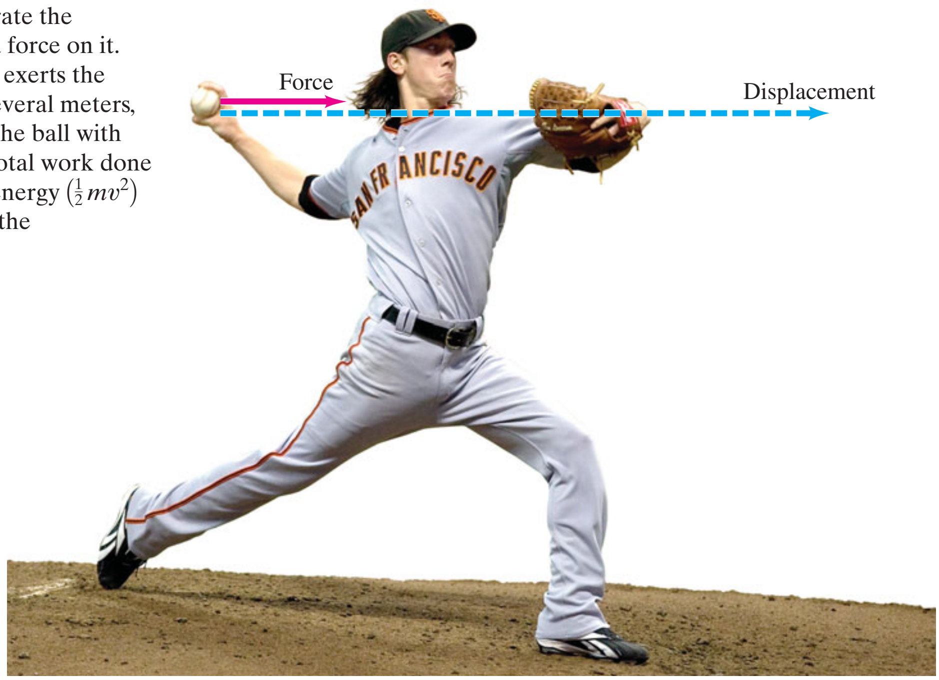
This baseball pitcher is about to accelerate the baseball to a high velocity by exerting a force on it. He will be doing work on the ball as he exerts the force over a displacement of perhaps several meters, from behind his head until he releases the ball with arm outstretched in front of him. The total work done on the ball will be equal to the kinetic energy $\left(\frac{1}{2}mv^2\right)$ acquired by the ball, a result known as the work-energy principle.

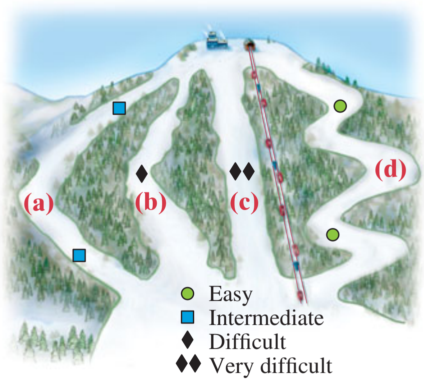


CHAPTER 6

Work and Energy

CHAPTER-OPENING QUESTION—Guess now!

A skier starts at the top of a hill. On which run does her gravitational potential energy change the most: **(a)**, **(b)**, **(c)**, or **(d)**; or are they **(e)** all the same? On which run would her speed at the bottom be the fastest if the runs are icy and we assume no friction or air resistance? Recognizing that there is always some friction, answer the above two questions again. List your four answers now.



CONTENTS

Until now we have been studying the translational motion of an object in terms of Newton's three laws of motion. In that analysis, *force* has played a central role as the quantity determining the motion. In this Chapter and the next, we discuss an alternative analysis of the translational motion of objects in terms of the quantities *energy* and *momentum*. The significance of energy and momentum is that they are *conserved*. That is, in quite general circumstances they remain constant. That conserved quantities exist gives us not only a deeper insight into the nature of the world, but also gives us another way to approach solving practical problems.

The conservation laws of energy and momentum are especially valuable in dealing with systems of many objects, in which a detailed consideration of the forces involved would be difficult or impossible. These laws apply to a wide range of phenomena. They even apply in the atomic and subatomic worlds, where Newton's laws are not sufficient.

This Chapter is devoted to the very important concept of *energy* and the closely related concept of *work*. These two quantities are scalars and so have no direction associated with them, which often makes them easier to work with than vector quantities such as acceleration and force.

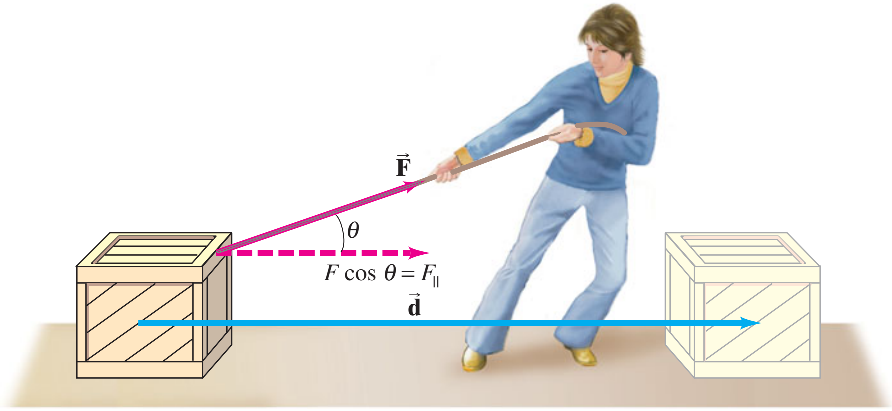


FIGURE 6–1 A person pulling a crate along the floor. The work done by the force $\vec{\mathbf{F}}$ is $W = Fd\cos\theta$, where $\vec{\mathbf{d}}$ is the displacement.

6–1 Work Done by a Constant Force

The word *work* has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished when a force acts on an object, and the object moves through a distance. We consider only translational motion for now and, unless otherwise explained, objects are assumed to be rigid with no complicating internal motion, and can be treated like particles. Then the **work** done on an object by a constant force (constant in both magnitude and direction) is defined to be *the product of the magnitude of the displacement times the component of the force parallel to the displacement*. In equation form, we can write

$$W = F_{\|}d,$$

where $F_{\|}$ is the component of the constant force $\vec{\mathbf{F}}$ parallel to the displacement $\vec{\mathbf{d}}$. We can also write

$$W = Fd\cos\theta, \qquad \textbf{(6–1)}$$

where F is the magnitude of the constant force, d is the magnitude of the displacement of the object, and θ is the angle between the directions of the force and the displacement (Fig. 6–1). The $\cos\theta$ factor appears in Eq. 6–1 because $F\cos\theta$ $(= F_{\|})$ is the component of $\vec{\mathbf{F}}$ that is parallel to $\vec{\mathbf{d}}$. Work is a scalar quantity—it has no direction, but only magnitude, which can be positive or negative.

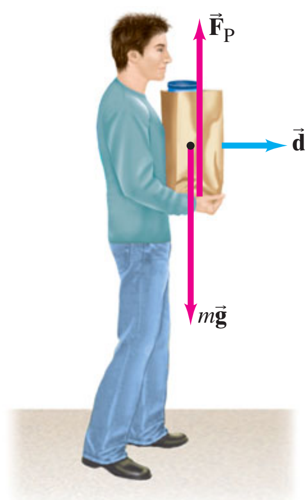
Let us consider the case in which the motion and the force are in the same direction, so $\theta = 0$ and $\cos\theta = 1$; in this case, $W = Fd$. For example, if you push a loaded grocery cart a distance of 50 m by exerting a horizontal force of 30 N on the cart, you do $30\,\mathrm{N} \times 50\,\mathrm{m} = 1500\,\mathrm{N\cdot m}$ of work on the cart.

As this example shows, in SI units work is measured in newton-meters ($\mathrm{N\cdot m}$). A special name is given to this unit, the **joule** (J): $1\,\mathrm{J} = 1\,\mathrm{N\cdot m}$.

[In the cgs system, the unit of work is called the *erg* and is defined as $1\,\mathrm{erg} = 1\,\mathrm{dyne\cdot cm}$. In British units, work is measured in foot-pounds. Their equivalence is $1\,\mathrm{J} = 10^7\,\mathrm{erg} = 0.7376\,\mathrm{ft\cdot lb}$.]

A force can be exerted on an object and yet do no work. If you hold a heavy bag of groceries in your hands at rest, you do no work on it. You do exert a force on the bag, but the displacement of the bag is zero, so the work done by you on the bag is $W = 0$. You need both a force and a displacement to do work. You also do no work on the bag of groceries if you carry it as you walk horizontally across the floor at constant velocity, as shown in Fig. 6–2. No horizontal force is required to move the bag at a constant velocity. The person shown in Fig. 6–2 exerts an upward force $\vec{\mathbf{F}}_{\mathrm{P}}$ on the bag equal to its weight. But this upward force is perpendicular to the horizontal displacement of the bag and thus is doing no work. This conclusion comes from our definition of work, Eq. 6–1: $W = 0$, because $\theta = 90^\circ$ and $\cos 90^\circ = 0$. Thus, when a particular force is perpendicular to the displacement, no work is done by that force. When you start or stop walking, there is a horizontal acceleration and you do briefly exert a horizontal force, and thus do work on the bag.

FIGURE 6–2 The person does no work on the bag of groceries because $\vec{\mathbf{F}}_{\mathrm{P}}$ is perpendicular to the displacement $\vec{\mathbf{d}}$.



CAUTION
Force without work

CAUTION
State that work is done on *or* by *an object*

When we deal with work, as with force, it is necessary to specify whether you are talking about work done *by* a specific object or done *on* a specific object. It is also important to specify whether the work done is due to one particular force (and which one), or the total (net) work done by the *net force* on the object.

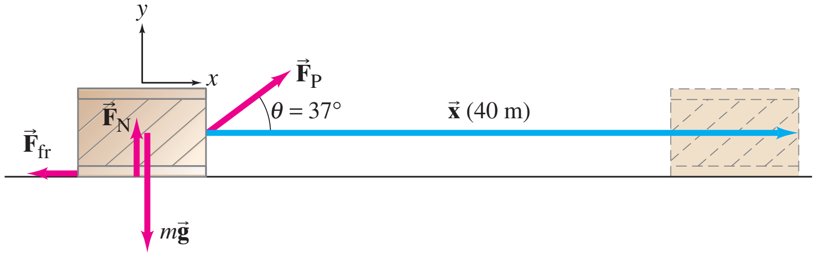


FIGURE 6–3 Example 6–1. A 50-kg crate is pulled along a floor.

EXAMPLE 6–1 **Work done on a crate.** A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\,\mathrm{N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $\vec{\mathbf{F}}_{fr} = 50\,\mathrm{N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.

APPROACH We choose our coordinate system so that the vector that represents the 40-m displacement is $\vec{\mathbf{x}}$ (that is, along the x axis). Four forces act on the crate, as shown in the free-body diagram in Fig. 6–3: the force exerted by the person $\vec{\mathbf{F}}_P$; the friction force $\vec{\mathbf{F}}_{fr}$; the gravitational force exerted by the Earth, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$; and the normal force $\vec{\mathbf{F}}_N$ exerted upward by the floor. The net force on the crate is the vector sum of these four forces.

SOLUTION (*a*) The work done by the gravitational force $(\vec{\mathbf{F}}_G)$ and by the normal force $(\vec{\mathbf{F}}_N)$ is zero, because they are perpendicular to the displacement $\vec{\mathbf{x}}$ ($\theta = 90°$ in Eq. 6–1):

$$W_G = mgx\cos 90° = 0$$
$$W_N = F_N x\cos 90° = 0.$$

The work done by $\vec{\mathbf{F}}_P$ is

$$W_P = F_P x\cos\theta = (100\,\mathrm{N})(40\,\mathrm{m})\cos 37° = 3200\,\mathrm{J}.$$

The work done by the friction force is

$$W_{fr} = F_{fr}x\cos 180° = (50\,\mathrm{N})(40\,\mathrm{m})(-1) = -2000\,\mathrm{J}.$$

The angle between the displacement $\vec{\mathbf{x}}$ and $\vec{\mathbf{F}}_{fr}$ is 180° because they point in opposite directions. Since the force of friction is opposing the motion (and $\cos 180° = -1$), the work done by friction on the crate is *negative*.

(*b*) The net work can be calculated in two equivalent ways.

