves to the right during the time the light is traveling to O_1 from A_1 and B_1 . As shown in Fig. 26–5b, we can see from our O_2 reference frame that the light from B_1 has already passed O_1 , whereas the light from A_1 has not yet reached O_1 . That is, O_1 observes the light coming from B_1 before observing the light coming from A_1 . Given (i) that light travels at the same speed c in any direction and in any reference frame, and (ii) that the distance O_1A_1 equals O_1B_1 , then observer O_1 can only conclude that the event at B_1 occurred before the event at A_1 . The two events are *not* simultaneous for O_1 , even though they are for O_2 .

We thus find that two events which take place at different locations and are simultaneous to one observer, are actually not simultaneous to a second observer who moves relative to the first.

It may be tempting to ask: “Which observer is right, O_1 or O_2 ?” The answer, according to relativity, is that they are *both* right. There is no “best” reference frame we can choose to determine which observer is right. Both frames are equally good. We can only conclude that *simultaneity is not an absolute concept*, but is relative. We are not aware of this lack of agreement on simultaneity in everyday life because the effect is noticeable only when the relative speed of the two reference frames is very large (near c), or the distances involved are very large.

26–4 Time Dilation and the Twin Paradox

The fact that two events simultaneous to one observer may not be simultaneous to a second observer suggests that time itself is not absolute. Could it be that time passes differently in one reference frame than in another? This is, indeed, just what Einstein’s theory of relativity predicts, as the following thought experiment shows.

Figure 26–6 shows a spaceship traveling past Earth at high speed. The point of view of an observer on the spaceship is shown in part (a), and that of an observer on Earth in part (b). Both observers have accurate clocks. The person on the spaceship (Fig. 26–6a) flashes a light and measures the time it takes the light to travel directly across the spaceship and return after reflecting from a mirror (the rays are drawn at a slight angle for clarity). In the reference frame of the spaceship, the

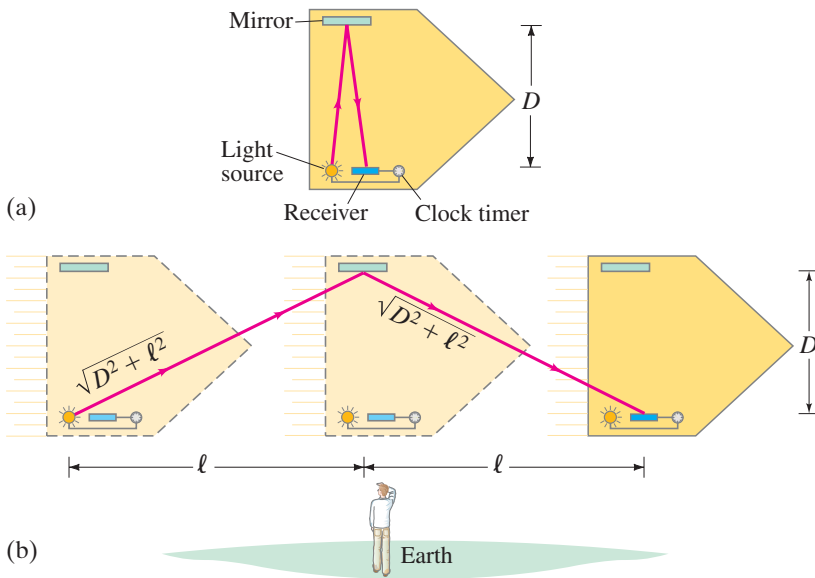


FIGURE 26-6 Time dilation can be shown by a thought experiment: the time it takes for light to travel across a spaceship and back is longer for the observer on Earth (b) than for the observer on the spaceship (a).

light travels a distance $2D$ at speed c , Fig. 26–6a; so the time required to go across and back, Δt_0 , is

$$\Delta t_0 = \frac{2D}{c}.$$

The observer on Earth, Fig. 26–6b, observes the same process. But to this observer, the spaceship is moving. So the light travels the diagonal path shown going across the spaceship, reflecting off the mirror, and returning to the sender. Although the light travels at the same speed to this observer (the second postulate), it travels a greater distance. Hence the time required, as measured by the observer on Earth, will be *greater* than that measured by the observer on the spaceship.

Let us determine the time interval Δt measured by the observer on Earth between sending and receiving the light. In time Δt , the spaceship travels a distance $2\ell = v \Delta t$ where v is the speed of the spaceship (Fig. 26–6b). The light travels a total distance on its diagonal path (Pythagorean theorem) of $2\sqrt{D^2 + \ell^2} = c \Delta t$, where $\ell = v \Delta t/2$. Therefore

$$c \Delta t = 2\sqrt{D^2 + \ell^2} = 2\sqrt{D^2 + v^2(\Delta t)^2/4}.$$

We square both sides to find $c^2(\Delta t)^2 = 4D^2 + v^2(\Delta t)^2$, and solve for $(\Delta t)^2$:

$$(\Delta t)^2 = 4D^2/(c^2 - v^2)$$

so

$$\Delta t = \frac{2D}{c \sqrt{1 - v^2/c^2}}.$$

We combine this equation for Δt with the formula for Δt_0 above, $\Delta t_0 = 2D/c$:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}. \quad (26-1a)$$

TIME DILATION

Since $\sqrt{1 - v^2/c^2}$ is always less than 1, we see that $\Delta t > \Delta t_0$. That is, the time interval between the two events (the sending of the light, and its reception on the spaceship) is *greater* for the observer on Earth than for the observer on the spaceship. This is a general result of the theory of relativity, and is known as **time dilation**. The time dilation effect can be stated as

clocks moving relative to an observer are measured to run more slowly, as compared to clocks at rest.

However, we should not think that the clocks are somehow at fault.

Time is actually measured to pass more slowly in any moving reference frame as compared to your own.

This remarkable result is an inevitable outcome of the two postulates of the special theory of relativity.

The factor $1/\sqrt{1 - v^2/c^2}$ occurs so often in relativity that we often give it the shorthand symbol γ (the Greek letter “gamma”), and write Eq. 26–1a as

$$\Delta t = \gamma \Delta t_0 \quad (26-1b)$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (26-2)$$

Note that γ is never less than one, and has no units. At normal speeds, $\gamma = 1$ to many decimal places. In general, $\gamma \geq 1$.

Values for $\gamma = 1/\sqrt{1 - v^2/c^2}$ at a few speeds v are given in Table 26–1. γ is never less than 1.00 and exceeds 1.00 significantly only at very high speeds, much above let’s say 10^6 m/s (for which $\gamma = 1.000006$).

The concept of time dilation may be hard to accept, for it contradicts our experience. We can see from Eq. 26–1 that the time dilation effect is indeed negligible unless v is reasonably close to c . If v is much less than c , then the term v^2/c^2 is much smaller than the 1 in the denominator of Eq. 26–1, and then $\Delta t \approx \Delta t_0$ (see Example 26–2). The speeds we experience in everyday life are much smaller than c , so it is little wonder we don’t ordinarily notice time dilation. But experiments that have tested the time dilation effect have confirmed Einstein’s predictions. In 1971, for example, extremely precise atomic clocks were flown around the Earth in jet planes. The speed of the planes (10^3 km/h) was much less than c , so the clocks had to be accurate to nanoseconds (10^{-9} s) in order to detect any time dilation. They were this accurate, and they confirmed Eqs. 26–1 to within experimental error. Time dilation had been confirmed decades earlier, however, by observations on “elementary particles” which have very small masses (typically 10^{-30} to 10^{-27} kg) and so require little energy to be accelerated to speeds close to the speed of light, c . Many of these elementary particles are not stable and decay after a time into lighter particles. One example is the muon, whose mean lifetime is $2.2 \mu\text{s}$ when at rest. Careful experiments showed that when a muon is traveling at high speeds, its lifetime is measured to be longer than when it is at rest, just as predicted by the time dilation formula.

TABLE 26–1 Values of γ

v	γ
0	1.00000 ...
0.01c	1.00005
0.10c	1.005
0.50c	1.15
0.90c	2.3
0.99c	7.1

EXAMPLE 26–1 Lifetime of a moving muon. (a) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8$ m/s with respect to the laboratory? A muon’s mean lifetime at rest is $2.20 \mu\text{s} = 2.20 \times 10^{-6}$ s. (b) How far does a muon travel in the laboratory, on average, before decaying?

APPROACH If an observer were to move along with the muon (the muon would be at rest to this observer), the muon would have a mean life of 2.20×10^{-6} s. To an observer in the lab, the muon lives longer because of time dilation. We find the mean lifetime using Eq. 26–1 and the average distance using $d = v \Delta t$.

SOLUTION (a) From Eq. 26–1 with $v = 0.60c$, we have

$$\begin{aligned} \Delta t &= \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \\ &= \frac{2.20 \times 10^{-6} \text{ s}}{\sqrt{1 - 0.36c^2/c^2}} = \frac{2.20 \times 10^{-6} \text{ s}}{\sqrt{0.64}} = 2.8 \times 10^{-6} \text{ s}. \end{aligned}$$

(b) Relativity predicts that a muon with speed 1.80×10^8 m/s would travel an average distance $d = v \Delta t = (1.80 \times 10^8 \text{ m/s})(2.8 \times 10^{-6} \text{ s}) = 500$ m, and this is the distance that is measured experimentally in the laboratory.

NOTE At a speed of 1.8×10^8 m/s, classical physics would tell us that with a mean life of $2.2 \mu\text{s}$, an average muon would travel $d = vt = (1.8 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 400$ m. This is shorter than the distance measured.

EXERCISE A What is the muon’s mean lifetime (Example 26–1) if it is traveling at $v = 0.90c$? (a) $0.42 \mu\text{s}$; (b) $2.3 \mu\text{s}$; (c) $5.0 \mu\text{s}$; (d) $5.3 \mu\text{s}$; (e) $12.0 \mu\text{s}$.

We need to clarify how to use Eq. 26-1, $\Delta t = \gamma \Delta t_0$, and the meaning of Δt and Δt_0 . The equation is true only when Δt_0 represents the time interval between the two events *in a reference frame where an observer at rest sees the two events occur at the same point in space* (as in Fig. 26-6a where the two events are the light flash being sent and being received). This time interval, Δt_0 , is called the **proper time**. Then Δt in Eqs. 26-1 represents the time interval between the two events as measured in a reference frame *moving* with speed v with respect to the first. In Example 26-1 above, Δt_0 (and not Δt) was set equal to 2.2×10^{-6} s because it is only in the rest frame of the muon that the two events (“birth” and “decay”) occur at the same point in space. The proper time Δt_0 is the shortest time between the events any observer can measure. In any other moving reference frame, the time Δt is greater.