(1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

$$\begin{aligned} W_{net} &= W_G + W_N + W_P + W_{fr} \\ &= 0 + 0 + 3200\,\mathrm{J} - 2000\,\mathrm{J} = 1200\,\mathrm{J}. \end{aligned}$$

(2) The net work can also be calculated by first determining the net force on the object and then taking the component of this net force along the displacement: $(F_{net})_x = F_P\cos\theta - F_{fr}$. Then the net work is

$$\begin{aligned} W_{net} = (F_{net})_x x &= (F_P\cos\theta - F_{fr})x \\ &= (100\,\mathrm{N}\cos 37° - 50\,\mathrm{N})(40\,\mathrm{m}) = 1200\,\mathrm{J}. \end{aligned}$$

In the vertical (y) direction, there is no displacement and no work done.

CAUTION
Negative work

In Example 6–1 we saw that friction did negative work. In general, the work done by a force is negative whenever the force (or the component of the force, $F_{\parallel}$) acts in the direction opposite to the direction of motion.

EXERCISE A A box is dragged a distance d across a floor by a force $\vec{\mathbf{F}}_P$ which makes an angle θ with the horizontal as in Fig. 6–1 or 6–3. If the magnitude of $\vec{\mathbf{F}}_P$ is held constant but the angle θ is increased, the work done by $\vec{\mathbf{F}}_P$ (*a*) remains the same; (*b*) increases; (*c*) decreases; (*d*) first increases, then decreases.

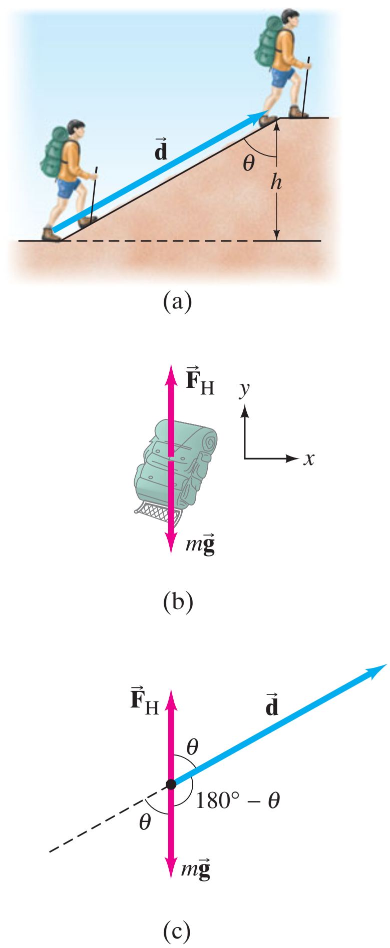
PROBLEM SOLVING

Work

1. **Draw a free-body diagram** showing all the forces acting on the object you choose to study.
2. **Choose** an xy **coordinate system**. If the object is in motion, it may be convenient to choose one of the coordinate directions as the direction of one of the forces, or as the direction of motion. [Thus, for an object on an incline, you might choose one coordinate axis to be parallel to the incline.]
3. **Apply Newton's laws** to determine unknown forces.
4. Find the **work done *by* a specific force** *on* the object by using $W = Fd\cos\theta$ for a constant force. The work done is negative when a force opposes the displacement.
5. To find the **net work** done on the object, either (a) find the work done by each force and add the results algebraically; or (b) find the net force on the object, F_{net}, and then use it to find the net work done, which for constant net force is:
$$W_{\text{net}} = F_{\text{net}}\,d\cos\theta.$$

EXAMPLE 6–2 **Work on a backpack.** (a) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0\,\text{m}$, as shown in Fig. 6–4a. Determine also (b) the work done by gravity on the backpack, and (c) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

FIGURE 6–4 Example 6–2.



APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

SOLUTION

1. **Draw a free-body diagram**. The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\vec{\mathbf{g}}$, acting downward; and $\vec{\mathbf{F}}_{\text{H}}$, the force the hiker must exert upward to support the backpack. The acceleration is zero, so horizontal forces on the backpack are negligible.
2. **Choose a coordinate system**. We are interested in the vertical motion of the backpack, so we choose the y coordinate as positive vertically upward.
3. **Apply Newton's laws**. Newton's second law applied in the vertical direction to the backpack gives (with $a_y = 0$)
$$\Sigma F_y = ma_y$$
$$F_{\text{H}} - mg = 0.$$
So,
$$F_{\text{H}} = mg = (15.0\,\text{kg})(9.80\,\text{m/s}^2) = 147\,\text{N}.$$
4. **Work done by a specific force**. (a) To calculate the work done by the hiker on the backpack, we use Eq. 6–1, where θ is shown in Fig. 6–4c,
$$W_{\text{H}} = F_{\text{H}}(d\cos\theta),$$
and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker is
$$W_{\text{H}} = F_{\text{H}}(d\cos\theta) = F_{\text{H}}h = mgh = (147\,\text{N})(10.0\,\text{m}) = 1470\,\text{J}.$$
The work done depends only on the elevation change and not on the angle of the hill, θ. The hiker would do the same work to lift the pack vertically by height h.
(b) The work done by gravity on the backpack is (from Eq. 6–1 and Fig. 6–4c)
$$W_{\text{G}} = mg\,d\cos(180^\circ - \theta).$$
Since $\cos(180^\circ - \theta) = -\cos\theta$ (Appendix A–7), we have
$$\begin{aligned} W_{\text{G}} &= mg(-d\cos\theta) \\ &= -mgh \\ &= -(15.0\,\text{kg})(9.80\,\text{m/s}^2)(10.0\,\text{m}) = -1470\,\text{J}. \end{aligned}$$

NOTE The work done by gravity (which is negative here) does not depend on the angle of the incline, only on the vertical height h of the hill.

PROBLEM SOLVING

Work done by gravity depends on height of hill (not on angle)

5. **Net work done**. (c) The *net* work done on the backpack is $W_{\text{net}} = 0$, because the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also get the net work done by adding the work done by each force:
$$W_{\text{net}} = W_{\text{G}} + W_{\text{H}} = -1470\,\text{J} + 1470\,\text{J} = 0.$$

NOTE Even though the *net* work done by all the forces on the backpack is zero, the hiker *does* do work on the backpack equal to 1470 J.

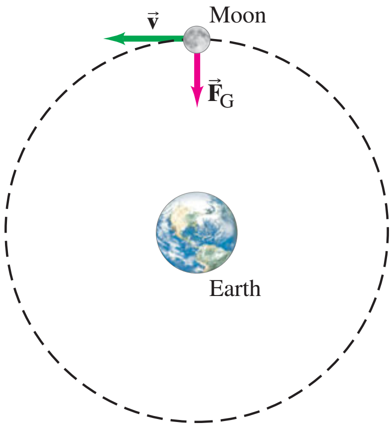
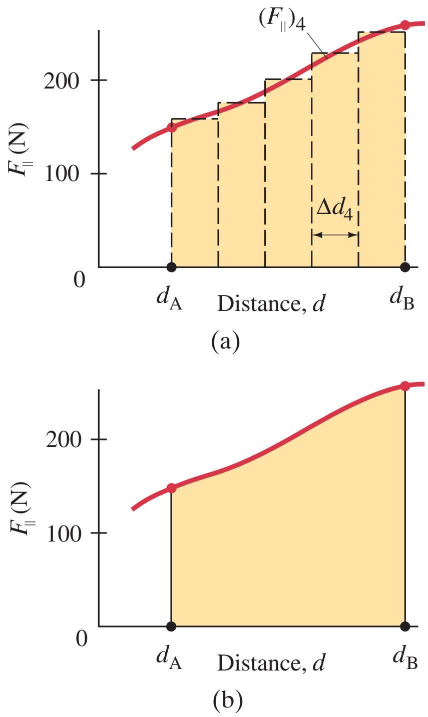


FIGURE 6–5 Example 6–3.

CONCEPTUAL EXAMPLE 6–3 **Does the Earth do work on the Moon?** The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

RESPONSE The gravitational force $\vec{\mathbf{F}}_G$ exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{\mathbf{F}}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero $(\cos 90° = 0)$. This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity.

FIGURE 6–6 Work done by a force F is (a) approximately equal to the sum of the areas of the rectangles, (b) exactly equal to the area under the curve of $F_{\parallel}$ vs. d.



*6–2 Work Done by a Varying Force

If the force acting on an object is constant, the work done by that force can be calculated using Eq. 6–1. But in many cases, the force varies in magnitude or direction during a process. For example, as a rocket moves away from Earth, work is done to overcome the force of gravity, which varies as the inverse square of the distance from the Earth's center. Other examples are the force exerted by a spring, which increases with the amount of stretch, or the work done by a varying force that pulls a box or cart up an uneven hill.

The work done by a varying force can be determined graphically. To do so, we plot $F_{\parallel}$ $(= F\cos\theta,$ the component of $\vec{\mathbf{F}}$ parallel to the direction of motion at any point) as a function of distance d, as in Fig. 6–6a. We divide the distance into small segments Δd. For each segment, we indicate the average of $F_{\parallel}$ by a horizontal dashed line. Then the work done for each segment is $\Delta W = F_{\parallel}\,\Delta d$, which is the area of a rectangle Δd wide and $F_{\parallel}$ high. The total work done to move the object a total distance $d = d_B - d_A$ is the sum of the areas of the rectangles (five in the case shown in Fig. 6–6a). Usually, the average value of $F_{\parallel}$ for each segment must be estimated, and a reasonable approximation of the work done can then be made.

If we subdivide the distance into many more segments, Δd can be made smaller and our estimate of the work done would be more accurate. In the limit as Δd approaches zero, the total area of the many narrow rectangles approaches the area under the curve, Fig. 6–6b. That is, *the work done by a variable force in moving an object between two points is equal to the area under the $F_{\parallel}$ vs. d curve between those two points*.

6–3 Kinetic Energy, and the Work-Energy Principle

Energy is one of the most important concepts in science. Yet we cannot give a simple general definition of energy in only a few words. Nonetheless, each specific type of energy can be defined fairly simply. In this Chapter we define translational kinetic energy and some types of potential energy. In later Chapters, we will examine other types of energy, such as that related to heat and electricity. The crucial aspect of energy is that the sum of all types, the *total energy*, is the same after any process as it was before: that is, energy is a conserved quantity.

For the purposes of this Chapter, we can define energy in the traditional way as "the ability to do work." This simple definition is not always applicable,† but it is valid for mechanical energy which we discuss in this Chapter. We now define and discuss one of the basic types of energy, kinetic energy.

† Energy associated with heat is often not available to do work, as we will discuss in Chapter 15.

A moving object can do work on another object it strikes. A flying cannonball does work on a brick wall it knocks down; a moving hammer does work on a nail it drives into wood. In either case, a moving object exerts a force on a second object which undergoes a displacement. An object in motion has the ability to do work and thus can be said to have energy. The energy of motion is called **kinetic energy**, from the Greek word *kinetikos*, meaning "motion."

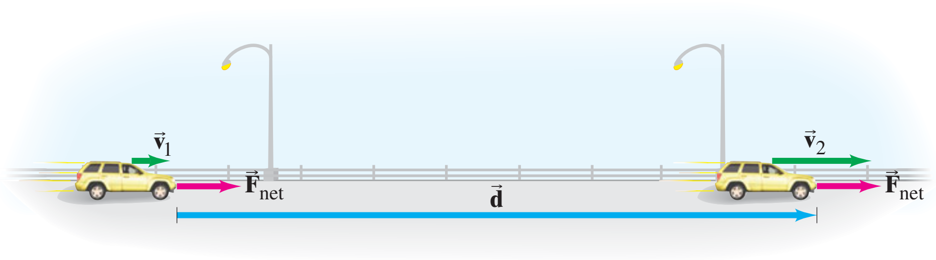


FIGURE 6–7 A constant net force F_{net} accelerates a car from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{net} = F_{net}d$.

To obtain a quantitative definition for kinetic energy, let us consider a simple rigid object of mass m (treated as a particle) that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a displacement d, Fig. 6–7. Then the net work done on the object is $W_{net} = F_{net}d$. We apply Newton's second law, $F_{net} = ma$, and use Eq. 2–11c $\left(v_2^2 = v_1^2 + 2ad\right)$, which we rewrite as

$$a = \frac{v_2^2 - v_1^2}{2d},$$

where v_1 is the initial speed and v_2 is the final speed. Substituting this into $F_{net} = ma$, we determine the work done:

$$W_{net} = F_{net}d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–2)}$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy (KE)** of the object:

$$\boxed{\text{KE} = \tfrac{1}{2}mv^2.} \qquad \textbf{(6–3)}$$

Kinetic energy (defined)

(We call this "translational" kinetic energy to distinguish it from rotational kinetic energy, which we will discuss in Chapter 8.) Equation 6–2, derived here for one-dimensional motion with a constant force, is valid in general for translational motion of an object in three dimensions and even if the force varies.

We can rewrite Eq. 6–2 as:

$$W_{net} = \text{KE}_2 - \text{KE}_1$$

or

$$\boxed{W_{net} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.} \qquad \textbf{(6–4)}$$

WORK-ENERGY PRINCIPLE

Equation 6–4 is a useful result known as the **work-energy principle**. It can be stated in words:

The net work done on an object is equal to the change in the object's kinetic energy.

WORK-ENERGY PRINCIPLE

Notice that we made use of Newton's second law, $F_{net} = ma$, where F_{net} is the *net* force—the sum of all forces acting on the object. Thus, the work-energy principle is valid only if W is the *net work* done on the object—that is, the work done by all forces acting on the object.

CAUTION
Work-energy valid only for net work

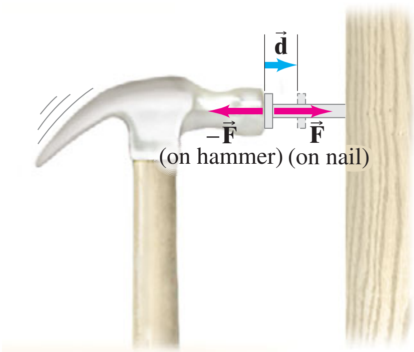


FIGURE 6–8 A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force $-F$ on the hammer (Newton's third law). The work done on the nail by the hammer is positive $(W_n = Fd > 0)$. The work done on the hammer by the nail is negative $(W_h = -Fd)$.

The work-energy principle is a very useful reformulation of Newton's laws. It tells us that if (positive) net work W is done on an object, the object's kinetic energy increases by an amount W. The principle also holds true for the reverse situation: if the net work W done on an object is negative, the object's kinetic energy decreases by an amount W. That is, a net force exerted on an object opposite to the object's direction of motion decreases its speed and its kinetic energy. An example is a moving hammer (Fig. 6–8) striking a nail. The net force on the hammer ($-\vec{\mathbf{F}}$ in Fig. 6–8, where $\vec{\mathbf{F}}$ is assumed constant for simplicity) acts toward the left, whereas the displacement $\vec{\mathbf{d}}$ of the hammer is toward the right. So the net work done on the hammer, $W_h = (F)(d)(\cos 180°) = -Fd$, is negative and the hammer's kinetic energy decreases (usually to zero).

Figure 6–8 also illustrates how energy can be considered the ability to do work. The hammer, as it slows down, does positive work on the nail: $W_n = (+F)(+d) = Fd$ and is positive. The decrease in kinetic energy of the hammer ($= Fd$ by Eq. 6–4) is equal to the work the hammer can do on another object, the nail in this case.

The translational kinetic energy $\left(= \frac{1}{2}mv^2\right)$ is directly proportional to the mass of the object, and it is also proportional to the *square* of the speed. Thus, if the mass is doubled, the kinetic energy is doubled. But if the speed is doubled, the object has four times as much kinetic energy and is therefore capable of doing four times as much work.

Because of the direct connection between work and kinetic energy, energy is measured in the same units as work: joules in SI units. [The energy unit is ergs in the cgs, and foot-pounds in the British system.] Like work, kinetic energy is a scalar quantity. The kinetic energy of a group of objects is the sum of the kinetic energies of the individual objects.

The work-energy principle can be applied to a particle, and also to an object that can be approximated as a particle, such as an object that is rigid or whose internal motions are insignificant. It is very useful in simple situations, as we will see in the Examples below.

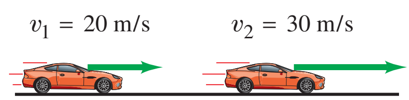


FIGURE 6–9 Example 6–4.

EXAMPLE 6–4 ESTIMATE **Work on a car, to increase its kinetic energy.** How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s (Fig. 6–9)?

APPROACH A car is a complex system. The engine turns the wheels and tires which push against the ground, and the ground pushes back (see Example 4–4). We aren't interested right now in those complications. Instead, we can get a useful result using the work-energy principle, but only if we model the car as a particle or simple rigid object.

SOLUTION The net work needed is equal to the increase in kinetic energy:

$$\begin{aligned} W &= \text{KE}_2 - \text{KE}_1 \\ &= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \\ &= \tfrac{1}{2}(1000\,\text{kg})(30\,\text{m/s})^2 - \tfrac{1}{2}(1000\,\text{kg})(20\,\text{m/s})^2 \\ &= 2.5 \times 10^5\,\text{J}. \end{aligned}$$

EXERCISE B (*a*) Make a guess: will the work needed to accelerate the car in Example 6–4 from rest to 20 m/s be more than, less than, or equal to the work already calculated to accelerate it from 20 m/s to 30 m/s? (*b*) Make the calculation.

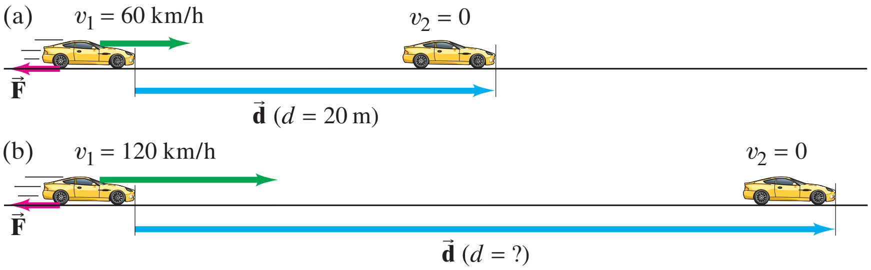


FIGURE 6–10 Example 6–5. A moving car comes to a stop. Initial velocity is (a) 60 km/h, (b) 120 km/h.

CONCEPTUAL EXAMPLE 6–5 **Work to stop a car.** A car traveling 60 km/h can brake to a stop in a distance d of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? Assume the maximum braking force is approximately independent of speed.

RESPONSE Again we model the car as if it were a particle. Because the net stopping force F is approximately constant, the work needed to stop the car, Fd, is proportional to the distance traveled. We apply the work-energy principle, noting that $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions and that the final speed of the car is zero:

$$W_{\text{net}} = Fd\cos 180° = -Fd.$$

Then

$$-Fd = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$
$$= 0 - \tfrac{1}{2}mv_1^2.$$

Thus, since the force and mass are constant, we see that the stopping distance, d, increases with the square of the speed:

$$d \propto v^2.$$

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

PHYSICS APPLIED
Car's stopping distance $\propto$ to initial speed squared

EXERCISE C Can kinetic energy ever be negative?

EXERCISE D (*a*) If the kinetic energy of a baseball is doubled, by what factor has its speed increased? (*b*) If its speed is doubled, by what factor does its kinetic energy increase?

6–4 Potential Energy

We have just discussed how an object is said to have energy by virtue of its motion, which we call kinetic energy. But it is also possible to have **potential energy**, which is the energy associated with forces that depend on the position or configuration of an object (or objects) relative to the surroundings. Various types of potential energy (PE) can be defined, and each type is associated with a particular force.

The spring of a wind-up toy is an example of an object with potential energy. The spring acquired its potential energy because work was done *on* it by the person winding the toy. As the spring unwinds, it exerts a force and does work to make the toy move.

Gravitational Potential Energy

Perhaps the most common example of potential energy is *gravitational potential energy*. A heavy brick held high above the ground has potential energy because of its position relative to the Earth. The raised brick has the ability to do work, for if it is released, it will fall to the ground due to the gravitational force, and can do work on, say, a stake, driving it into the ground.

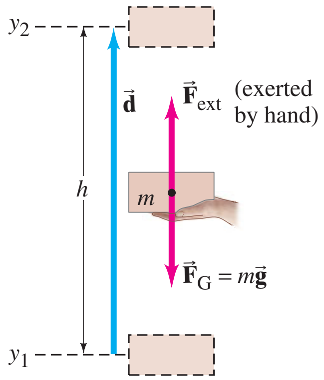


FIGURE 6–11 A person exerts an upward force $F_{ext} = mg$ to lift a brick from y_1 to y_2.

Let us seek the form for the gravitational potential energy of an object near the surface of the Earth. For an object of mass m to be lifted vertically, an upward force at least equal to its weight, mg, must be exerted on it, say by a person's hand. To lift the object without acceleration, the person exerts an "external force" $F_{ext} = mg$. If it is raised a vertical height h, from position y_1 to y_2 in Fig. 6–11 (upward direction chosen positive), a person does work equal to the product of the "external" force she exerts, $F_{ext} = mg$ upward, multiplied by the vertical displacement h. That is,

$$\begin{aligned} W_{ext} &= F_{ext}\, d \cos 0° = mgh \\ &= mg(y_2 - y_1). \end{aligned} \tag{6–5a}$$

Gravity is also acting on the object as it moves from y_1 to y_2, and does work on the object equal to

$$W_G = F_G\, d \cos\theta = mgh \cos 180°,$$

where $\theta = 180°$ because $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{d}}$ point in opposite directions. So

$$\begin{aligned} W_G &= -mgh \\ &= -mg(y_2 - y_1). \end{aligned} \tag{6–5b}$$

Next, if we allow the object to start from rest at y_2 and fall freely under the action of gravity, it acquires a velocity given by $v^2 = 2gh$ (Eq. 2–11c) after falling a height h. It then has kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$, and if it strikes a stake, it can do work on the stake equal to mgh (Section 6–3).

Thus, to raise an object of mass m to a height h *requires* an amount of work equal to mgh (Eq. 6–5a). And once at height h, the object has the *ability* to do an amount of work equal to mgh. We can say that the work done in lifting the object has been stored as gravitational potential energy.

We therefore define the **gravitational potential energy** of an object, due to Earth's gravity, as the product of the object's weight mg and its height y above some reference level (such as the ground):

$$\boxed{\mathrm{PE}_G = mgy.} \tag{6–6}$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. 6–5a with Eq. 6–6:

$$\begin{aligned} W_{ext} &= mg(y_2 - y_1) \\ W_{ext} &= \mathrm{PE}_2 - \mathrm{PE}_1 = \Delta\mathrm{PE}_G. \end{aligned} \tag{6–7a}$$

CAUTION
$\Delta\mathrm{PE}_G$ = work done by net external force

That is, the change in potential energy when an object moves from a height y_1 to a height y_2 is equal to the work done by a net external force to move the object from position 1 to position 2 without acceleration.

Equivalently, we can define the change in gravitational potential energy, $\Delta\mathrm{PE}_G$, in terms of the work done by gravity itself. Starting from Eq. 6–5b, we obtain

$$\begin{aligned} W_G &= -mg(y_2 - y_1) \\ W_G &= -(\mathrm{PE}_2 - \mathrm{PE}_1) = -\Delta\mathrm{PE}_G \end{aligned}$$

or

CAUTION
$\Delta\mathrm{PE}_G = -W_G$

$$\Delta\mathrm{PE}_G = -W_G. \tag{6–7b}$$

That is, the change in gravitational potential energy as the object moves from position 1 to position 2 is equal to the negative of the work done by gravity itself.

Gravitational potential energy depends on the *vertical height* of the object *above some reference level* (Eq. 6–6). In some situations, you may wonder from what point to measure the height y. The gravitational potential energy of a book held high above a table, for example, depends on whether we measure y from the top of the table, from the floor, or from some other reference point. What is physically important in any situation is the *change* in potential energy, $\Delta\mathrm{PE}$, because that is what is related to the work done, Eqs. 6–7; and it is $\Delta\mathrm{PE}$ that can be measured. We can thus choose to measure y from any reference level that is convenient, but we must choose the reference level at the start and be consistent throughout. The *change* in potential energy between any two points does not depend on this choice.

CAUTION
Change in PE is what is physically meaningful

An important result we discussed earlier (see Example 6–2 and Fig. 6–4) concerns the gravity force, which does work only in the vertical direction: the work done by gravity depends only on the vertical height h, and not on the path taken, whether it be purely vertical motion or, say, motion along an incline. Thus, from Eqs. 6–7 we see that changes in gravitational potential energy depend only on the change in vertical height and not on the path taken.

Potential energy belongs to a system, and not to a single object alone. Potential energy is associated with a force, and a force on one object is always exerted by some other object. Thus potential energy is a property of the system as a whole. For an object raised to a height y above the Earth's surface, the change in gravitational potential energy is mgy. The system here is the object plus the Earth, and properties of both are involved: object (m) and Earth (g).

CAUTION

Potential energy belongs to a system, not to a single object

EXAMPLE 6–6 **Potential energy changes for a roller coaster.** A 1000-kg roller-coaster car moves from point 1, Fig. 6–12, to point 2 and then to point 3. (*a*) What is the gravitational potential energy at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (*b*) What is the change in potential energy when the car goes from point 2 to point 3? (*c*) Repeat parts (*a*) and (*b*), but take the reference point $(y = 0)$ to be at point 3.

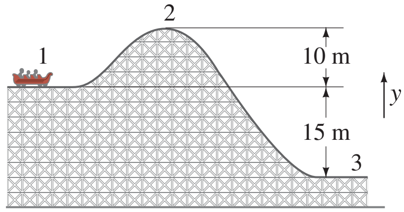


FIGURE 6–12 Example 6–6.