CAUTION
Proper time Δt_0 is for 2 events at the same point in space

CAUTION
Proper time is shortest:
 $\Delta t > \Delta t_0$

EXAMPLE 26-2 **Time dilation at 100 km/h.** Let us check time dilation for everyday speeds. A car traveling 100 km/h covers a certain distance in 10.00 s according to the driver’s watch. What does an observer at rest on Earth measure for the time interval?

APPROACH The car’s speed relative to Earth, written in meters per second, is $100 \text{ km/h} = (1.00 \times 10^5 \text{ m})/(3600 \text{ s}) = 27.8 \text{ m/s}$. The driver is at rest in the reference frame of the car, so we set $\Delta t_0 = 10.00 \text{ s}$ in the time dilation formula.

SOLUTION We use Eq. 26-1a:

$$\begin{aligned} \Delta t &= \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10.00 \text{ s}}{\sqrt{1 - \left(\frac{27.8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} \\ &= \frac{10.00 \text{ s}}{\sqrt{1 - (8.59 \times 10^{-15})}}. \end{aligned}$$

If you put these numbers into a calculator, you will obtain $\Delta t = 10.00 \text{ s}$, because the denominator differs from 1 by such a tiny amount. The time measured by an observer fixed on Earth would show no difference from that measured by the driver, even with the best instruments. A computer that could calculate to a large number of decimal places would reveal a slight difference between Δt and Δt_0 .

NOTE We can estimate the difference using the binomial expansion (Appendix A-5),

PROBLEM SOLVING
Use of the binomial expansion

$$(1 \pm x)^n \approx 1 \pm nx. \quad [\text{for } x \ll 1]$$

In our time dilation formula, we have the factor $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$. Thus[†]

$$\begin{aligned} \Delta t &= \gamma \Delta t_0 = \Delta t_0 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx \Delta t_0 \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) \\ &\approx 10.00 \text{ s} \left[1 + \frac{1}{2} \left(\frac{27.8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2\right] \\ &\approx 10.00 \text{ s} + 4 \times 10^{-14} \text{ s}. \end{aligned}$$

So the difference between Δt and Δt_0 is predicted to be 4×10^{-14} s, an extremely small amount.

EXERCISE B A certain atomic clock keeps precise time on Earth. If the clock is taken on a spaceship traveling at a speed $v = 0.60c$, does this clock now run slow according to the people (a) on the spaceship, (b) on Earth?

[†]Recall that $1/x^n$ is written as x^{-n} , such as $1/x^2 = x^{-2}$, Appendix A-2.

EXAMPLE 26-3 **Reading a magazine on a spaceship.** A passenger on a fictional high-speed spaceship traveling between Earth and Jupiter at a steady speed of $0.75c$ reads a magazine which takes 10.0 min according to her watch. (a) How long does this take as measured by Earth-based clocks? (b) How much farther is the spaceship from Earth at the end of reading the article than it was at the beginning?

APPROACH (a) The time interval in one reference frame is related to the time interval in the other by Eq. 26-1a or b. (b) At constant speed, distance is speed \times time. Because there are two time intervals (Δt and Δt_0) we will get two distances, one for each reference frame. [This surprising result is explored in the next Section (26-5).]

SOLUTION (a) The given 10.0-min time interval is the proper time Δt_0 —starting and finishing the magazine happen at the same place on the spaceship. Earth clocks measure

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - (v^2/c^2)}} = \frac{10.00 \text{ min}}{\sqrt{1 - (0.75)^2}} = 15.1 \text{ min.}$$

(b) In the Earth frame, the rocket travels a distance $D = v \Delta t = (0.75c)(15.1 \text{ min}) = (0.75)(3.0 \times 10^8 \text{ m/s})(15.1 \text{ min} \times 60 \text{ s/min}) = 2.04 \times 10^{11} \text{ m}$. In the spaceship's frame, the Earth is moving away from the spaceship at $0.75c$, but the time is only 10.0 min, so the distance is measured to be $D_0 = v \Delta t_0 = (2.25 \times 10^8 \text{ m/s})(600 \text{ s}) = 1.35 \times 10^{11} \text{ m}$.

Space Travel?

Time dilation has aroused interesting speculation about space travel. According to classical (Newtonian) physics, to reach a star 100 light-years away would not be possible for ordinary mortals (1 light-year is the distance light can travel in 1 year $= 3.0 \times 10^8 \text{ m/s} \times 3.16 \times 10^7 \text{ s} = 9.5 \times 10^{15} \text{ m}$). Even if a spaceship could travel at close to the speed of light, it would take over 100 years to reach such a star. But time dilation tells us that the time involved could be less. In a spaceship traveling at $v = 0.999c$, the time for such a trip would be only about $\Delta t_0 = \Delta t \sqrt{1 - v^2/c^2} = (100 \text{ yr})\sqrt{1 - (0.999)^2} = 4.5 \text{ yr}$. Thus time dilation allows such a trip, but the enormous practical problems of achieving such speeds may not be possible to overcome, certainly not in the near future.

When we talk in this Chapter and in the Problems about spaceships moving at speeds close to c , it is for understanding and for fun, but not realistic, although for tiny elementary particles such high speeds *are* realistic.

In this example, 100 years would pass on Earth, whereas only 4.5 years would pass for the astronaut on the trip. Is it just the clocks that would slow down for the astronaut? No.

All processes, including aging and other life processes, run more slowly for the astronaut as measured by an Earth observer. But to the astronaut, time would pass in a normal way.

The astronaut would experience 4.5 years of normal sleeping, eating, reading, and so on. And people on Earth would experience 100 years of ordinary activity.

Twin Paradox

Not long after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. According to this **twin paradox**, suppose one of a pair of 20-year-old twins takes off in a spaceship traveling at very high speed to a distant star and back again, while the other twin remains on Earth. According to the Earth twin, the astronaut twin will age less. Whereas 20 years might pass for the Earth twin, perhaps only 1 year (depending on the spacecraft's speed) would pass for the traveler. Thus, when the traveler returns, the earthbound twin could expect to be 40 years old whereas the traveling twin would be only 21.

This is the viewpoint of the twin on the Earth. But what about the traveling twin? If all inertial reference frames are equally good, won't the traveling twin make all the claims the Earth twin does, only in reverse? Can't the astronaut twin claim that since the Earth is moving away at high speed, time passes more slowly on Earth and the twin on Earth will age less? This is the opposite of what the Earth twin predicts. They cannot both be right, for after all the spacecraft returns to Earth and a direct comparison of ages and clocks can be made.

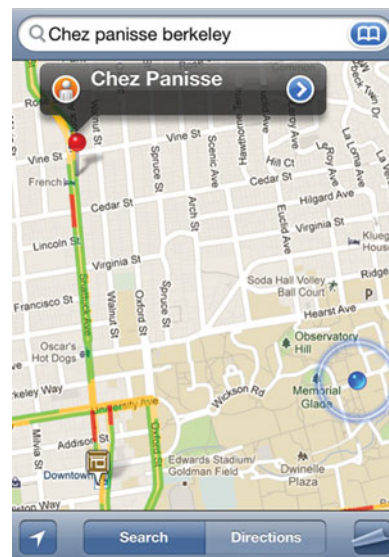
There is, however, no contradiction here. One of the viewpoints is indeed incorrect. The consequences of the special theory of relativity—in this case, time dilation—can be applied only by observers in an inertial reference frame. The Earth is such a frame (or nearly so), whereas the spacecraft is not. The spacecraft accelerates at the start and end of its trip and when it turns around at the far point of its journey. Part of the time, the astronaut twin may be in an inertial frame (and is justified in saying the Earth twin's clocks run slow). But during the accelerations, the twin on the spacecraft is not in an inertial frame. So she cannot use special relativity to predict their relative ages when she returns to Earth. The Earth twin stays in the same inertial frame, and we can thus trust her predictions based on special relativity. Thus, there is no paradox. The prediction of the Earth twin that the traveling twin ages less is the correct one.

* Global Positioning System (GPS)

Airplanes, cars, boats, and hikers use **global positioning system (GPS)** receivers to tell them quite accurately where they are at a given moment (Fig. 26–7). There are more than 30 global positioning system satellites that send out precise time signals using atomic clocks. Your receiver compares the times received from at least four satellites, all of whose times are carefully synchronized to within 1 part in 10^{13} . By comparing the time differences with the known satellite positions and the fixed speed of light, the receiver can determine how far it is from each satellite and thus where it is on the Earth. It can do this to an accuracy of a few meters, if it has been constructed to make corrections such as the one below due to relativity.



FIGURE 26–7 A visiting professor of physics uses the GPS on her smart phone to find a restaurant (red dot). Her location in the physics department is the blue dot. Traffic on some streets is also shown (green = good, orange = slow, red = heavy traffic) which comes in part by tracking cell phone movements.



CONCEPTUAL EXAMPLE 26–4 **A relativity correction to GPS.** GPS satellites move at about $4 \text{ km/s} = 4000 \text{ m/s}$. Show that a good GPS receiver needs to correct for time dilation if it is to produce results consistent with atomic clocks accurate to 1 part in 10^{13} .