APPROACH We are interested in the potential energy of the car–Earth system. We take upward as the positive y direction, and use the definition of gravitational potential energy to calculate the potential energy.

SOLUTION (*a*) We measure heights from point 1 $(y_1 = 0)$, which means initially that the gravitational potential energy is zero. At point 2, where $y_2 = 10\text{ m}$,

$$\text{PE}_2 = mgy_2 = (1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 9.8 \times 10^4\text{ J}.$$

At point 3, $y_3 = -15\text{ m}$, since point 3 is below point 1. Therefore,

$$\text{PE}_3 = mgy_3 = (1000\text{ kg})(9.8\text{ m/s}^2)(-15\text{ m}) = -1.5 \times 10^5\text{ J}.$$

(*b*) In going from point 2 to point 3, the potential energy change $(\text{PE}_{\text{final}} - \text{PE}_{\text{initial}})$ is

$$\text{PE}_3 - \text{PE}_2 = (-1.5 \times 10^5\text{ J}) - (9.8 \times 10^4\text{ J}) = -2.5 \times 10^5\text{ J}.$$

The gravitational potential energy decreases by 2.5×10^5 J.

(*c*) Now we set $y_3 = 0$. Then $y_1 = +15\text{ m}$ at point 1, so the potential energy initially is

$$\text{PE}_1 = (1000\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 1.5 \times 10^5\text{ J}.$$

At point 2, $y_2 = 25\text{ m}$, so the potential energy is

$$\text{PE}_2 = 2.5 \times 10^5\text{ J}.$$

At point 3, $y_3 = 0$, so the potential energy is zero. The change in potential energy going from point 2 to point 3 is

$$\text{PE}_3 - \text{PE}_2 = 0 - 2.5 \times 10^5\text{ J} = -2.5 \times 10^5\text{ J},$$

which is the same as in part (*b*).

NOTE Work done by gravity depends only on the vertical height, so changes in gravitational potential energy do not depend on the path taken.

Potential Energy Defined in General

There are other kinds of potential energy besides gravitational. Each form of potential energy is associated with a particular force, and can be defined analogously to gravitational potential energy. In general, the *change in potential energy associated with a particular force is equal to the negative of the work done by that force when the object is moved from one point to a second point* (as in Eq. 6–7b for gravity). Alternatively, we can define the *change in potential energy as the work required of an external force to move the object without acceleration between the two points*, as in Eq. 6–7a.

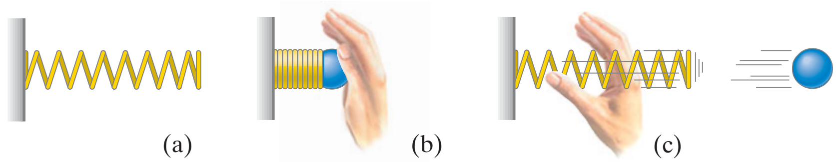


FIGURE 6–13 A spring (a) can store energy (elastic PE) when compressed as in (b) and can do work when released (c).

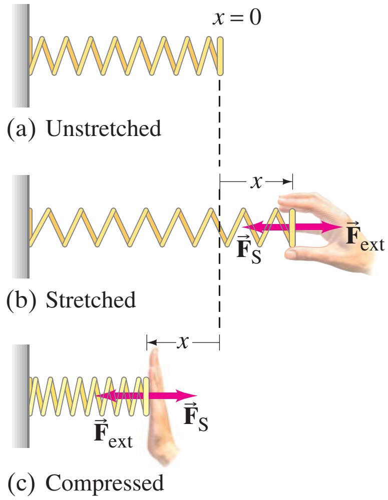
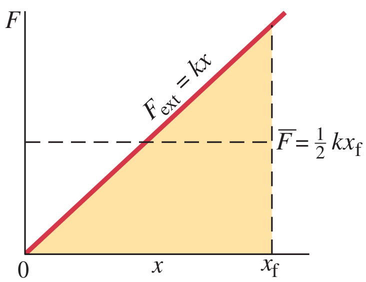


FIGURE 6–14 (a) Spring in natural (unstretched) position. (b) Spring is stretched by a person exerting a force $\vec{\mathbf{F}}_{\text{ext}}$ to the right (positive direction). The spring pulls back with a force $\vec{\mathbf{F}}_{\text{S}}$, where $F_{\text{S}} = -kx$. (c) Person compresses the spring $(x < 0)$ by exerting an external force $\vec{\mathbf{F}}_{\text{ext}}$ to the left; the spring pushes back with a force $F_{\text{S}} = -kx$, where $F_{\text{S}} > 0$ because $x < 0$.

FIGURE 6–15 As a spring is stretched (or compressed), the magnitude of the force needed increases linearly as x increases: graph of $F = kx$ vs. x from $x = 0$ to $x = x_{\text{f}}$.



Potential Energy of Elastic Spring

We now consider potential energy associated with elastic materials, which includes a great variety of practical applications. Consider the simple coil spring shown in Fig. 6–13. The spring has potential energy when compressed (or stretched), because when it is released, it can do work on a ball as shown. To hold a spring either stretched or compressed an amount x from its natural (unstretched) length requires the hand to exert an external force on the spring of magnitude F_{ext} which is directly proportional to x. That is,

$$F_{\text{ext}} = kx,$$

where k is a constant, called the *spring stiffness constant* (or simply **spring constant**), and is a measure of the stiffness of the particular spring. The stretched or compressed spring itself exerts a force F_{S} in the opposite direction on the hand, as shown in Fig. 6–14:

$$F_{\text{S}} = -kx. \qquad \text{[spring force]} \quad \textbf{(6–8)}$$

This force is sometimes called a "restoring force" because the spring exerts its force in the direction opposite the displacement (hence the minus sign), acting to return it to its natural length. Equation 6–8 is known as the **spring equation** and also as **Hooke's law**, and is accurate for springs as long as x is not too great.

To calculate the potential energy of a stretched spring, let us calculate the work required to stretch it (Fig. 6–14b). We might hope to use Eq. 6–1 for the work done on it, $W = Fx$, where x is the amount it is stretched from its natural length. But this would be incorrect since the force F_{ext} $(= kx)$ is not constant but varies over the distance x, becoming greater the more the spring is stretched, as shown graphically in Fig. 6–15. So let us use the average force, $\bar{F}$. Since F_{ext} varies linearly, from zero at the unstretched position to kx when stretched to x, the average force is $\bar{F} = \frac{1}{2}[0 + kx] = \frac{1}{2}kx$, where x here is the final amount stretched (shown as x_{f} in Fig. 6–15 for clarity). The work done is then

$$W_{\text{ext}} = \bar{F}x = \left(\tfrac{1}{2}kx\right)(x) = \tfrac{1}{2}kx^2.$$

Hence the **elastic potential energy**, PE_{el}, is proportional to the square of the amount stretched:

$$\text{PE}_{\text{el}} = \tfrac{1}{2}kx^2. \qquad \text{[elastic spring]} \quad \textbf{(6–9)}$$

If a spring is *compressed* a distance x from its natural ("equilibrium") length, the average force again has magnitude $\bar{F} = \frac{1}{2}kx$, and again the potential energy is given by Eq. 6–9. Thus x can be either the amount compressed or amount stretched from the spring's natural length.† Note that for a spring, we choose the reference point for zero PE at the spring's natural position.

Potential Energy as Stored Energy

In the above examples of potential energy—from a brick held at a height y, to a stretched or compressed spring—an object has the capacity or *potential* to do work even though it is not yet actually doing it. These examples show that energy can be *stored*, for later use, in the form of potential energy (as in Fig. 6–13, for a spring).

Note that there is a single universal formula for the translational kinetic energy of an object, $\frac{1}{2}mv^2$, but there is no single formula for potential energy. Instead, the mathematical form of the potential energy depends on the force involved.

†We can also obtain Eq. 6–9 using Section 6–2. The work done, and hence ΔPE, equals the area under the F vs. x graph of Fig. 6–15. This area is a triangle (colored in Fig. 6–15) of altitude kx and base x, and hence of area (for a triangle) equal to $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$.

6–5 Conservative and Nonconservative Forces

The work done against gravity in moving an object from one point to another does not depend on the path taken. For example, it takes the same work ($= mgh$) to lift an object of mass m vertically a height h as to carry it up an incline of the same vertical height, as in Fig. 6–4 (see Example 6–2). Forces such as gravity, for which the work done does not depend on the path taken but only on the initial and final positions, are called **conservative forces**. The elastic force of a spring (or other elastic material), in which $F = -kx$, is also a conservative force. An object that starts at a given point and returns to that same point under the action of a conservative force has no net work done on it because the potential energy is the same at the start and the finish of such a round trip.

Many forces, such as friction and a push or pull exerted by a person, are **nonconservative forces** since any work they do depends on the path. For example, if you push a crate across a floor from one point to another, the work you do depends on whether the path taken is straight or is curved. As shown in Fig. 6–16, if a crate is pushed slowly from point 1 to point 2 along the longer semicircular path, you do more work against friction than if you push it along the straight path.

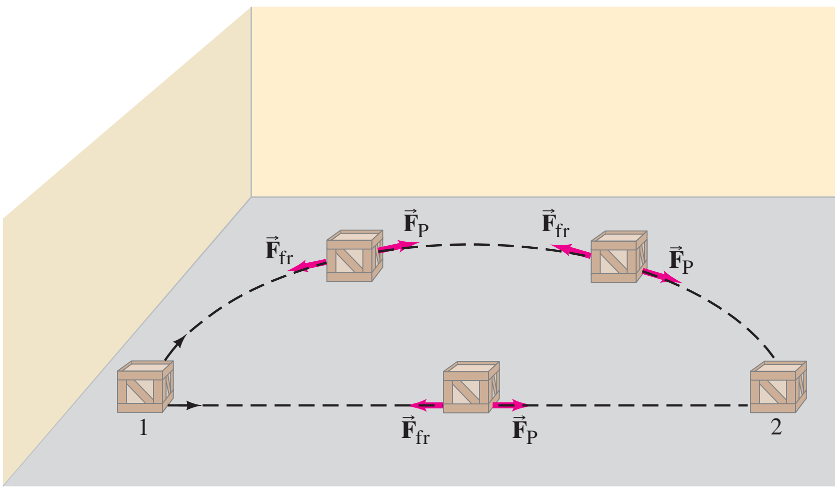


FIGURE 6–16 A crate is pushed slowly at constant speed across a rough floor from position 1 to position 2 via two paths, one straight and one curved. The pushing force $\vec{\mathbf{F}}_{\mathrm{P}}$ is in the direction of motion at each point. (The friction force opposes the motion.) Hence for a constant magnitude pushing force, the work it does is $W = F_{\mathrm{P}}d$, so if the distance traveled d is greater (as for the curved path), then W is greater. The work done does not depend only on points 1 and 2; it also depends on the path taken.

You do more work on the curved path because the distance is greater and, unlike the gravitational force, the pushing force $\vec{\mathbf{F}}_{\mathrm{P}}$ is in the direction of motion at each point. Thus the work done by the person in Fig. 6–16 does not depend *only* on points 1 and 2; it depends also on the path taken. The force of kinetic friction, also shown in Fig. 6–16, always opposes the motion; it too is a nonconservative force, and we discuss how to treat it later in this Chapter (Section 6–9). Table 6–1 lists a few conservative and nonconservative forces.

Because potential energy is energy associated with the position or configuration of objects, potential energy can only make sense if it can be stated uniquely for a given point. This cannot be done with nonconservative forces because the work done depends on the path taken (as in Fig. 6–16). Hence, *potential energy can be defined only for a conservative force*. Thus, although potential energy is always associated with a force, not all forces have a potential energy. For example, there is no potential energy for friction.

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

EXERCISE E An object acted on by a constant force F moves from point 1 to point 2 and back again. The work done by the force F in this round trip is 60 J. Can you determine from this information if F is a conservative or nonconservative force?

Work-Energy Extended

We can extend the **work-energy principle** (discussed in Section 6–3) to include potential energy. Suppose several forces act on an object which can undergo translational motion. And suppose only some of these forces are conservative. We write the total (net) work W_{net} as a sum of the work done by conservative forces, W_{C}, and the work done by nonconservative forces, W_{NC}:

$$W_{\text{net}} = W_{\text{C}} + W_{\text{NC}}.$$

Then, from the work-energy principle, Eq. 6–4, we have

$$\begin{aligned} W_{\text{net}} &= \Delta\text{KE} \\ W_{\text{C}} + W_{\text{NC}} &= \Delta\text{KE} \end{aligned}$$

where $\Delta\text{KE} = \text{KE}_2 - \text{KE}_1$. Then

$$W_{\text{NC}} = \Delta\text{KE} - W_{\text{C}}.$$

Work done by a conservative force can be written in terms of potential energy, as we saw in Eq. 6–7b for gravitational potential energy:

$$W_{\text{C}} = -\Delta\text{PE}.$$

We combine these last two equations:

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}. \qquad \textbf{(6–10)}$$

Thus, *the work W_{NC} done by the nonconservative forces acting on an object is equal to the total change in kinetic and potential energies*.