RESPONSE Let us calculate the magnitude of the time dilation effect by inserting $v = 4000 \text{ m/s}$ into Eq. 26–1a:

$$\begin{aligned} \Delta t &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta t_0 = \frac{1}{\sqrt{1 - \left(\frac{4 \times 10^3 \text{ m/s}}{3 \times 10^8 \text{ m/s}}\right)^2}} \Delta t_0 \\ &= \frac{1}{\sqrt{1 - 1.8 \times 10^{-10}}} \Delta t_0. \end{aligned}$$

We use the binomial expansion: $(1 \pm x)^n \approx 1 \pm nx$ for $x \ll 1$ (see Appendix A–5) which here is $(1 - x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x$. That is

$$\Delta t = \left(1 + \frac{1}{2}(1.8 \times 10^{-10})\right) \Delta t_0 = (1 + 9 \times 10^{-11}) \Delta t_0.$$

The time “error” divided by the time interval is

$$\frac{(\Delta t - \Delta t_0)}{\Delta t_0} = 1 + 9 \times 10^{-11} - 1 = 9 \times 10^{-11} \approx 1 \times 10^{-10}.$$

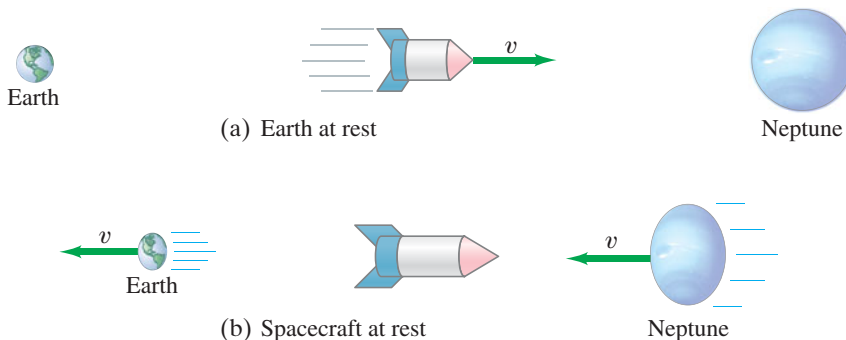
Time dilation, if not accounted for, would introduce an error of about 1 part in 10^{10} , which is 1000 times greater than the precision of the atomic clocks. Not correcting for time dilation means a receiver could give much poorer position accuracy.

NOTE GPS devices must make other corrections as well, including effects associated with general relativity.

26–5 Length Contraction

Time intervals are not the only things different in different reference frames. Space intervals—lengths and distances—are different as well, according to the special theory of relativity, and we illustrate this with a thought experiment.

FIGURE 26–8 (a) A spaceship traveling at very high speed v from Earth to the planet Neptune, as seen from Earth's frame of reference. (b) According to an observer on the spaceship, Earth and Neptune are moving at the very high speed v : Earth leaves the spaceship, and a time Δt_0 later Neptune arrives at the spaceship.



Observers on Earth watch a spacecraft traveling at speed v from Earth to, say, Neptune, Fig. 26–8a. The distance between the planets, as measured by the Earth observers, is ℓ_0 . The time required for the trip, measured from Earth, is

$$\Delta t = \frac{\ell_0}{v}. \quad \text{[Earth observer]}$$

In Fig. 26–8b we see the point of view of observers on the spacecraft. In this frame of reference, the spaceship is at rest; Earth and Neptune move[†] with speed v . The time between departure of Earth and arrival of Neptune, as observed from the spacecraft, is the “proper time” Δt_0 (page 753), because these two events occur at the same point in space (i.e., at the spacecraft). Therefore the time interval is less for the spacecraft observers than for the Earth observers. That is, because of time dilation (Eq. 26–1a), the time for the trip as viewed by the spacecraft is

$$\begin{aligned} \Delta t_0 &= \Delta t \sqrt{1 - v^2/c^2} \\ &= \Delta t/\gamma. \end{aligned} \quad \text{[spacecraft observer]}$$

Because the spacecraft observers measure the same speed but less time between these two events, they also measure the distance as less. If we let ℓ be the distance between the planets as viewed by the spacecraft observers, then $\ell = v \Delta t_0$, which we can rewrite as $\ell = v \Delta t_0 = v \Delta t \sqrt{1 - v^2/c^2} = \ell_0 \sqrt{1 - v^2/c^2}$. Thus we have the important result that

LENGTH CONTRACTION

$$\ell = \ell_0 \sqrt{1 - v^2/c^2} \quad (26-3a)$$

or, using γ (Eq. 26–2),

$$\ell = \frac{\ell_0}{\gamma}. \quad (26-3b)$$

This is a general result of the special theory of relativity and applies to lengths of objects as well as to distance between objects. The result can be stated most simply in words as:

the length of an object moving relative to an observer is measured to be shorter along its direction of motion than when it is at rest.



CAUTION

Proper length is measured in reference frame where the two positions are at rest

This is called **length contraction**. The length ℓ_0 in Eqs. 26–3 is called the **proper length**. It is the length of the object (or distance between two points whose positions are measured at the same time) as determined by *observers at rest* with respect to the object. Equations 26–3 give the length ℓ that will be measured by observers when the object travels past them at speed v .

[†]We assume v is much greater than the relative speed of Neptune and Earth (which we thus ignore).

It is important to note that length contraction occurs *only along the direction of motion*. For example, the moving spaceship in Fig. 26–8a is shortened in length, but its height is the same as when it is at rest.

Length contraction, like time dilation, is not noticeable in everyday life because the factor $\sqrt{1 - v^2/c^2}$ in Eq. 26–3a differs significantly from 1.00 only when v is very large.

EXAMPLE 26–5 **Painting’s contraction.** A rectangular painting measures 1.00 m tall and 1.50 m wide, Fig. 26–9a. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of $0.90c$. (a) What are the dimensions of the picture according to the captain of the spaceship? (b) What are the dimensions as seen by an observer on the Earth?

APPROACH We apply the length contraction formula, Eq. 26–3a, to the dimension parallel to the motion; v is the speed of the painting relative to the Earth observer.

SOLUTION (a) The painting is at rest ($v = 0$) on the spaceship so it (as well as everything else in the spaceship) looks perfectly normal to everyone on the spaceship. The captain sees a 1.00-m by 1.50-m painting.

(b) Only the dimension in the direction of motion is shortened, so the height is unchanged at 1.00 m, Fig. 26–9b. The length, however, is contracted to

$$\begin{aligned} \ell &= \ell_0 \sqrt{1 - \frac{v^2}{c^2}} \\ &= (1.50 \text{ m}) \sqrt{1 - (0.90)^2} = 0.65 \text{ m}. \end{aligned}$$

So the picture has dimensions $1.00 \text{ m} \times 0.65 \text{ m}$ to an observer on Earth.

EXAMPLE 26–6 **A fantasy supertrain.** A very fast train with a “proper length” of $\ell_0 = 500 \text{ m}$ (measured by people at rest on the train) is passing through a tunnel that is 200 m long according to observers on the ground. Let us imagine the train’s speed to be so great that the train fits completely within the tunnel as seen by observers on the ground. That is, the engine is just about to emerge from one end of the tunnel at the time the last car disappears into the other end. What is the train’s speed?

APPROACH Since the train just fits inside the tunnel, its length measured by the person on the ground is $\ell = 200 \text{ m}$. The length contraction formula, Eq. 26–3a or b, can thus be used to solve for v .

SOLUTION Substituting $\ell = 200 \text{ m}$ and $\ell_0 = 500 \text{ m}$ into Eq. 26–3a gives

$$200 \text{ m} = 500 \text{ m} \sqrt{1 - \frac{v^2}{c^2}};$$

dividing both sides by 500 m and squaring, we get

$$(0.40)^2 = 1 - \frac{v^2}{c^2}$$

or

$$\frac{v}{c} = \sqrt{1 - (0.40)^2}$$

and

$$v = 0.92c.$$

NOTE No real train could go this fast. But it is fun to think about.

NOTE An observer on the *train* would *not* see the two ends of the train inside the tunnel at the same time. Recall that observers moving relative to each other do not agree about simultaneity. (See Example 26–7, next.)

EXERCISE C What is the length of the tunnel as measured by observers on the train in Example 26–6?

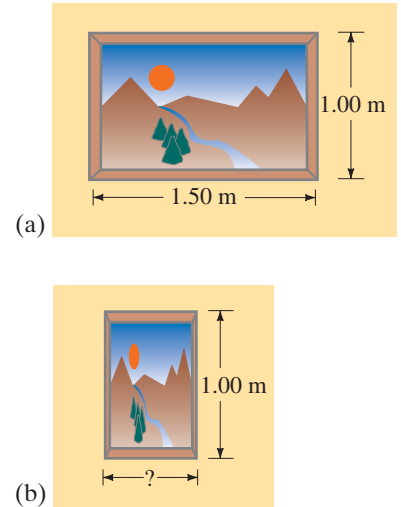


FIGURE 26–9 Example 26–5.

CONCEPTUAL EXAMPLE 26-7 Resolving the train and tunnel length.

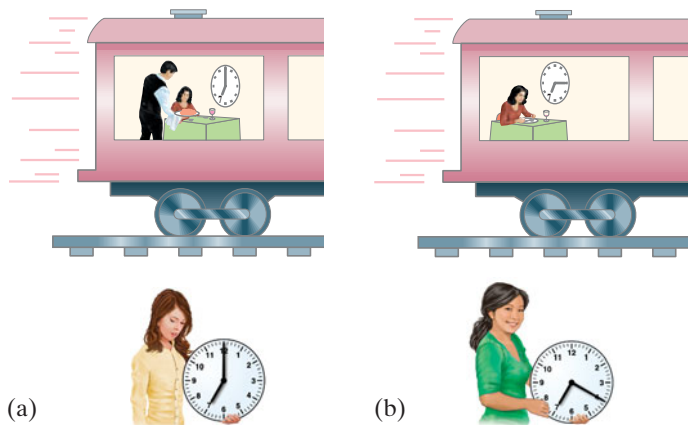
Observers at rest on the Earth see a very fast 200-m-long train pass through a 200-m-long tunnel (as in Example 26-6) so that the train momentarily disappears from view inside the tunnel. Observers on the train measure the train's length to be 500 m and the tunnel's length to be only 80 m (Exercise C, using Eq. 26-3a). Clearly a 500-m-long train cannot fit inside an 80-m-long tunnel. How is this apparent inconsistency explained?

RESPONSE Events simultaneous in one reference frame may not be simultaneous in another. Let the engine emerging from one end of the tunnel be “event A,” and the last car disappearing into the other end of the tunnel “event B.” To observers in the Earth frame, events A and B are simultaneous. To observers on the train, however, the events are not simultaneous. In the train's frame, event A occurs before event B. As the engine emerges from the tunnel, observers on the train observe the last car as still $500\text{ m} - 80\text{ m} = 420\text{ m}$ from the entrance to the tunnel.