It must be emphasized that *all* the forces acting on an object must be included in Eq. 6–10, either in the potential energy term on the right (if it is a conservative force), or in the work term on the left (but not in both!).

6–6 Mechanical Energy and Its Conservation

If we can ignore friction and other nonconservative forces, or if only conservative forces do work on a system, we arrive at a particularly simple and beautiful relation involving energy.

When no nonconservative forces do work, then $W_{\text{NC}} = 0$ in the general form of the work-energy principle (Eq. 6–10). Then we have

$$\Delta\text{KE} + \Delta\text{PE} = 0 \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11a)}$$

or

$$(\text{KE}_2 - \text{KE}_1) + (\text{PE}_2 - \text{PE}_1) = 0. \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11b)}$$

We now define a quantity E, called the **total mechanical energy** of our system, as the sum of the kinetic and potential energies at any moment:

$$E = \text{KE} + \text{PE}.$$

Now we can rewrite Eq. 6–11b as

CONSERVATION OF MECHANICAL ENERGY

$$\boxed{\text{KE}_2 + \text{PE}_2 = \text{KE}_1 + \text{PE}_1} \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12a)}$$

or

$$\boxed{E_2 = E_1 = \text{constant.}} \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12b)}$$

Equations 6–12 express a useful and profound principle regarding the total mechanical energy of a system—namely, that it is a **conserved quantity**. The total mechanical energy E remains constant as long as no nonconservative forces do work: $\text{KE} + \text{PE}$ at some initial time 1 is equal to the $\text{KE} + \text{PE}$ at any later time 2.

To say it another way, consider Eq. 6–11a which tells us $\Delta \text{PE} = -\Delta \text{KE}$; that is, if the kinetic energy KE of a system increases, then the potential energy PE must decrease by an equivalent amount to compensate. Thus, the total, $\text{KE} + \text{PE}$, remains constant:

If only conservative forces do work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

CONSERVATION OF MECHANICAL ENERGY

This is the **principle of conservation of mechanical energy** for conservative forces.

In the next Section we shall see the great usefulness of the conservation of mechanical energy principle in a variety of situations, and how it is often easier to use than the kinematic equations or Newton's laws. After that we will discuss how other forms of energy can be included in the general conservation of energy law, such as energy associated with friction.

6–7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy (neglecting air resistance) is a rock allowed to fall due to Earth's gravity from a height h above the ground, as shown in Fig. 6–17. If the rock starts from rest, all of the initial energy is potential energy. As the rock falls, the potential energy mgy decreases (because the rock's height above the ground y decreases), but the rock's kinetic energy increases to compensate, so that the sum of the two remains constant. At any point along the path, the total mechanical energy is given by

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

where v is its speed at that point. If we let the subscript 1 represent the rock at one point along its path (for example, the initial point), and the subscript 2 represent it at some other point, then we can write

$$\text{total mechanical energy at point 1} = \text{total mechanical energy at point 2}$$

or (see also Eq. 6–12a)

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[gravity only]} \quad \textbf{(6–13)}$$

Just before the rock hits the ground, where we chose $y = 0$, all of the initial potential energy will have been transformed into kinetic energy.

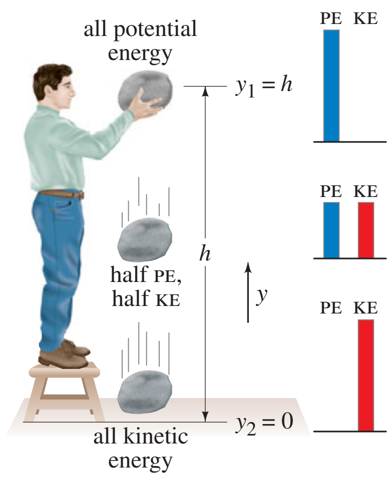


FIGURE 6–17 The rock's potential energy changes to kinetic energy as it falls. Note bar graphs representing potential energy PE and kinetic energy KE for the three different positions.

EXAMPLE 6–7 **Falling rock.** If the initial height of the rock in Fig. 6–17 is $y_1 = h = 3.0\text{ m}$, calculate the rock's velocity when it has fallen to 1.0 m above the ground.

APPROACH We apply the principle of conservation of mechanical energy, Eq. 6–13, with only gravity acting on the rock. We choose the ground as our reference level $(y = 0)$.

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\text{ m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\text{ m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2.$$

The m's cancel out and $v_1 = 0$, so

$$gy_1 = \tfrac{1}{2}v_2^2 + gy_2.$$

Solving for v_2 we find

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\text{ m/s}^2)[(3.0\text{ m}) - (1.0\text{ m})]} = 6.3\text{ m/s}.$$

The rock's velocity 1.0 m above the ground is 6.3 m/s downward.

NOTE The velocity of the rock is independent of the rock's mass.

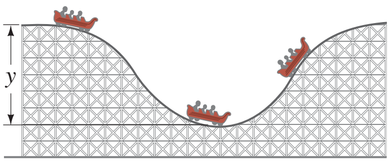


FIGURE 6–18 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

Equation 6–13 can be applied to any object moving without friction under the action of gravity. For example, Fig. 6–18 shows a roller-coaster car starting from rest at the top of a hill and coasting without friction to the bottom and up the hill on the other side. True, there is another force besides gravity acting on the car, the normal force exerted by the tracks. But the normal force acts perpendicular to the direction of motion at each point and so does zero work. We ignore rotational motion of the car's wheels and treat the car as a particle undergoing simple translation. Initially, the car has only potential energy. As it coasts down the hill, it loses potential energy and gains in kinetic energy, but the sum of the two remains constant. At the bottom of the hill it has its maximum kinetic energy, and as it climbs up the other side the kinetic energy changes back to potential energy. When the car comes to rest again at the same height from which it started, all of its energy will be potential energy. Given that the gravitational potential energy is proportional to the vertical height, energy conservation tells us that (in the absence of friction) the car comes to rest at a height equal to its original height. If the two hills are the same height, the car will just barely reach the top of the second hill when it stops. If the second hill is lower than the first, not all of the car's kinetic energy will be transformed to potential energy and the car can continue over the top and down the other side. If the second hill is higher, the car will reach a maximum height on it equal to its original height on the first hill. This is true (in the absence of friction) no matter how steep the hill is, since potential energy depends only on the vertical height (Eq. 6–6).

EXAMPLE 6–8 Roller-coaster car speed using energy conservation. Assuming the height of the hill in Fig. 6–18 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

APPROACH We use conservation of mechanical energy. We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\,\text{m})$. In part (*a*), point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. In part (*b*) we let y_2 be the unknown.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$, which gives

$$mgy_1 = \tfrac{1}{2}mv_2^2$$

or

$$\begin{aligned} v_2 &= \sqrt{2gy_1} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(40\,\text{m})} = 28\,\text{m/s}. \end{aligned}$$

(*b*) Now y_2 will be an unknown. We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = \frac{1}{2}(28\,\text{m/s}) = 14\,\text{m/s}$ and $v_1 = 0$. Solving for the unknown y_2 gives

$$y_2 = y_1 - \frac{v_2^2}{2g} = 40\,\text{m} - \frac{(14\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 30\,\text{m}.$$

That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

The mathematics of the roller-coaster Example 6–8 is almost the same as in Example 6–7. But there is an important difference between them. In Example 6–7 the motion is all vertical and could have been solved using force, acceleration, and the kinematic equations (Eqs. 2–11). For the roller coaster, where the motion is not vertical, we could *not* have used Eqs. 2–11 because a is not constant on the curved track of Example 6–8. But energy conservation readily gives us the answer.

CONCEPTUAL EXAMPLE 6–9 **Speeds on two water slides.** Two water slides at a pool are shaped differently, but start at the same height h (Fig. 6–19). Two riders start from rest at the same time on different slides. (a) Which rider, Paul or Corinne, is traveling faster at the bottom? (b) Which rider makes it to the bottom first? Ignore friction and assume both slides have the same path length.

RESPONSE (a) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\frac{1}{2}mv^2 = mgh$. The mass cancels and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed. (b) Note that Corinne is consistently at a lower elevation than Paul at any instant, until the end. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, and because the distance is the same, Corinne gets to the bottom first.

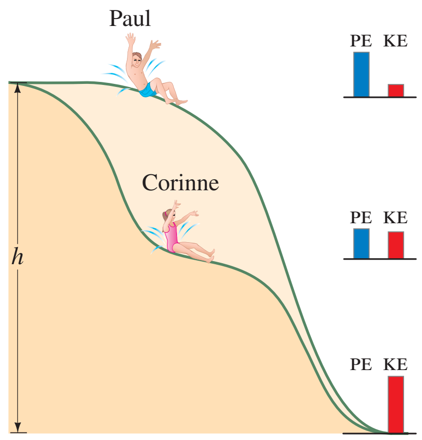


FIGURE 6–19 Example 6–9.

FIGURE 6–20 Transformation of energy during a pole vault: KE $\rightarrow$ PE$_{\text{el}}$ $\rightarrow$ PE$_{\text{G}}$.

There are many interesting examples of the conservation of energy in sports, such as the pole vault illustrated in Fig. 6–20. We often have to make approximations, but the sequence of events in broad outline for the pole vault is as follows. The initial kinetic energy of the running athlete is transformed into elastic potential energy of the bending pole and, as the athlete leaves the ground, into gravitational potential energy. When the vaulter reaches the top and the pole has straightened out again, the energy has all been transformed into gravitational potential energy (if we ignore the vaulter's low horizontal speed over the bar). The pole does not supply any energy, but it acts as a device to *store* energy and thus aid in the transformation of kinetic energy into gravitational potential energy, which is the net result. The energy required to pass over the bar depends on how high the center of mass (CM) of the vaulter must be raised. By bending their bodies, pole vaulters keep their CM so low that it can actually pass slightly beneath the bar (Fig. 6–21), thus enabling them to cross over a higher bar than would otherwise be possible. (Center of mass is covered in Chapter 7.)

As another example of the conservation of mechanical energy, let us consider an object of mass m connected to a compressed horizontal spring (Fig. 6–13b) whose own mass can be neglected and whose spring stiffness constant is k. When the spring is released, the mass m has speed v at any moment. The potential energy of the system (object plus spring) is $\frac{1}{2}kx^2$, where x is the displacement of the spring from its unstretched length (Eq. 6–9). If neither friction nor any other force is acting, conservation of mechanical energy tells us that

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2, \qquad \text{[elastic PE only]} \quad \textbf{(6–14)}$$

where the subscripts 1 and 2 refer to the velocity and displacement at two different moments.

FIGURE 6–21 By bending her body, a pole vaulter can keep her center of mass so low that it may even pass below the bar.

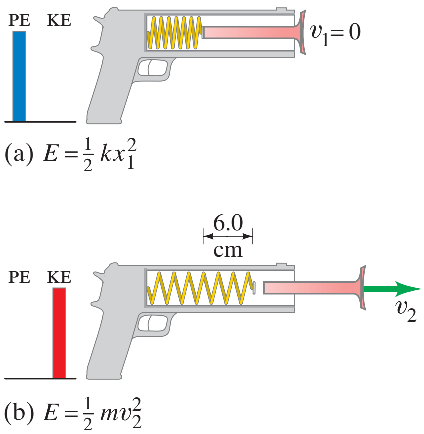


FIGURE 6–22 Example 6–10. (a) A dart is pushed against a spring, compressing it 6.0 cm. The dart is then released, and in (b) it leaves the spring at velocity v_2.

EXAMPLE 6–10 **Toy dart gun.** A dart of mass 0.100 kg is pressed against the spring of a toy dart gun as shown in Fig. 6–22a. The spring, with spring stiffness constant $k = 250\,\mathrm{N/m}$ and ignorable mass, is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length $(x = 0)$, what speed does the dart acquire?

APPROACH The dart is initially at rest (point 1), so $\mathrm{KE}_1 = 0$. We ignore friction and use conservation of mechanical energy; the only potential energy is elastic.

SOLUTION We use Eq. 6–14 with point 1 being at the maximum compression of the spring, so $v_1 = 0$ (dart not yet released) and $x_1 = -0.060\,\mathrm{m}$. Point 2 we choose to be the instant the dart flies off the end of the spring (Fig. 6–22b), so $x_2 = 0$ and we want to find v_2. Thus Eq. 6–14 can be written

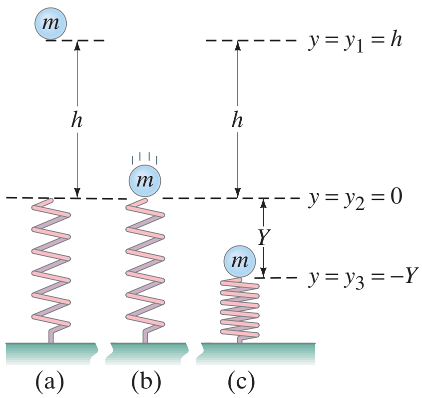
$$0 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0.$$

Then

$$v_2^2 = \frac{kx_1^2}{m} = \frac{(250\,\mathrm{N/m})(-0.060\,\mathrm{m})^2}{(0.100\,\mathrm{kg})} = 9.0\,\mathrm{m^2/s^2},$$

and $v_2 = \sqrt{v_2^2} = 3.0\,\mathrm{m/s}$.

FIGURE 6–23 Example 6–11. A falling ball compresses a spring.



EXAMPLE 6–11 **Two kinds of potential energy.** A ball of mass $m = 2.60\,\mathrm{kg}$, starting from rest, falls a vertical distance $h = 55.0\,\mathrm{cm}$ before striking a vertical coiled spring, which it compresses an amount $Y = 15.0\,\mathrm{cm}$ (Fig. 6–23). Determine the spring stiffness constant k of the spring. Assume the spring has negligible mass, and ignore air resistance. Measure all distances from the point where the ball first touches the uncompressed spring $(y = 0$ at this point).

APPROACH The forces acting on the ball are the gravitational pull of the Earth and the elastic force exerted by the spring. Both forces are conservative, so we can use conservation of mechanical energy, including both types of potential energy. We must be careful, however: gravity acts throughout the fall (Fig. 6–23), whereas the elastic force does not act until the ball touches the spring (Fig. 6–23b). We choose y positive upward, and $y = 0$ at the end of the spring in its natural (uncompressed) state.

SOLUTION We divide this solution into two parts. (An alternate solution follows.) *Part 1*: Let us first consider the energy changes as the ball falls from a height $y_1 = h = 0.550\,\mathrm{m}$, Fig. 6–23a, to $y_2 = 0$, just as it touches the spring, Fig. 6–23b. Our system is the ball acted on by gravity plus the spring (which up to this point doesn't do anything). Thus

$$\begin{aligned} \tfrac{1}{2}mv_1^2 + mgy_1 &= \tfrac{1}{2}mv_2^2 + mgy_2 \\ 0 + mgh &= \tfrac{1}{2}mv_2^2 + 0. \end{aligned}$$

We solve for $v_2 = \sqrt{2gh} = \sqrt{2(9.80\,\mathrm{m/s^2})(0.550\,\mathrm{m})} = 3.283\,\mathrm{m/s} \approx 3.28\,\mathrm{m/s}$. This is the speed of the ball just as it touches the top of the spring, Fig. 6–23b.

Part 2: As the ball compresses the spring, Figs. 6–23b to c, there are two conservative forces on the ball—gravity and the spring force. So our conservation of energy equation is

$$\begin{aligned} E_2\,(\text{ball touches spring}) &= E_3\,(\text{spring compressed}) \\ \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 &= \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2. \end{aligned}$$

Substituting $y_2 = 0$, $v_2 = 3.283\,\mathrm{m/s}$, $v_3 = 0$ (the ball comes to rest for an instant), and $y_3 = -Y = -0.150\,\mathrm{m}$, we have

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 - mgY + \tfrac{1}{2}k(-Y)^2.$$

We know m, v_2, and Y, so we can solve for k:

$$\begin{aligned} k &= \frac{2}{Y^2}\left[\tfrac{1}{2}mv_2^2 + mgY\right] = \frac{m}{Y^2}\left[v_2^2 + 2gY\right] \\ &= \frac{(2.60\,\mathrm{kg})}{(0.150\,\mathrm{m})^2}\left[(3.283\,\mathrm{m/s})^2 + 2(9.80\,\mathrm{m/s^2})(0.150\,\mathrm{m})\right] = 1590\,\mathrm{N/m}. \end{aligned}$$

PROBLEM SOLVING
Quicker Solution

Alternate Solution Instead of dividing the solution into two parts, we can do it all at once. After all, we get to choose what two points are used on the left and right of the energy equation. Let us write the energy equation for points 1 and 3 in Fig. 6–23. Point 1 is the initial point just before the ball starts to fall (Fig. 6–23a), so $v_1 = 0$, and $y_1 = h = 0.550\,\text{m}$. Point 3 is when the spring is fully compressed (Fig. 6–23c), so $v_3 = 0$, $y_3 = -Y = -0.150\,\text{m}$. The forces on the ball in this process are gravity and (at least part of the time) the spring. So conservation of energy tells us

$$\begin{aligned} \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(0)^2 &= \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2 \\ 0 + mgh + 0 &= 0 - mgY + \tfrac{1}{2}kY^2 \end{aligned}$$

where we have set $y = 0$ for the spring at point 1 because it is not acting and is not compressed or stretched. We solve for k:

$$k = \frac{2mg(h+Y)}{Y^2} = \frac{2(2.60\,\text{kg})(9.80\,\text{m/s}^2)(0.550\,\text{m} + 0.150\,\text{m})}{(0.150\,\text{m})^2} = 1590\,\text{N/m}$$

just as in our first method of solution.

6–8 Other Forms of Energy and Energy Transformations; The Law of Conservation of Energy

FIGURE 6–24 Gravitational potential energy of water at the top of Yosemite Falls gets transformed into kinetic energy as the water falls. (Some of the energy is transformed into heat by air resistance, and some into sound.)

FIGURE 6–25 Potential energy of a bent bow about to be transformed into kinetic energy of an arrow.

Besides the kinetic energy and potential energy of mechanical systems, other forms of energy can be defined as well. These include electric energy, nuclear energy, thermal energy, and the chemical energy stored in food and fuels. These other forms of energy are considered to be kinetic or potential energy at the atomic or molecular level. For example, according to atomic theory, thermal energy is the kinetic energy of rapidly moving molecules—when an object is heated, the molecules that make up the object move faster. On the other hand, the energy stored in food or in a fuel such as gasoline is regarded as potential energy stored by virtue of the relative positions of the atoms within a molecule due to electric forces between the atoms (chemical bonds). The energy in chemical bonds can be released through chemical reactions. This is analogous to a compressed spring which, when released, can do work. Electric, magnetic, and nuclear energies also can be considered examples of kinetic and potential (or stored) energies. We will deal with these other forms of energy in later Chapters.

Energy can be transformed from one form to another. For example, a rock held high in the air has potential energy; as it falls, it loses potential energy and gains in kinetic energy. Potential energy is being transformed into kinetic energy.

Often the transformation of energy involves a transfer of energy from one object to another. The potential energy stored in the spring of Fig. 6–13b is transformed into the kinetic energy of the ball, Fig. 6–13c. Water at the top of a waterfall (Fig. 6–24) or a dam has potential energy, which is transformed into kinetic energy as the water falls. At the base of a dam, the kinetic energy of the water can be transferred to turbine blades and further transformed into electric energy, as discussed later. The potential energy stored in a bent bow can be transformed into kinetic energy of the arrow (Fig. 6–25).

In each of these examples, the transfer of energy is accompanied by the performance of work. The spring of Fig. 6–13 does work on the ball. Water does work on turbine blades. A bow does work on an arrow. This observation gives us a further insight into the relation between work and energy: *work is done when energy is transferred from one object to another*.†

† If the objects are at different temperatures, heat can flow between them instead, or in addition. See Chapters 14 and 15.

One of the great results of physics is that whenever energy is transferred or transformed, it is found that no energy is gained or lost in the process.

This is the **law of conservation of energy**, one of the most important principles in physics; it can be stated as:

LAW OF CONSERVATION OF ENERGY

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

We have already discussed the conservation of energy for mechanical systems involving conservative forces, and we saw how it could be derived from Newton's laws and thus is equivalent to them. But in its full generality, the validity of the law of conservation of energy, encompassing all forms of energy including those associated with nonconservative forces like friction, rests on experimental observation. Even though Newton's laws are found to fail in the submicroscopic world of the atom, the law of conservation of energy has been found to hold in every experimental situation so far tested.

6–9 Energy Conservation with Dissipative Forces: Solving Problems

In our applications of energy conservation in Section 6–7, we neglected friction and other nonconservative forces. But in many situations they cannot be ignored. In a real situation, the roller-coaster car in Fig. 6–18, for example, will not in fact reach the same height on the second hill as it had on the first hill because of friction. In this, and in other natural processes, the mechanical energy (sum of the kinetic and potential energies) does not remain constant but decreases. Because frictional forces reduce the mechanical energy (but *not* the total energy), they are called **dissipative forces**. Historically, the presence of dissipative forces hindered the formulation of a comprehensive conservation of energy law until well into the nineteenth century. It was only then that heat, which is always produced when there is friction (try rubbing your hands together), was interpreted in terms of energy. Quantitative studies by nineteenth-century scientists (discussed in Chapters 14 and 15) demonstrated that if heat is considered as a transfer of energy (thermal energy), then the total energy is conserved in any process. For example, if the roller-coaster car in Fig. 6–18 is subject to frictional forces, then the initial total energy of the car will be equal to the kinetic plus potential energy of the car at any subsequent point along its path plus the amount of thermal energy produced in the process (equal to the work done by friction).

Let us recall the general form of the work-energy principle, Eq. 6–10:

$$W_{\mathrm{NC}} = \Delta\mathrm{KE} + \Delta\mathrm{PE},$$

where W_{NC} is the work done by nonconservative forces such as friction. Consider an object, such as a roller-coaster car, as a particle moving under gravity with nonconservative forces like friction acting on it. When the object moves from some point 1 to another point 2, then

$$W_{\mathrm{NC}} = \mathrm{KE}_2 - \mathrm{KE}_1 + \mathrm{PE}_2 - \mathrm{PE}_1 .$$

We can rewrite this as

$$\mathrm{KE}_1 + \mathrm{PE}_1 + W_{\mathrm{NC}} = \mathrm{KE}_2 + \mathrm{PE}_2 . \qquad \textbf{(6–15)}$$

For the case of friction, $W_{\mathrm{NC}} = -F_{\mathrm{fr}}\,d$, where d is the distance over which the friction (assumed constant) acts as the object moves from point 1 to point 2. ($\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions, hence the minus sign from $\cos 180° = -1$ in Eq. 6–1.)

With KE $= \frac{1}{2}mv^2$ and PE $= mgy$, Eq. 6–15 with $W_{NC} = -F_{fr}d$ becomes

$$\tfrac{1}{2}mv_1^2 + mgy_1 - F_{fr}d = \tfrac{1}{2}mv_2^2 + mgy_2. \quad \begin{bmatrix}\text{gravity and}\\ \text{friction acting}\end{bmatrix} \tag{6–16a}$$

That is, the initial mechanical energy is reduced by the amount $F_{fr}d$. We could also write this equation as

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}d$$

or

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2 + F_{fr}d, \quad \begin{bmatrix}\text{gravity and}\\ \text{friction}\\ \text{acting}\end{bmatrix} \tag{6–16b}$$

and state equally well that the initial mechanical energy of the car (point 1) equals the (reduced) final mechanical energy of the car plus the energy transformed by friction into thermal energy.

Equations 6–16 can be seen to be Eq. 6–13 modified to include nonconservative forces such as friction. As such, they are statements of conservation of energy. When other forms of energy are involved, such as chemical or electrical energy, the total amount of energy is always found to be conserved. Hence the law of conservation of energy is believed to be universally valid.

EXERCISE F Return to the Chapter-Opening Question, page 138, and answer it again now. Try to explain why you may have answered differently the first time.

Work-Energy versus Energy Conservation

The law of conservation of energy is more general and more powerful than the work-energy principle. Indeed, the work-energy principle should not be viewed as a statement of conservation of energy. It is nonetheless useful for mechanical problems; and whether you use it, or use the more powerful conservation of energy, can depend on your *choice of the system* under study. If you choose as your system a particle or rigid object on which external forces do work, then you can use the work-energy principle: the work done by the external forces on your object equals the change in its kinetic energy.

PROBLEM SOLVING
Choosing the system

On the other hand, if you choose a system on which no external forces do work, then you need to apply conservation of energy to that system directly.

Consider, for example, a spring connected to a block on a frictionless table (Fig. 6–26). If you choose the block as your system, then the work done on the block by the spring equals the change in kinetic energy of the block: the work-energy principle. (Energy conservation does not apply to this system—the block's energy changes.) If instead you choose the block plus the spring as your system, no external forces do work (since the spring is part of the chosen system). To this system you need to apply conservation of energy: if you compress the spring and then release it, the spring still exerts a force[†] on the block, but the subsequent motion can be discussed in terms of kinetic energy $(\frac{1}{2}mv^2)$ plus potential energy $(\frac{1}{2}kx^2)$, whose total remains constant.