26-6 Four-Dimensional Space-Time

Let us imagine a person is on a train moving at a very high speed, say $0.65c$, Fig. 26-10. This person begins a meal at 7:00 and finishes at 7:15, according to a clock on the train. The two events, beginning and ending the meal, take place at the same point on the train, so the “proper time” between these two events is 15 min. To observers on Earth, the plate is moving and the meal will take longer—20 min according to Eqs. 26-1. Let us assume that the meal was served on a 20-cm-diameter plate (its “proper length”). To observers on the Earth, the plate is moving and is only 15 cm wide (length contraction). Thus, to observers on the Earth, the meal looks smaller but lasts longer.

FIGURE 26-10 According to an accurate clock on a fast-moving train, a person (a) begins dinner at 7:00 and (b) finishes at 7:15. At the beginning of the meal, two observers on Earth set their watches to correspond with the clock on the train. These observers measure the eating time as 20 minutes.



In a sense the two effects, time dilation and length contraction, balance each other. When viewed from the Earth, what an object seems to lose in size it gains in length of time it lasts. Space, or length, is exchanged for time.

Considerations like this led to the idea of **four-dimensional space-time**: space takes up three dimensions and time is a fourth dimension. Space and time are intimately connected. Just as when we squeeze a balloon we make one dimension larger and another smaller, so when we examine objects and events from different reference frames, a certain amount of space is exchanged for time, or vice versa.

Although the idea of four dimensions may seem strange, it refers to the idea that any object or event is specified by four quantities—three to describe where in space, and one to describe when in time. The really unusual aspect of four-dimensional space–time is that space and time can intermix: a little of one can be exchanged for a little of the other when the reference frame is changed.

[In Galilean–Newtonian relativity, the time interval between two events, Δt , and the distance between two events or points, Δx , are invariant quantities no matter what inertial reference frame they are viewed from. Neither of these quantities is invariant according to Einstein’s relativity. But there is an invariant quantity in four-dimensional space–time, called the **space–time interval**, which is $(\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2$.]

26–7 Relativistic Momentum

So far in this Chapter, we have seen that two basic mechanical quantities, length and time intervals, need modification because they are relative—their value depends on the reference frame from which they are measured. We might expect that other physical quantities might need some modification according to the theory of relativity, such as momentum and energy.

The analysis of collisions between two particles shows that if we want to preserve the law of conservation of momentum in relativity, we must redefine momentum as

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv. \quad (26-4)$$

Here γ is shorthand for $1/\sqrt{1 - v^2/c^2}$ as before (Eq. 26–2). For speeds much less than the speed of light, Eq. 26–4 gives the classical momentum, $p = mv$.

Relativistic momentum has been tested many times on tiny elementary particles (such as muons), and it has been found to behave in accord with Eq. 26–4.

EXAMPLE 26–8 Momentum of moving electron. Compare the momentum of an electron to its classical value when it has a speed of (a) 4.00×10^7 m/s in the CRT of an old TV set, and (b) $0.98c$ in an accelerator used for cancer therapy.

APPROACH We use Eq. 26–4 for the momentum of a moving electron.

SOLUTION (a) At $v = 4.00 \times 10^7$ m/s, the electron’s momentum is

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mv}{\sqrt{1 - \frac{(4.00 \times 10^7 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}}} = 1.01mv.$$

The factor $\gamma = 1/\sqrt{1 - v^2/c^2} \approx 1.01$, so the momentum is only about 1% greater than the classical value. (If we put in the mass of an electron, $m = 9.11 \times 10^{-31}$ kg, the momentum is $p = 1.01mv = 3.68 \times 10^{-23}$ kg·m/s, compared to 3.64×10^{-23} kg·m/s classically.)

(b) With $v = 0.98c$, the momentum is

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mv}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}} = \frac{mv}{\sqrt{1 - (0.98)^2}} = 5.0mv.$$

An electron traveling at 98% the speed of light has $\gamma = 5.0$ and a momentum 5.0 times its classical value.

**CAUTION**

Most physicists prefer to consider the mass of a particle as fixed

* Rest Mass and Relativistic Mass

The relativistic definition of momentum, Eq. 26–4, has sometimes been interpreted as an increase in the mass of an object. In this interpretation, a particle can have a **relativistic mass**, m_{rel} , which increases with speed according to

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - v^2/c^2}}$$

In this “mass-increase” formula, m is referred to as the **rest mass** of the object. With this interpretation, *the mass of an object appears to increase as its speed increases*. But there are problems with relativistic mass. If we plug it into formulas like $F = ma$ or $\text{KE} = \frac{1}{2}mv^2$, we obtain formulas that do not agree with experiment. (If we write Newton’s second law in its more general form, $\vec{F} = \Delta\vec{p}/\Delta t$, that would get a correct result.) Also, be careful *not* to think a mass acquires more particles or more molecules as its speed becomes very large. It doesn’t. Today, most physicists prefer not to use relativistic mass, so an object has only one mass (its rest mass), and it is only the momentum that increases with speed.

Whenever we talk about the mass of an object, we will always mean its rest mass (a fixed value). [But see Problem 46.]

26–8 The Ultimate Speed

A basic result of the special theory of relativity is that the speed of an object cannot equal or exceed the speed of light. That the speed of light is a natural speed limit in the universe can be seen from any of Eqs. 26–1, 26–3, or 26–4. It is perhaps easiest to see from Eq. 26–4. As an object is accelerated to greater and greater speeds, its momentum becomes larger and larger. Indeed, if v were to equal c , the denominator in this equation would be zero, and the momentum would be infinite. To accelerate an object up to $v = c$ would thus require infinite energy, and so is not possible.

26–9 $E = mc^2$; Mass and Energy

If momentum needs to be modified to fit with relativity as we just saw in Eq. 26–4, then we might expect that energy would also need to be rethought. Indeed, Einstein not only developed a new formula for kinetic energy, but also found a new relation between mass and energy, and the startling idea that mass is a form of energy.

We start with the work-energy principle (Chapter 6), hoping it is still valid in relativity and will give verifiable results. That is, we assume the net work done on a particle is equal to its change in kinetic energy (KE). Using this principle, Einstein showed that at high speeds the formula $\text{KE} = \frac{1}{2}mv^2$ is not correct. Instead, Einstein showed that the kinetic energy of a particle of mass m traveling at speed v is given by

$$\text{KE} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2. \quad (26-5a)$$

In terms of $\gamma = 1/\sqrt{1 - v^2/c^2}$ we can rewrite Eq. 26–5a as

$$\text{KE} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2. \quad (26-5b)$$

Equation 26–5a requires some interpretation. The first term increases with the speed v of the particle. The second term, mc^2 , is constant; it is called the **rest energy** of the particle, and represents a form of energy that a particle has even when at rest. Note that if a particle is at rest ($v = 0$) the first term in Eq. 26–5a becomes mc^2 , so $\text{KE} = 0$ as it should.

We can rearrange Eq. 26–5b to get

$$\gamma mc^2 = mc^2 + \text{KE}.$$

We call γmc^2 the *total energy* E of the particle (assuming no potential energy), because it equals the rest energy plus the kinetic energy:

$$E = \text{KE} + mc^2. \quad (26-6a)$$

The total energy[†] can also be written, using Eqs. 26–5, as

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}. \quad (26-6b)$$

For a particle at rest in a given reference frame, KE is zero in Eq. 26–6a, so the total energy is its rest energy:

$$E = mc^2. \quad (26-7)$$

MASS RELATED
TO ENERGY

Here we have Einstein’s famous formula, $E = mc^2$. This formula mathematically relates the concepts of energy and mass. But if this idea is to have any physical meaning, then mass ought to be convertible to other forms of energy and vice versa. Einstein suggested that this might be possible, and indeed changes of mass to other forms of energy, and vice versa, have been experimentally confirmed countless times in nuclear and elementary particle physics. For example, an electron and a positron (= a positive electron, see Section 32–3) have often been observed to collide and disappear, producing pure electromagnetic radiation. The amount of electromagnetic energy produced is found to be exactly equal to that predicted by Einstein’s formula, $E = mc^2$. The reverse process is also commonly observed in the laboratory: electromagnetic radiation under certain conditions can be converted into material particles such as electrons (see Section 27–6 on pair production). On a larger scale, the energy produced in nuclear power plants is a result of the loss in mass of the uranium fuel as it undergoes the process called fission (Chapter 31). Even the radiant energy we receive from the Sun is an example of $E = mc^2$; the Sun’s mass is continually decreasing as it radiates electromagnetic energy outward.

The relation $E = mc^2$ is now believed to apply to all processes, although the changes are often too small to measure. That is, when the energy of a system changes by an amount ΔE , the mass of the system changes by an amount Δm given by

$$\Delta E = (\Delta m)(c^2). \quad (26-8)$$

In a nuclear reaction where an energy E is required or released, the masses of the reactants and the products will be different by $\Delta m = \Delta E/c^2$.

EXAMPLE 26–9 Pion’s kinetic energy. A π^0 meson ($m = 2.4 \times 10^{-28}$ kg) travels at a speed $v = 0.80c = 2.4 \times 10^8$ m/s. What is its kinetic energy? Compare to a classical calculation.

APPROACH We use Eq. 26–5 and compare to $\frac{1}{2}mv^2$.

SOLUTION We substitute values into Eq. 26–5a

$$\begin{aligned} \text{KE} &= mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \\ &= (2.4 \times 10^{-28} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 \left(\frac{1}{(1 - 0.64)^{1/2}} - 1 \right) \\ &= 1.4 \times 10^{-11} \text{ J}. \end{aligned}$$

 **PROBLEM SOLVING**
Relativistic kinetic energy

Notice that the units of mc^2 are $\text{kg} \cdot \text{m}^2/\text{s}^2$, which is the joule.

NOTE Classically $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(2.4 \times 10^{-28} \text{ kg})(2.4 \times 10^8 \text{ m/s})^2 = 6.9 \times 10^{-12} \text{ J}$, about half as much, but this is not a correct result. Note that $\frac{1}{2}\gamma mv^2$ also does not work.

[†]This is for a “free particle,” without forces and potential energy. Potential energy terms can be added.

EXAMPLE 26–10 Energy from nuclear decay. The energy required or released in nuclear reactions and decays comes from a change in mass between the initial and final particles. In one type of radioactive decay (Chapter 30), an atom of uranium ($m = 232.03716$ u) decays to an atom of thorium ($m = 228.02874$ u) plus an atom of helium ($m = 4.00260$ u) where the masses given are in atomic mass units ($1 \text{ u} = 1.6605 \times 10^{-27}$ kg). Calculate the energy released in this decay.