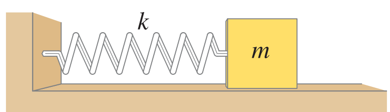


FIGURE 6–26 A spring connected to a block on a frictionless table. If you choose your system to be the block plus spring, then

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

is conserved.

You may also wonder sometimes whether to approach a problem using work and energy, or instead to use Newton's laws. As a rough guideline, if the force(s) involved are constant, either approach may succeed. If the forces are not constant, and/or the path is not simple, energy may be the better approach because it is a scalar.

PROBLEM SOLVING
Use energy, or Newton's laws?

Problem solving is not a process that can be done by simply following a set of rules. The Problem Solving Strategy on the next page, like all others, is thus *not* a prescription, but is a summary to help you get started solving problems involving energy.

[†]The force the spring exerts on the block, and the force the block exerts back on the spring, are not "external" forces—they are within the system.

PROBLEM SOLVING

Conservation of Energy

1. **Draw a picture** of the physical situation.
2. Determine **the system** for which you will apply energy conservation: the object or objects and the forces acting.
3. Ask yourself what quantity you are looking for, and **choose initial** (point 1) **and final** (point 2) **positions**.
4. If the object under investigation changes its height during the problem, then **choose a reference frame** with a convenient $y = 0$ level for gravitational potential energy; the lowest point in the situation is often a good choice.

   If springs are involved, choose the unstretched spring position to be x (or y) $= 0$.
5. **Is mechanical energy conserved?** If no friction or other nonconservative forces act, then conservation of mechanical energy holds:
$$\mathrm{KE}_1 + \mathrm{PE}_1 = \mathrm{KE}_2 + \mathrm{PE}_2. \qquad \textbf{(6–12a)}$$
6. **Apply conservation of energy.** If friction (or other nonconservative forces) are present, then an additional term (W_{NC}) will be needed:
$$W_{\mathrm{NC}} = \Delta\mathrm{KE} + \Delta\mathrm{PE}. \qquad \textbf{(6–10)}$$
   For a constant friction force acting over a distance d
$$\mathrm{KE}_1 + \mathrm{PE}_1 = \mathrm{KE}_2 + \mathrm{PE}_2 + F_{\mathrm{fr}}\,d. \qquad \textbf{(6–16b)}$$
   For other nonconservative forces use your intuition for the sign of W_{NC}: is the total mechanical energy increased or decreased in the process?
7. Use the equation(s) you develop to **solve** for the unknown quantity.

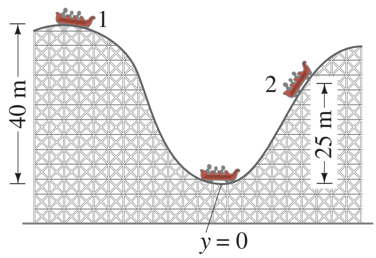


FIGURE 6–27 Example 6–12. Because of friction, a roller-coaster car does not reach the original height on the second hill. (Not to scale.)

EXAMPLE 6–12 **ESTIMATE** **Friction on the roller-coaster car.** The roller-coaster car in Example 6–8 reaches a vertical height of only 25 m on the second hill, where it slows to a momentary stop, Fig. 6–27. It traveled a total distance of 400 m. Determine the thermal energy produced and estimate the average friction force (assume it is roughly constant) on the car, whose mass is 1000 kg.

APPROACH We explicitly follow the Problem Solving Strategy above.

SOLUTION

1. **Draw a picture**. See Fig. 6–27.
2. **The system**. The system is the roller-coaster car and the Earth (which exerts the gravitational force). The forces acting on the car are gravity and friction. (The normal force also acts on the car, but does no work, so it does not affect the energy.) Gravity is accounted for as potential energy, and friction as a term $F_{\mathrm{fr}}\,d$.
3. **Choose initial and final positions**. We take point 1 to be the instant when the car started coasting (at the top of the first hill), and point 2 to be the instant it stopped at a height of 25 m up the second hill.
4. **Choose a reference frame**. We choose the lowest point in the motion to be $y = 0$ for the gravitational potential energy.
5. **Is mechanical energy conserved**? No. Friction is present.
6. **Apply conservation of energy**. There is friction acting on the car, so we use conservation of energy in the form of Eq. 6–16b, with $v_1 = 0$, $y_1 = 40$ m, $v_2 = 0$, $y_2 = 25$ m, and $d = 400$ m. Thus
$$0 + (1000\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(40\,\mathrm{m}) = 0 + (1000\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(25\,\mathrm{m}) + F_{\mathrm{fr}}\,d.$$
7. **Solve**. We solve the above equation for $F_{\mathrm{fr}}\,d$, the energy dissipated to thermal energy:
$$F_{\mathrm{fr}}\,d = mg\,\Delta h = (1000\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(40\,\mathrm{m} - 25\,\mathrm{m}) = 147{,}000\,\mathrm{J}.$$
   The friction force, which acts over a distance of 400 m, averages out to be
$$F_{\mathrm{fr}} = (1.47 \times 10^5\,\mathrm{J})/400\,\mathrm{m} = 370\,\mathrm{N}.$$

NOTE This result is only a rough average: the friction force at various points depends on the normal force, which varies with slope.

6–10 Power

Power is defined as the *rate at which work is done*. Average power equals the work done divided by the time to do it. Power can also be defined as the *rate at which energy is transformed*. Thus

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}. \qquad \textbf{(6–17)}$$

The power rating of an engine refers to how much chemical or electrical energy can be transformed into mechanical energy per unit time. In SI units, power is measured in joules per second, and this unit is given a special name, the **watt** (W): $1\text{ W} = 1\text{ J/s}$. We are most familiar with the watt for electrical devices, such as the rate at which an electric lightbulb or heater changes electric energy into light or thermal energy. But the watt is used for other types of energy transformations as well.

In the British system, the unit of power is the foot-pound per second (ft·lb/s). For practical purposes, a larger unit is often used, the **horsepower**. One horsepower† (hp) is defined as 550 ft·lb/s, which equals 746 W. An engine's power is usually specified in hp or in kW $\left(1\text{ kW} \approx 1\tfrac{1}{3}\text{hp}\right)$‡.

To see the distinction between energy and power, consider the following example. A person is limited in the work he or she can do, not only by the total energy required, but also by how fast this energy is transformed: that is, by power. For example, a person may be able to walk a long distance or climb many flights of stairs before having to stop because so much energy has been expended. On the other hand, a person who runs very quickly up stairs may feel exhausted after only a flight or two. He or she is limited in this case by power, the rate at which his or her body can transform chemical energy into mechanical energy.

CAUTION
Distinguish between power and energy

EXAMPLE 6–13 **Stair-climbing power.** A 60-kg jogger runs up a long flight of stairs in 4.0 s (Fig. 6–28). The vertical height of the stairs is 4.5 m. (*a*) Estimate the jogger's power output in watts and horsepower. (*b*) How much energy did this require?

APPROACH The work done by the jogger is against gravity, and equals $W = mgy$. To get her average power output, we divide W by the time it took.

SOLUTION (*a*) The average power output was

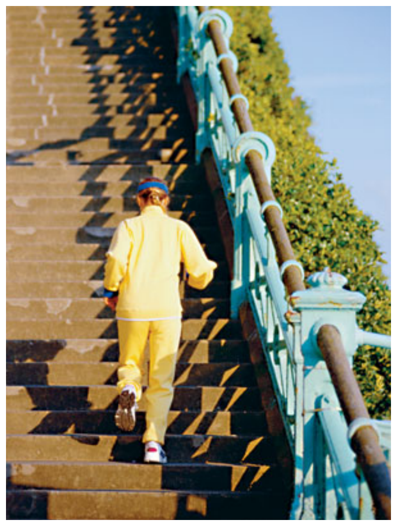
$$\overline{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(60\text{ kg})(9.8\text{ m/s}^2)(4.5\text{ m})}{4.0\text{ s}} = 660\text{ W}.$$

Since there are 746 W in 1 hp, the jogger is doing work at a rate of just under 1 hp. A human cannot do work at this rate for very long.

(*b*) The energy required is $E = \overline{P}t = (660\text{ J/s})(4.0\text{ s}) = 2600\text{ J}$. This result equals $W = mgy$.

NOTE The person had to transform more energy than this 2600 J. The total energy transformed by a person or an engine always includes some thermal energy (recall how hot you get running up stairs).

FIGURE 6–28 Example 6–13.



Automobiles do work to overcome the force of friction and air resistance, to climb hills, and to accelerate. A car is limited by the rate at which it can do work, which is why automobile engines are rated in horsepower or kilowatts.

PHYSICS APPLIED
Power needs of a car

†The unit was chosen by James Watt (1736–1819), who needed a way to specify the power of his newly developed steam engines. He found by experiment that a good horse can work all day at an average rate of about 360 ft·lb/s. So as not to be accused of exaggeration in the sale of his steam engines, he multiplied this by $1\tfrac{1}{2}$ when he defined the hp.

‡$1\text{ kW} = (1000\text{ W})/(746\text{ W/hp}) \approx 1\tfrac{1}{3}\text{hp}$.

A car needs power most when climbing hills and when accelerating. In the next Example, we will calculate how much power is needed in these situations for a car of reasonable size. Even when a car travels on a level road at constant speed, it needs some power just to do work to overcome the retarding forces of internal friction and air resistance. These forces depend on the conditions and speed of the car, but are typically in the range 400 N to 1000 N.

It is often convenient to write power in terms of the net force F applied to an object and its speed v. This is readily done because $\overline{P} = W/t$ and $W = Fd$, where d is the distance traveled. Then

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v}, \qquad \textbf{(6–18)}$$

where $\overline{v} = d/t$ is the average speed of the object.

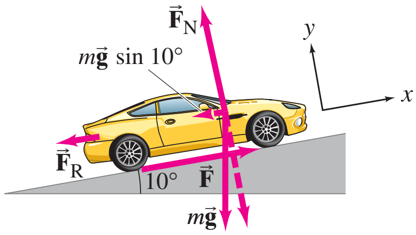


FIGURE 6–29 Example 6–14. Calculation of power needed for a car to climb a hill.

EXAMPLE 6–14 **Power needs of a car.** Calculate the power required of a 1400-kg car under the following circumstances: (*a*) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (*b*) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume the average retarding force on the car is $F_R = 700\text{ N}$ throughout. See Fig. 6–29.

APPROACH First we must be careful not to confuse $\vec{\mathbf{F}}_R$, which is due to air resistance and friction that retards the motion, with the force $\vec{\mathbf{F}}$ needed to accelerate the car, which is the frictional force exerted by the road on the tires—the reaction to the motor-driven tires pushing against the road. We must determine the magnitude of the force F before calculating the power.

SOLUTION (*a*) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, $mg \sin 10°$, Fig. 6–29. Thus

$$\begin{aligned} F &= 700\text{ N} + mg \sin 10° \\ &= 700\text{ N} + (1400\text{ kg})(9.80\text{ m/s}^2)(0.174) = 3100\text{ N}. \end{aligned}$$

Since $\overline{v} = 80\text{ km/h} = 22\text{ m/s}^{\dagger}$ and is parallel to $\vec{\mathbf{F}}$, then (Eq. 6–18) the power is

$$\overline{P} = F\overline{v} = (3100\text{ N})(22\text{ m/s}) = 6.8 \times 10^4\text{ W} = 68\text{ kW} = 91\text{ hp}.$$

(*b*) The car accelerates from 25.0 m/s to 30.6 m/s (90 to 110 km/h) on the flat. The car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration

$$\overline{a}_x = \frac{(30.6\text{ m/s} - 25.0\text{ m/s})}{6.0\text{ s}} = 0.93\text{ m/s}^2.$$

We apply Newton's second law with x being the horizontal direction of motion (no component of gravity):

$$ma_x = \Sigma F_x = F - F_R.$$

We solve for the force required, F:

$$\begin{aligned} F &= ma_x + F_R \\ &= (1400\text{ kg})(0.93\text{ m/s}^2) + 700\text{ N} = 1300\text{ N} + 700\text{ N} = 2000\text{ N}. \end{aligned}$$

Since $\overline{P} = F\overline{v}$, the required power increases with speed and the motor must be able to provide a maximum power output in this case of

$$\overline{P} = (2000\text{ N})(30.6\text{ m/s}) = 6.1 \times 10^4\text{ W} = 61\text{ kW} = 82\text{ hp}.$$

NOTE Even taking into account the fact that only 60 to 80% of the engine's power output reaches the wheels, it is clear from these calculations that an engine of 75 to 100 kW (100 to 130 hp) is adequate from a practical point of view.