APPROACH The initial mass minus the total final mass gives the mass loss in atomic mass units (u); we convert that to kg, and multiply by c^2 to find the energy released, $\Delta E = \Delta m c^2$.

SOLUTION The initial mass is 232.03716 u, and after the decay the mass is $228.02874 \text{ u} + 4.00260 \text{ u} = 232.03134 \text{ u}$, so there is a loss of mass of 0.00582 u. This mass, which equals $(0.00582 \text{ u})(1.66 \times 10^{-27} \text{ kg}) = 9.66 \times 10^{-30} \text{ kg}$, is changed into energy. By $\Delta E = \Delta m c^2$, we have

$$\begin{aligned}\Delta E &= (9.66 \times 10^{-30} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2 \\ &= 8.70 \times 10^{-13} \text{ J}.\end{aligned}$$

Since $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$ (Section 17–4), the energy released is 5.4 MeV.

In the tiny world of atoms and nuclei, it is common to quote energies in eV (electron volts) or multiples such as MeV (10^6 eV). Momentum (see Eq. 26–4) can be quoted in units of eV/c (or MeV/c). And mass can be quoted (from $E = mc^2$) in units of eV/c² (or MeV/c²). Note the use of c to keep the units correct. The masses of the electron and the proton can be shown to be $0.511 \text{ MeV}/c^2$ and $938 \text{ MeV}/c^2$, respectively. For example, for the electron, $mc^2 = (9.11 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 / (1.602 \times 10^{-13} \text{ J/MeV}) = 0.511 \text{ MeV}$. See also the Table inside the front cover.

EXAMPLE 26–11 A 1-TeV proton. The Tevatron accelerator at Fermilab in Illinois can accelerate protons to a kinetic energy of 1.0 TeV (10^{12} eV). What is the speed of such a proton?

APPROACH We solve the kinetic energy formula, Eq. 26–5a, for v .

SOLUTION The rest energy of a proton is $mc^2 = 938 \text{ MeV}$ or $9.38 \times 10^8 \text{ eV}$. Compared to the kinetic energy of 10^{12} eV, the rest energy can be neglected, so we simplify Eq. 26–5a to

$$\text{KE} \approx \frac{mc^2}{\sqrt{1 - v^2/c^2}}.$$

We solve this for v in the following steps:

$$\begin{aligned}\sqrt{1 - \frac{v^2}{c^2}} &= \frac{mc^2}{\text{KE}}; \\ 1 - \frac{v^2}{c^2} &= \left(\frac{mc^2}{\text{KE}}\right)^2; \\ \frac{v^2}{c^2} &= 1 - \left(\frac{mc^2}{\text{KE}}\right)^2 = 1 - \left(\frac{9.38 \times 10^8 \text{ eV}}{1.0 \times 10^{12} \text{ eV}}\right)^2; \\ v &= \sqrt{1 - (9.38 \times 10^{-4})^2} c \\ &= 0.99999956 c.\end{aligned}$$

So the proton is traveling at a speed very nearly equal to c .

At low speeds, $v \ll c$, the relativistic formula for kinetic energy reduces to the classical one, as we now show by using the binomial expansion (Appendix A): $(1 \pm x)^n = 1 \pm nx + \dots$, keeping only two terms because $x = v/c$ is very much less than 1. With $n = -\frac{1}{2}$ we expand the square root in Eq. 26-5a

$$\text{KE} = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

so that

$$\text{KE} \approx mc^2 \left(1 + \frac{1}{2} \frac{v^2}{c^2} + \dots - 1 \right) \approx \frac{1}{2} mv^2.$$

The dots in the first expression represent very small terms in the expansion which we neglect since we assumed that $v \ll c$. Thus at low speeds, the relativistic form for kinetic energy reduces to the classical form, $\text{KE} = \frac{1}{2} mv^2$. This makes relativity a viable theory in that it can predict accurate results at low speed as well as at high. Indeed, the other equations of special relativity also reduce to their classical equivalents at ordinary speeds: length contraction, time dilation, and modifications to momentum as well as kinetic energy, all disappear for $v \ll c$ since $\sqrt{1 - v^2/c^2} \approx 1$.

A useful relation between the total energy E of a particle and its momentum p can also be derived. The momentum of a particle of mass m and speed v is given by Eq. 26-4

$$p = \gamma mv = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

The total energy is

$$E = \text{KE} + mc^2$$

or

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}.$$

We square this equation (and we insert “ $v^2 - v^2$ ” which is zero, but will help us):

$$\begin{aligned} E^2 &= \frac{m^2 c^2 c^2}{1 - v^2/c^2} = \frac{m^2 c^2 (c^2 - v^2 + v^2)}{1 - v^2/c^2} = \frac{m^2 c^2 v^2}{1 - v^2/c^2} + \frac{m^2 c^2 (c^2 - v^2)}{1 - v^2/c^2} \\ &= p^2 c^2 + \frac{m^2 c^4 (1 - v^2/c^2)}{1 - v^2/c^2} \end{aligned}$$

or

$$E^2 = p^2 c^2 + m^2 c^4. \quad (26-9)$$

Thus, the total energy can be written in terms of the momentum p , or in terms of the kinetic energy (Eq. 26-6a), where we have assumed there is no potential energy.

* Invariant Energy–Momentum

We can rewrite Eq. 26-9 as $E^2 - p^2 c^2 = m^2 c^4$. Since the mass m of a given particle is the same in any reference frame, we see that the quantity $E^2 - p^2 c^2$ must also be the same in any reference frame. Thus, at any given moment the total energy E and momentum p of a particle will be different in different reference frames, but the quantity $E^2 - p^2 c^2$ will have the same value in all inertial reference frames. We say that the quantity $E^2 - p^2 c^2$ is **invariant**.

When Do We Use Relativistic Formulas?

From a practical point of view, we do not have much opportunity in our daily lives to use the mathematics of relativity. For example, the γ factor, $\gamma = 1/\sqrt{1 - v^2/c^2}$, has a value of 1.005 when $v = 0.10c$. Thus, for speeds even as high as $0.10c = 3.0 \times 10^7$ m/s, the factor $\sqrt{1 - v^2/c^2}$ in relativistic formulas gives a numerical correction of less than 1%. For speeds less than $0.10c$, or unless mass and energy are interchanged, we don't usually need the more complicated relativistic formulas, and can use the simpler classical formulas.

If you are given a particle's mass m and its kinetic energy KE , you can do a quick calculation to determine if you need to use relativistic formulas or if classical ones are good enough. You simply compute the ratio KE/mc^2 because (Eq. 26–5b)

$$\frac{\text{KE}}{mc^2} = \gamma - 1 = \frac{1}{\sqrt{1 - v^2/c^2}} - 1.$$

If this ratio comes out to be less than, say, 0.01, then $\gamma \leq 1.01$ and relativistic equations will correct the classical ones by about 1%. If your expected precision is no better than 1%, classical formulas are good enough. But if your precision is 1 part in 1000 (0.1%) then you would want to use relativistic formulas. If your expected precision is only 10%, you need relativity if $(\text{KE}/mc^2) \gtrsim 0.1$.

EXERCISE D For 1% accuracy, does an electron with $\text{KE} = 100 \text{ eV}$ need to be treated relativistically? [Hint: The mass of an electron is 0.511 MeV.]

26–10 Relativistic Addition of Velocities

Consider a rocket ship that travels away from the Earth with speed v , and assume that this rocket has fired off a second rocket that travels at speed u' with respect to the first (Fig. 26–11). We might expect that the speed u of rocket 2 with respect to Earth is $u = v + u'$, which in the case shown in Fig. 26–11 is $u = 0.60c + 0.60c = 1.20c$. But, as discussed in Section 26–8, no object can travel faster than the speed of light in any reference frame. Indeed, Einstein showed that since length and time are different in different reference frames, the classical addition-of-velocities formula is no longer valid. Instead, the correct formula is

$$u = \frac{v + u'}{1 + vu'/c^2} \quad \left[\begin{array}{l} \vec{u} \text{ and } \vec{v} \text{ along} \\ \text{the same direction} \end{array} \right] \quad (26-10)$$

for motion along a straight line. We derive this formula in Appendix E. If u' is in the opposite direction from v , then u' must have a minus sign in the above equation so $u = (v - u')/(1 - vu'/c^2)$.

EXAMPLE 26–12 **Relative velocity, relativistically.** Calculate the speed of rocket 2 in Fig. 26–11 with respect to Earth.

APPROACH We combine the speed of rocket 2 relative to rocket 1 with the speed of rocket 1 relative to Earth, using the relativistic Eq. 26–10 because the speeds are high and they are along the same line.

SOLUTION Rocket 2 moves with speed $u' = 0.60c$ with respect to rocket 1. Rocket 1 has speed $v = 0.60c$ with respect to Earth. The speed of rocket 2 with respect to Earth is (Eq. 26–10)

$$u = \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c.$$

NOTE The speed of rocket 2 relative to Earth is less than c , as it must be.

We can see that Eq. 26–10 reduces to the classical form for velocities small compared to the speed of light since $1 + vu'/c^2 \approx 1$ for v and $u' \ll c$. Thus, $u \approx v + u'$, as in classical physics (Chapter 3).

Let us test our formula at the other extreme, that of the speed of light. Suppose that rocket 1 in Fig. 26–11 sends out a beam of light so that $u' = c$. Equation 26–10 tells us that the speed of this light relative to Earth is

$$u = \frac{0.60c + c}{1 + \frac{(0.60c)(c)}{c^2}} = \frac{1.60c}{1.60} = c,$$

which is fully consistent with the second postulate of relativity.

EXERCISE E Use Eq. 26–10 to calculate the speed of rocket 2 in Fig. 26–11 relative to Earth if it was shot from rocket 1 at a speed $u' = 3000 \text{ km/s} = 0.010c$. Assume rocket 1 had a speed $v = 6000 \text{ km/s} = 0.020c$.

EXERCISE F Return to the Chapter-Opening Question, page 744, and answer it again now. Try to explain why you may have answered differently the first time.

CAUTION

Relative velocities do not add simply, as in classical mechanics ($v \ll c$)

Relativistic addition of velocities formula (\vec{u} and \vec{v} along same line)

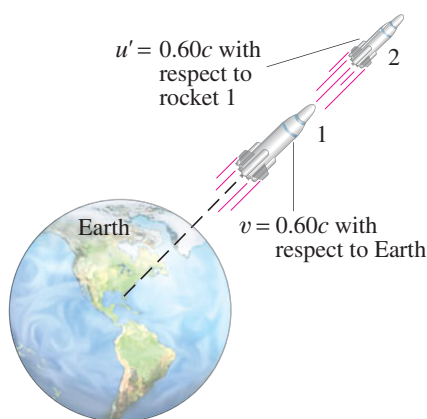


FIGURE 26–11 Rocket 1 leaves Earth at $v = 0.60c$. Rocket 2 is fired from rocket 1 with speed $u' = 0.60c$. What is the speed of rocket 2 with respect to the Earth? Example 26–12.

26–11 The Impact of Special Relativity

A great many experiments have been performed to test the predictions of the special theory of relativity. Within experimental error, no contradictions have been found. Scientists have therefore accepted relativity as an accurate description of nature.

At speeds much less than the speed of light, the relativistic formulas reduce to the old classical ones, as we have discussed. We would, of course, hope—or rather, insist—that this be true since Newtonian mechanics works so well for objects moving with speeds $v \ll c$. This insistence that a more general theory (such as relativity) give the same results as a more restricted theory (such as classical mechanics which works for $v \ll c$) is called the **correspondence principle**. The two theories must correspond where their realms of validity overlap. Relativity thus does not contradict classical mechanics. Rather, it is a more general theory, of which classical mechanics is now considered to be a limiting case.

The importance of relativity is not simply that it gives more accurate results, especially at very high speeds. Much more than that, it has changed the way we view the world. The concepts of space and time are now seen to be relative, and intertwined with one another, whereas before they were considered absolute and separate. Even our concepts of matter and energy have changed: either can be converted to the other. The impact of relativity extends far beyond physics. It has influenced the other sciences, and even the world of art and literature; it has, indeed, entered the general culture.

The special theory of relativity we have studied in this Chapter deals with inertial (nonaccelerating) reference frames. In Chapter 33 we will discuss briefly the more complicated “general theory of relativity” which can deal with non-inertial reference frames.

Summary

An **inertial reference frame** is one in which Newton’s law of inertia holds. Inertial reference frames move at constant velocity relative to one another. Accelerating reference frames are **noninertial**.

The **special theory of relativity** is based on two principles: the **relativity principle**, which states that the laws of physics are the same in all inertial reference frames, and the principle of the **constancy of the speed of light**, which states that the speed of light in empty space has the same value in all inertial reference frames.

One consequence of relativity theory is that two events that are simultaneous in one reference frame may not be simultaneous in another. Other effects are **time dilation**: moving clocks are measured to run slow; and **length contraction**: the length of a moving object is measured to be shorter (in its direction of motion) than when it is at rest. Quantitatively,

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_0 \quad (26-1)$$

$$\ell = \ell_0 \sqrt{1 - v^2/c^2} = \frac{\ell_0}{\gamma} \quad (26-3)$$

where ℓ and Δt are the length and time interval of objects (or events) observed as they move by at the speed v ; ℓ_0 and Δt_0 are the **proper length** and **proper time**—that is, the same quantities as measured in the rest frame of the objects or events. The quantity γ is shorthand for

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (26-2)$$

The theory of relativity has changed our notions of space and time, and of momentum, energy, and mass. Space and time

are seen to be intimately connected, with time being the fourth dimension in addition to the three dimensions of space.

The **momentum** of an object is given by

$$p = \gamma mv = \frac{mv}{\sqrt{1 - v^2/c^2}}. \quad (26-4)$$

Mass and energy are interconvertible. The equation

$$E = mc^2 \quad (26-7)$$

tells how much energy E is needed to create a mass m , or vice versa. Said another way, $E = mc^2$ is the amount of energy an object has because of its mass m . The law of conservation of energy must include mass as a form of energy.

The kinetic energy KE of an object moving at speed v is given by

$$\text{KE} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2 \quad (26-5)$$

where m is the mass of the object. The total energy E , if there is no potential energy, is

$$E = \text{KE} + mc^2 = \gamma mc^2. \quad (26-6)$$

The momentum p of an object is related to its total energy E (assuming no potential energy) by

$$E^2 = p^2c^2 + m^2c^4. \quad (26-9)$$

Velocity addition also must be done in a special way. All these relativistic effects are significant only at high speeds, close to the speed of light, which itself is the ultimate speed in the universe.

Questions

1. You are in a windowless car in an exceptionally smooth train moving at constant velocity. Is there any physical experiment you can do in the train car to determine whether you are moving? Explain.
2. You might have had the experience of being at a red light when, out of the corner of your eye, you see the car beside you creep forward. Instinctively you stomp on the brake pedal, thinking that you are rolling backward. What does this say about absolute and relative motion?
3. A worker stands on top of a railroad car moving at constant velocity and throws a heavy ball straight up (from his point of view). Ignoring air resistance, explain whether the ball will land back in his hand or behind him.
4. Does the Earth really go around the Sun? Or is it also valid to say that the Sun goes around the Earth? Discuss in view of the relativity principle (that there is no best reference frame). Explain. See Section 5–8.
5. If you were on a spaceship traveling at $0.6c$ away from a star, at what speed would the starlight pass you?
6. The time dilation effect is sometimes expressed as “moving clocks run slowly.” Actually, this effect has nothing to do with motion affecting the functioning of clocks. What then does it deal with?
7. Does time dilation mean that time actually passes more slowly in moving reference frames or that it only *seems* to pass more slowly?
8. A young-looking woman astronaut has just arrived home from a long trip. She rushes up to an old gray-haired man and in the ensuing conversation refers to him as her son. How might this be possible?
9. If you were traveling away from Earth at speed $0.6c$, would you notice a change in your heartbeat? Would your mass, height, or waistline change? What would observers on Earth using telescopes say about you?
10. Do time dilation and length contraction occur at ordinary speeds, say 90 km/h ?
11. Suppose the speed of light were infinite. What would happen to the relativistic predictions of length contraction and time dilation?
12. Explain how the length contraction and time dilation formulas might be used to indicate that c is the limiting speed in the universe.
13. Discuss how our everyday lives would be different if the speed of light were only 25 m/s .
14. The drawing at the start of this Chapter shows the street as seen by Mr Tompkins, for whom the speed of light is $c = 20 \text{ mi/h}$. What does Mr Tompkins look like to the people standing on the street (Fig. 26–12)? Explain.



FIGURE 26–12
Question 14.
Mr Tompkins as seen by people on the sidewalk. See also Chapter-Opening Figure on page 744.

15. An electron is limited to travel at speeds less than c . Does this put an upper limit on the momentum of an electron? If so, what is this upper limit? If not, explain.
16. Can a particle of nonzero mass attain the speed of light? Explain.
17. Does the equation $E = mc^2$ conflict with the conservation of energy principle? Explain.
18. If mass is a form of energy, does this mean that a spring has more mass when compressed than when relaxed? Explain.
19. It is not correct to say that “matter can neither be created nor destroyed.” What must we say instead?
20. Is our intuitive notion that velocities simply add, as in Section 3–8, completely wrong?

MisConceptual Questions

1. The fictional rocket ship *Adventure* is measured to be 50 m long by the ship’s captain inside the rocket. When the rocket moves past a space dock at $0.5c$, space-dock personnel measure the rocket ship to be 43.3 m long. What is its proper length?
(a) 50 m . (b) 43.3 m . (c) 93.3 m . (d) 13.3 m .
2. As rocket ship *Adventure* (MisConceptual Question 1) passes by the space dock, the ship’s captain flashes a flashlight at 1.00-s intervals as measured by space-dock personnel. How often does the flashlight flash relative to the captain?
(a) Every 1.15 s . (b) Every 1.00 s . (c) Every 0.87 s .
(d) We need to know the distance between the ship and the space dock.
3. For the flashing of the flashlight in MisConceptual Question 2, what time interval is the proper time interval?
(a) 1.15 s . (b) 1.00 s . (c) 0.87 s . (d) 0.13 s .
4. The rocket ship of MisConceptual Question 1 travels to a star many light-years away, then turns around and returns at the same speed. When it returns to the space dock, who would have aged less: the space-dock personnel or ship’s captain?
(a) The space-dock personnel.
(b) The ship’s captain.
(c) Both the same amount, because both sets of people were moving relative to each other.
(d) We need to know how far away the star is.
5. An Earth observer notes that clocks on a passing spacecraft run slowly. The person on the spacecraft
(a) agrees her clocks move slower than those on Earth.
(b) feels normal, and her heartbeat and eating habits are normal.
(c) observes that Earth clocks are moving slowly.
(d) The real time is in between the times measured by the two observers.
(e) Both (a) and (b).
(f) Both (b) and (c).

6. Spaceships A and B are traveling directly toward each other at a speed $0.5c$ relative to the Earth, and each has a headlight aimed toward the other ship. What value do technicians on ship B get by measuring the speed of the light emitted by ship A's headlight?
(a) $0.5c$. (b) $0.75c$. (c) $1.0c$. (d) $1.5c$.
7. Relativistic formulas for time dilation, length contraction, and mass are valid
(a) only for speeds less than $0.10c$.
(b) only for speeds greater than $0.10c$.
(c) only for speeds very close to c .
(d) for all speeds.
8. Which of the following will two observers in inertial reference frames always agree on? (Choose all that apply.)
(a) The time an event occurred.
(b) The distance between two events.
(c) The time interval between the occurrence of two events.
(d) The speed of light.
(e) The validity of the laws of physics.
(f) The simultaneity of two events.
9. Two observers in different inertial reference frames moving relative to each other at nearly the speed of light see the same two events but, using precise equipment, record different time intervals between the two events. Which of the following is true of their measurements?
(a) One observer is incorrect, but it is impossible to tell which one.
(b) One observer is incorrect, and it is possible to tell which one.
(c) Both observers are incorrect.
(d) Both observers are correct.
10. You are in a rocket ship going faster and faster. As your speed increases and your velocity gets closer to the speed of light, which of the following do you observe in your frame of reference?
(a) Your mass increases.
(b) Your length shortens in the direction of motion.
(c) Your wristwatch slows down.
(d) All of the above.
(e) None of the above.
11. You are in a spaceship with no windows, radios, or other means to check outside. How could you determine whether your spaceship is at rest or moving at constant velocity?
(a) By determining the apparent velocity of light in the spaceship.
(b) By checking your precision watch. If it's running slow, then the ship is moving.
(c) By measuring the lengths of objects in the spaceship. If they are shortened, then the ship is moving.
(d) Give up, because you can't tell.
12. The period of a pendulum attached in a spaceship is 2 s while the spaceship is parked on Earth. What is the period to an observer on Earth when the spaceship moves at $0.6c$ with respect to the Earth?
(a) Less than 2 s.
(b) More than 2 s.
(c) 2 s.
13. Two spaceships, each moving at a speed $0.75c$ relative to the Earth, are headed directly toward each other. What do occupants of one ship measure the speed of other ship to be?
(a) $0.96c$. (b) $1.0c$. (c) $1.5c$. (d) $1.75c$. (e) $0.75c$.

For assigned homework and other learning materials, go to the MasteringPhysics website.



Problems

26–4 and 26–5 Time Dilation, Length Contraction

1. (I) A spaceship passes you at a speed of $0.850c$. You measure its length to be 44.2 m. How long would it be when at rest?
2. (I) A certain type of elementary particle travels at a speed of 2.70×10^8 m/s. At this speed, the average lifetime is measured to be 4.76×10^{-6} s. What is the particle's lifetime at rest?
3. (II) You travel to a star 135 light-years from Earth at a speed of 2.90×10^8 m/s. What do you measure this distance to be?
4. (II) What is the speed of a pion if its average lifetime is measured to be 4.40×10^{-8} s? At rest, its average lifetime is 2.60×10^{-8} s.
5. (II) In an Earth reference frame, a star is 49 light-years away. How fast would you have to travel so that to you the distance would be only 35 light-years?
6. (II) At what speed v will the length of a 1.00-m stick look 10.0% shorter (90.0 cm)?
7. (II) At what speed do the relativistic formulas for (a) length and (b) time intervals differ from classical values by 1.00%? (This is a reasonable way to estimate when to use relativistic calculations rather than classical.)
8. (II) You decide to travel to a star 62 light-years from Earth at a speed that tells you the distance is only 25 light-years. How many years would it take you to make the trip?
9. (II) A friend speeds by you in her spacecraft at a speed of $0.720c$. It is measured in your frame to be 4.80 m long and 1.35 m high. (a) What will be its length and height at rest? (b) How many seconds elapsed on your friend's watch when 20.0 s passed on yours? (c) How fast did you appear to be traveling according to your friend? (d) How many seconds elapsed on your watch when she saw 20.0 s pass on hers?
10. (II) A star is 21.6 light-years from Earth. How long would it take a spacecraft traveling $0.950c$ to reach that star as measured by observers: (a) on Earth, (b) on the spacecraft? (c) What is the distance traveled according to observers on the spacecraft? (d) What will the spacecraft occupants compute their speed to be from the results of (b) and (c)?
11. (II) A fictional news report stated that starship *Enterprise* had just returned from a 5-year voyage while traveling at $0.70c$. (a) If the report meant 5.0 years of *Earth time*, how much time elapsed on the ship? (b) If the report meant 5.0 years of *ship time*, how much time passed on Earth?
12. (II) A box at rest has the shape of a cube 2.6 m on a side. This box is loaded onto the flat floor of a spaceship and the spaceship then flies past us with a horizontal speed of $0.80c$. What is the volume of the box as we observe it?
13. (III) Escape velocity from the Earth is 11.2 km/s. What would be the percent decrease in length of a 68.2-m-long spacecraft traveling at that speed as seen from Earth?

14. (III) An unstable particle produced in an accelerator experiment travels at constant velocity, covering 1.00 m in 3.40 ns in the lab frame before changing (“decaying”) into other particles. In the rest frame of the particle, determine (a) how long it lived before decaying, (b) how far it moved before decaying.
15. (III) How fast must a pion be moving on average to travel 32 m before it decays? The average lifetime, at rest, is 2.6×10^{-8} s.

26–7 Relativistic Momentum

16. (I) What is the momentum of a proton traveling at $v = 0.68c$?
17. (II) (a) A particle travels at $v = 0.15c$. By what percentage will a calculation of its momentum be wrong if you use the classical formula? (b) Repeat for $v = 0.75c$.
18. (II) A particle of mass m travels at a speed $v = 0.22c$. At what speed will its momentum be doubled?
19. (II) An unstable particle is at rest and suddenly decays into two fragments. No external forces act on the particle or its fragments. One of the fragments has a speed of $0.60c$ and a mass of 6.68×10^{-27} kg, while the other has a mass of 1.67×10^{-27} kg. What is the speed of the less massive fragment?
20. (II) What is the percent change in momentum of a proton that accelerates from (a) $0.45c$ to $0.85c$, (b) $0.85c$ to $0.98c$?

26–9 $E = mc^2$; Mass and Energy

21. (I) Calculate the rest energy of an electron in joules and in MeV ($1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$).
22. (I) When a uranium nucleus at rest breaks apart in the process known as *fission* in a nuclear reactor, the resulting fragments have a total kinetic energy of about 200 MeV. How much mass was lost in the process?
23. (I) The total annual energy consumption in the United States is about 1×10^{20} J. How much mass would have to be converted to energy to fuel this need?
24. (I) Calculate the mass of a proton (1.67×10^{-27} kg) in MeV/c^2 .
25. (I) A certain chemical reaction requires 4.82×10^4 J of energy input for it to go. What is the increase in mass of the products over the reactants?
26. (II) Calculate the kinetic energy and momentum of a proton traveling 2.90×10^8 m/s.
27. (II) What is the momentum of a 950-MeV proton (that is, its kinetic energy is 950 MeV)?
28. (II) What is the speed of an electron whose kinetic energy is 1.12 MeV?
29. (II) (a) How much work is required to accelerate a proton from rest up to a speed of $0.985c$? (b) What would be the momentum of this proton?
30. (II) At what speed will an object’s kinetic energy be 33% of its rest energy?
31. (II) Determine the speed and the momentum of an electron ($m = 9.11 \times 10^{-31}$ kg) whose KE equals its rest energy.
32. (II) A proton is traveling in an accelerator with a speed of 1.0×10^8 m/s. By what factor does the proton’s kinetic energy increase if its speed is doubled?
33. (II) How much energy can be obtained from conversion of 1.0 gram of mass? How much mass could this energy raise to a height of 1.0 km above the Earth’s surface?
34. (II) To accelerate a particle of mass m from rest to speed $0.90c$ requires work W_1 . To accelerate the particle from speed $0.90c$ to $0.99c$ requires work W_2 . Determine the ratio W_2/W_1 .
35. (II) Suppose there was a process by which two photons, each with momentum $0.65 \text{ MeV}/c$, could collide and make a single particle. What is the maximum mass that the particle could possess?
36. (II) What is the speed of a proton accelerated by a potential difference of 165 MV?
37. (II) What is the speed of an electron after being accelerated from rest by 31,000 V?
38. (II) The kinetic energy of a particle is 45 MeV. If the momentum is $121 \text{ MeV}/c$, what is the particle’s mass?
39. (II) Calculate the speed of a proton ($m = 1.67 \times 10^{-27}$ kg) whose kinetic energy is exactly half (a) its total energy, (b) its rest energy.
40. (II) Calculate the kinetic energy and momentum of a proton ($m = 1.67 \times 10^{-27}$ kg) traveling 8.65×10^7 m/s. By what percentages would your calculations have been in error if you had used classical formulas?
41. (II) Suppose a spacecraft of mass 17,000 kg is accelerated to $0.15c$. (a) How much kinetic energy would it have? (b) If you used the classical formula for kinetic energy, by what percentage would you be in error?
42. (II) A negative muon traveling at 53% the speed of light collides head on with a positive muon traveling at 65% the speed of light. The two muons (each of mass $105.7 \text{ MeV}/c^2$) annihilate, and produce how much electromagnetic energy?
43. (II) Two identical particles of mass m approach each other at equal and opposite speeds, v . The collision is completely inelastic and results in a single particle at rest. What is the mass of the new particle? How much energy was lost in the collision? How much kinetic energy was lost in this collision?
44. (III) The americium nucleus, $^{241}_{95}\text{Am}$, decays to a neptunium nucleus, $^{237}_{93}\text{Np}$, by emitting an alpha particle of mass 4.00260 u and kinetic energy 5.5 MeV. Estimate the mass of the neptunium nucleus, ignoring its recoil, given that the americium mass is 241.05682 u.
45. (III) Show that the kinetic energy KE of a particle of mass m is related to its momentum p by the equation
- $$p = \sqrt{\text{KE}^2 + 2\text{KE} mc^2}/c.$$
- *46. (III) What magnetic field B is needed to keep 998-GeV protons revolving in a circle of radius 1.0 km? Use the relativistic mass. The proton’s “rest mass” is $0.938 \text{ GeV}/c^2$. ($1 \text{ GeV} = 10^9 \text{ eV}$.) [Hint: In relativity, $m_{\text{rel}}v^2/r = qvB$ is still valid in a magnetic field, where $m_{\text{rel}} = \gamma m$.]

26–10 Relativistic Addition of Velocities

47. (I) A person on a rocket traveling at $0.40c$ (with respect to the Earth) observes a meteor come from behind and pass her at a speed she measures as $0.40c$. How fast is the meteor moving with respect to the Earth?
48. (II) Two spaceships leave Earth in opposite directions, each with a speed of $0.60c$ with respect to Earth. (a) What is the velocity of spaceship 1 relative to spaceship 2? (b) What is the velocity of spaceship 2 relative to spaceship 1?

49. (II) A spaceship leaves Earth traveling at $0.65c$. A second spaceship leaves the first at a speed of $0.82c$ with respect to the first. Calculate the speed of the second ship with respect to Earth if it is fired (a) in the same direction the first spaceship is already moving, (b) directly backward toward Earth.
50. (II) An observer on Earth sees an alien vessel approach at a speed of $0.60c$. The fictional starship *Enterprise* comes to the rescue (Fig. 26–13), overtaking the aliens while moving directly toward Earth at a speed of $0.90c$ relative to Earth. What is the relative speed of one vessel as seen by the other?

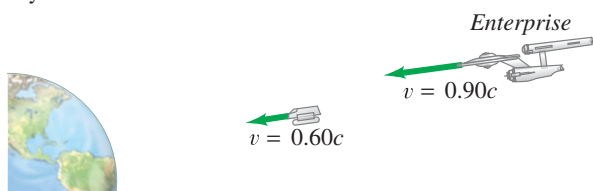


FIGURE 26–13 Problem 50.

51. (II) A spaceship in distress sends out two escape pods in opposite directions. One travels at a speed $v_1 = +0.70c$ in one direction, and the other travels at a speed $v_2 = -0.80c$ in the other direction, as observed from the spaceship. What speed does the first escape pod measure for the second escape pod?
52. (II) Rocket A passes Earth at a speed of $0.65c$. At the same time, rocket B passes Earth moving $0.95c$ relative to Earth in the same direction as A. How fast is B moving relative to A when it passes A?
53. (II) Your spaceship, traveling at $0.90c$, needs to launch a probe out the forward hatch so that its speed relative to the planet that you are approaching is $0.95c$. With what speed must it leave your ship?

General Problems

54. What is the speed of a particle when its kinetic energy equals its rest energy? Does the mass of the particle affect the result?
55. The nearest star to Earth is Proxima Centauri, 4.3 light-years away. (a) At what constant velocity must a spacecraft travel from Earth if it is to reach the star in 4.9 years, as measured by travelers on the spacecraft? (b) How long does the trip take according to Earth observers?
56. According to the special theory of relativity, the factor γ that determines the length contraction and the time dilation is given by $\gamma = 1/\sqrt{1 - v^2/c^2}$. Determine the numerical values of γ for an object moving at speed $v = 0.01c, 0.05c, 0.10c, 0.20c, 0.30c, 0.40c, 0.50c, 0.60c, 0.70c, 0.80c, 0.90c, 0.95c, \text{ and } 0.99c$. Make a graph of γ versus v .
57. A healthy astronaut's heart rate is 60 beats/min. Flight doctors on Earth can monitor an astronaut's vital signs remotely while in flight. How fast would an astronaut be flying away from Earth if the doctor measured her having a heart rate of 25 beats/min?
58. (a) What is the speed v of an electron whose kinetic energy is 14,000 times its rest energy? You can state the answer as the difference $c - v$. Such speeds are reached in the Stanford Linear Accelerator, SLAC. (b) If the electrons travel in the lab through a tube 3.0 km long (as at SLAC), how long is this tube in the electrons' reference frame? [Hint: Use the binomial expansion.]
59. What minimum amount of electromagnetic energy is needed to produce an electron and a positron together? A positron is a particle with the same mass as an electron, but has the opposite charge. (Note that electric charge is conserved in this process. See Section 27–6.)
60. How many grams of matter would have to be totally destroyed to run a 75-W lightbulb for 1.0 year?
61. A free neutron can decay into a proton, an electron, and a neutrino. Assume the neutrino's mass is zero; the other masses can be found in the Table inside the front cover. Determine the total kinetic energy shared among the three particles when a neutron decays at rest.
62. An electron ($m = 9.11 \times 10^{-31} \text{ kg}$) is accelerated from rest to speed v by a conservative force. In this process, its potential energy decreases by $6.20 \times 10^{-14} \text{ J}$. Determine the electron's speed, v .
63. The Sun radiates energy at a rate of about $4 \times 10^{26} \text{ W}$. (a) At what rate is the Sun's mass decreasing? (b) How long does it take for the Sun to lose a mass equal to that of Earth? (c) Estimate how long the Sun could last if it radiated constantly at this rate.
64. How much energy would be required to break a helium nucleus into its constituents, two protons and two neutrons? The masses of a proton (including an electron), a neutron, and neutral helium are, respectively, 1.00783 u, 1.00867 u, and 4.00260 u. (This energy difference is called the *total binding energy* of the ${}^4\text{He}$ nucleus.)
65. Show analytically that a particle with momentum p and energy E has a speed given by
- $$v = \frac{pc^2}{E} = \frac{pc}{\sqrt{m^2c^2 + p^2}}.$$
66. Two protons, each having a speed of $0.990c$ in the laboratory, are moving toward each other. Determine (a) the momentum of each proton in the laboratory, (b) the total momentum of the two protons in the laboratory, and (c) the momentum of one proton as seen by the other proton.
67. When two moles of hydrogen molecules (H_2) and one mole of oxygen molecules (O_2) react to form two moles of water (H_2O), the energy released is 484 kJ. How much does the mass decrease in this reaction? What % of the total original mass is this?
68. The fictional starship *Enterprise* obtains its power by combining matter and antimatter, achieving complete conversion of mass into energy. If the mass of the *Enterprise* is approximately $6 \times 10^9 \text{ kg}$, how much mass must be converted into kinetic energy to accelerate it from rest to one-tenth the speed of light?
69. Make a graph of the kinetic energy versus momentum for (a) a particle of nonzero mass, and (b) a particle with zero mass.

70. A spaceship and its occupants have a total mass of 160,000 kg. The occupants would like to travel to a star that is 35 light-years away at a speed of $0.70c$. To accelerate, the engine of the spaceship changes mass directly to energy. (a) Estimate how much mass will be converted to energy to accelerate the spaceship to this speed. (b) Assuming the acceleration is rapid, so the speed for the entire trip can be taken to be $0.70c$, determine how long the trip will take according to the astronauts on board.
71. In a nuclear reaction two identical particles are created, traveling in opposite directions. If the speed of each particle is $0.82c$, relative to the laboratory frame of reference, what is one particle's speed relative to the other particle?
72. A 36,000-kg spaceship is to travel to the vicinity of a star 6.6 light-years from Earth. Passengers on the ship want the (one-way) trip to take no more than 1.0 year. How much work must be done on the spaceship to bring it to the speed necessary for this trip?
73. Suppose a 14,500-kg spaceship left Earth at a speed of $0.90c$. What is the spaceship's kinetic energy? Compare with the total U.S. annual energy consumption (about 10^{20} J).
74. A pi meson of mass m_π decays at rest into a muon (mass m_μ) and a neutrino of negligible or zero mass. Show that the kinetic energy of the muon is $\text{KE}_\mu = (m_\pi - m_\mu)^2 c^2 / (2m_\pi)$.
75. An astronaut on a spaceship traveling at $0.75c$ relative to Earth measures his ship to be 23 m long. On the ship, he eats his lunch in 28 min. (a) What length is the spaceship according to observers on Earth? (b) How long does the astronaut's lunch take to eat according to observers on Earth?
76. Astronomers measure the distance to a particular star to be 6.0 light-years (1 ly = distance light travels in 1 year). A spaceship travels from Earth to the vicinity of this star at steady speed, arriving in 3.50 years as measured by clocks on the spaceship. (a) How long does the trip take as measured by clocks in Earth's reference frame? (b) What distance does the spaceship travel as measured in its own reference frame?
77. An electron is accelerated so that its kinetic energy is greater than its rest energy mc^2 by a factor of (a) 5.00, (b) 999. What is the speed of the electron in each case?
78. You are traveling in a spaceship at a speed of $0.70c$ away from Earth. You send a laser beam toward the Earth traveling at velocity c relative to you. What do observers on the Earth measure for the speed of the laser beam?
79. A farm boy studying physics believes that he can fit a 13.0-m-long pole into a 10.0-m-long barn if he runs fast enough, carrying the pole. Can he do it? Explain in detail. How does this fit with the idea that when he is running the barn looks even shorter than 10.0 m?
80. An atomic clock is taken to the North Pole, while another stays at the Equator. How far will they be out of synchronization after 2.0 years has elapsed? [Hint: Use the binomial expansion, Appendix A.]
81. An airplane travels 1300 km/h around the Earth in a circle of radius essentially equal to that of the Earth, returning to the same place. Using special relativity, estimate the difference in time to make the trip as seen by Earth and by airplane observers. [Hint: Use the binomial expansion, Appendix A.]

Search and Learn

- Determine about how fast Mr Tompkins is traveling in the Chapter-Opening Photograph. Do you agree with the picture in terms of the way Mr Tompkins would see the world? Explain. [Hint: Assume the bank clock and Stop sign facing us are round according to the people on the sidewalk.]
- Examine the experiment of Fig. 26–5 from O_1 's reference frame. In this case, O_1 will be at rest and will see the lightning bolt at B_1 and B_2 , before the lightning bolt at A_1 and A_2 . Will O_1 recognize that O_2 , who is moving with speed v to the left, will see the two events as simultaneous? Explain in detail, drawing diagrams equivalent to Fig. 26–5. [Hint: Include length contraction.]
- Using Example 26–2 as a guide, show that for objects that move slowly in comparison to c , the length contraction formula is roughly $\ell \approx \ell_0(1 - \frac{1}{2}v^2/c^2)$. Use this approximation to find the “length shortening” $\Delta\ell = \ell_0 - \ell$ of the train in Example 26–6 if the train travels at 100 km/h (rather than $0.92c$).
- In Example 26–5, the spaceship is moving at $0.90c$ in the horizontal direction relative to an observer on the Earth. If instead the spaceship moved at $0.90c$ directed at 30° above the horizontal, what would be the painting's dimensions as seen by the observer on Earth?
- Protons from outer space crash into the Earth's atmosphere at a high rate. These protons create particles that eventually decay into other particles called *muons*. This cosmic debris travels through the atmosphere. Every second, dozens of muons pass through your body. If a muon is created 30 km above the Earth's surface, what minimum speed and kinetic energy must the muon have in order to hit Earth's surface? A muon's mean lifetime (at rest) is $2.20 \mu\text{s}$ and its mass is $105.7 \text{ MeV}/c^2$.
- As a rough rule, anything traveling faster than about $0.1c$ is called *relativistic*—that is, special relativity is a significant effect. Determine the speed of an electron in a hydrogen atom (radius $0.53 \times 10^{-10} \text{ m}$) and state whether or not it is relativistic. (Treat the electron as though it were in a circular orbit around the proton. See hint for Problem 46.)

ANSWERS TO EXERCISES

A: (c).

B: (a) No; (b) yes.

C: 80 m.

D: No; $\text{KE}/mc^2 \approx 2 \times 10^{-4}$.

E: $0.030c$, same as classical, to an accuracy of better than 0.1%.

F: (d).