†Recall $1\text{ km/h} = 1000\text{ m}/3600\text{ s} = 0.278\text{ m/s}$.

We mentioned in Example 6–14 that only part of the energy output of a car engine reaches the wheels. Not only is some energy wasted in getting from the engine to the wheels, in the engine itself most of the input energy (from the burning of gasoline or other fuel) does not do useful work. An important characteristic of all engines is their overall **efficiency** e, defined as the ratio of the useful power output of the engine, P_{out}, to the power input, P_{in} (provided by burning of gasoline, for example):

$$e = \frac{P_{\text{out}}}{P_{\text{in}}}.$$

The efficiency is always less than 1.0 because no engine can create energy, and no engine can even transform energy from one form to another without some energy going to friction, thermal energy, and other nonuseful forms of energy. For example, an automobile engine converts chemical energy released in the burning of gasoline into mechanical energy that moves the pistons and eventually the wheels. But nearly 85% of the input energy is "wasted" as thermal energy that goes into the cooling system or out the exhaust pipe, plus friction in the moving parts. Thus car engines are roughly only about 15% efficient. We will discuss efficiency in more detail in Chapter 15.

Summary

Work is done on an object by a force when the object moves through a distance d. If the direction of a constant force $\vec{\mathbf{F}}$ makes an angle θ with the direction of motion, the work done by this force is

$$W = Fd\cos\theta. \tag{6–1}$$

Energy can be defined as the ability to do work. In SI units, work and energy are measured in **joules** $(1\text{ J} = 1\text{ N}\cdot\text{m})$.

Kinetic energy (KE) is energy of motion. An object of mass m and speed v has **translational kinetic energy**

$$\text{KE} = \tfrac{1}{2}mv^2. \tag{6–3}$$

The **work-energy principle** states that the *net* work done on an object (by the *net* force) equals the change in kinetic energy of that object:

$$W_{\text{net}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \tag{6–4}$$

Potential energy (PE) is energy associated with forces that depend on the position or configuration of objects. Gravitational potential energy is

$$\text{PE}_{\text{G}} = mgy, \tag{6–6}$$

where y is the height of the object of mass m above an arbitrary reference point. Elastic potential energy is given by

$$\text{PE}_{\text{el}} = \tfrac{1}{2}kx^2 \tag{6–9}$$

for a stretched or compressed spring, where x is the displacement from the unstretched position and k is the **spring stiffness constant**. Other potential energies include chemical, electrical, and nuclear energy. The *change in potential energy* when an object changes position *is equal to the external work* needed to take the object from one position to the other.

Potential energy is associated only with **conservative forces**, for which the work done by the force in moving an object from one position to another depends only on the two positions and not on the path taken. **Nonconservative forces** like friction are different—work done by them does depend on the path taken and potential energy cannot be defined for them.

The **law of conservation of energy** states that energy can be transformed from one type to another, but the total energy remains constant. It is valid even when friction is present, because the heat generated can be considered a form of energy transfer. When only *conservative forces* act, the **total mechanical energy** is conserved:

$$\text{KE} + \text{PE} = \text{constant}. \tag{6–12}$$

When nonconservative forces such as friction act, then

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}, \tag{6–10, 6–15}$$

where W_{NC} is the work done by nonconservative forces.

Power is defined as the rate at which work is done, or the rate at which energy is transformed. The SI unit of power is the **watt** $(1\text{ W} = 1\text{ J/s})$.

Questions

1. In what ways is the word "work" as used in everyday language the same as it is defined in physics? In what ways is it different? Give examples of both.
2. Can a centripetal force ever do work on an object? Explain.
3. Why is it tiring to push hard against a solid wall even though you are doing no work?
4. Can the normal force on an object ever do work? Explain.
5. You have two springs that are identical except that spring 1 is stiffer than spring 2 $(k_1 > k_2)$. On which spring is more work done: (*a*) if they are stretched using the same force; (*b*) if they are stretched the same distance?
6. If the speed of a particle triples, by what factor does its kinetic energy increase?
7. List some everyday forces that are not conservative, and explain why they aren't.

8. A hand exerts a constant horizontal force on a block that is free to slide on a frictionless surface (Fig. 6–30). The block starts from rest at point A, and by the time it has traveled a distance d to point B it is traveling with speed v_B. When the block has traveled another distance d to point C, will its speed be greater than, less than, or equal to $2v_B$? Explain your reasoning.

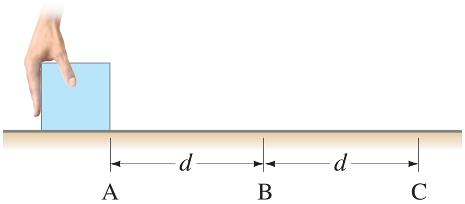


FIGURE 6–30 Question 8.

9. You lift a heavy book from a table to a high shelf. List the forces on the book during this process, and state whether each is conservative or nonconservative.

10. A hill has a height h. A child on a sled (total mass m) slides down starting from rest at the top. Does the speed at the bottom depend on the angle of the hill if (a) it is icy and there is no friction, and (b) there is friction (deep snow)? Explain your answers.

11. Analyze the motion of a simple swinging pendulum in terms of energy, (a) ignoring friction, and (b) taking friction into account. Explain why a grandfather clock has to be wound up.

12. In Fig. 6–31, water balloons are tossed from the roof of a building, all with the same speed but with different launch angles. Which one has the highest speed when it hits the ground? Ignore air resistance. Explain your answer.

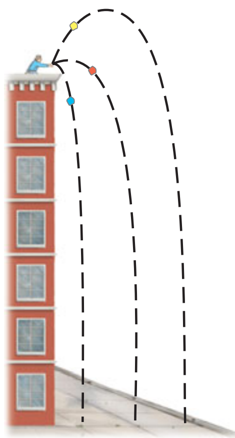


FIGURE 6–31 Question 12.

13. What happens to the gravitational potential energy when water at the top of a waterfall falls to the pool below?

14. Experienced hikers prefer to step over a fallen log in their path rather than stepping on top and stepping down on the other side. Explain.

15. The energy transformations in pole vaulting and archery are discussed in this Chapter. In a similar fashion, discuss the energy transformations related to: (a) hitting a golf ball; (b) serving a tennis ball; and (c) shooting a basket in basketball.

16. Describe precisely what is "wrong" physically in the famous Escher drawing shown in Fig. 6–32.

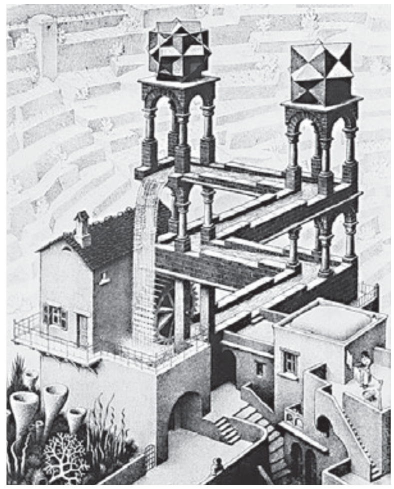


FIGURE 6–32 Question 16.

17. Two identical arrows, one with twice the speed of the other, are fired into a bale of hay. Assuming the hay exerts a constant "frictional" force on the arrows, the faster arrow will penetrate how much farther than the slower arrow? Explain.

18. A heavy ball is hung from the ceiling by a steel wire. The instructor pulls the ball back and stands against the wall with the ball against his chin. To avoid injury the instructor is supposed to release the ball without pushing it (Fig. 6–33). Why?

FIGURE 6–33 Question 18.

19. Describe the energy transformations when a child hops around on a pogo stick (there is a spring inside).

20. Describe the energy transformations that take place when a skier starts skiing down a hill, but after a time is brought to rest by striking a snowdrift.

21. Suppose you lift a suitcase from the floor to a table. The work you do on the suitcase depends on which of the following: (a) whether you lift it straight up or along a more complicated path, (b) the time the lifting takes, (c) the height of the table, and (d) the weight of the suitcase?

22. Repeat Question 21 for the *power* needed instead of the work.

23. Why is it easier to climb a mountain via a zigzag trail rather than to climb straight up?

MisConceptual Questions

1. You push very hard on a heavy desk, trying to move it. You do work on the desk:
(*a*) whether or not it moves, as long as you are exerting a force.
(*b*) only if it starts moving.
(*c*) only if it doesn't move.
(*d*) never—it does work on you.
(*e*) None of the above.

2. A satellite in circular orbit around the Earth moves at constant speed. This orbit is maintained by the force of gravity between the Earth and the satellite, yet no work is done on the satellite. How is this possible?
(*a*) No work is done if there is no contact between objects.
(*b*) No work is done because there is no gravity in space.
(*c*) No work is done if the direction of motion is perpendicular to the force.
(*d*) No work is done if objects move in a circle.

3. When the speed of your car is doubled, by what factor does its kinetic energy increase?
(*a*) $\sqrt{2}$. (*b*) 2. (*c*) 4. (*d*) 8.

4. A car traveling at a velocity v can stop in a minimum distance d. What would be the car's minimum stopping distance if it were traveling at a velocity of $2v$?
(*a*) d. (*b*) $\sqrt{2}\,d$. (*c*) $2d$. (*d*) $4d$. (*e*) $8d$.

5. A bowling ball is dropped from a height h onto the center of a trampoline, which launches the ball back up into the air. How high will the ball rise?
(*a*) Significantly less than h.
(*b*) More than h. The exact amount depends on the mass of the ball and the springiness of the trampoline.
(*c*) No more than h—probably a little less.
(*d*) Cannot tell without knowing the characteristics of the trampoline.

6. A ball is thrown straight up. At what point does the ball have the most energy? Ignore air resistance.
(*a*) At the highest point of its path.
(*b*) When it is first thrown.
(*c*) Just before it hits the ground.
(*d*) When the ball is halfway to the highest point of its path.
(*e*) Everywhere; the energy of the ball is the same at all of these points.

7. A car accelerates from rest to 30 km/h. Later, on a highway it accelerates from 30 km/h to 60 km/h. Which takes more energy, going from 0 to 30, or from 30 to 60?
(*a*) 0 to 30 km/h.
(*b*) 30 to 60 km/h.
(*c*) Both are the same.

8. Engines, including car engines, are rated in horsepower. What is horsepower?
(*a*) The force needed to start the engine.
(*b*) The force needed to keep the engine running at a steady rate.
(*c*) The energy the engine needs to obtain from gasoline or some other source.
(*d*) The rate at which the engine can do work.
(*e*) The amount of work the engine can perform.

9. Two balls are thrown off a building with the same speed, one straight up and one at a 45° angle. Which statement is true if air resistance can be ignored?
(*a*) Both hit the ground at the same time.
(*b*) Both hit the ground with the same speed.
(*c*) The one thrown at an angle hits the ground with a lower speed.
(*d*) The one thrown at an angle hits the ground with a higher speed.
(*e*) Both (*a*) and (*b*).

10. A skier starts from rest at the top of each of the hills shown in Fig. 6–34. On which hill will the skier have the highest speed at the bottom if we ignore friction:
(*a*), (*b*), (*c*), (*d*), or (*e*) *c* and *d* equally?

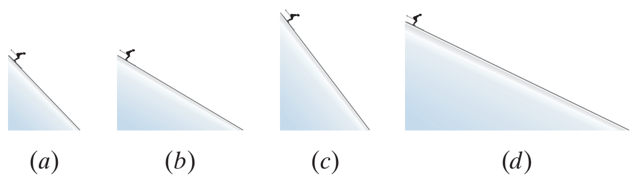


FIGURE 6–34
MisConceptual Questions 10 and 11.

11. Answer MisConceptual Question 10 assuming a small amount of friction.

12. A man pushes a block up an incline at a constant speed. As the block moves up the incline,
(*a*) its kinetic energy and potential energy both increase.
(*b*) its kinetic energy increases and its potential energy remains the same.
(*c*) its potential energy increases and its kinetic energy remains the same.
(*d*) its potential energy increases and its kinetic energy decreases by the same amount.

13. You push a heavy crate *down* a ramp at a constant velocity. Only four forces act on the crate. Which force does the greatest magnitude of work on the crate?
(*a*) The force of friction.
(*b*) The force of gravity.
(*c*) The normal force.
(*d*) The force of you pushing.
(*e*) The net force.

14. A ball is thrown straight up. Neglecting air resistance, which statement is *not* true regarding the energy of the ball?
(*a*) The potential energy decreases while the ball is going up.
(*b*) The kinetic energy decreases while the ball is going up.
(*c*) The sum of the kinetic energy and potential energy is constant.
(*d*) The potential energy decreases when the ball is coming down.
(*e*) The kinetic energy increases when the ball is coming down.

For assigned homework and other learning materials, go to the MasteringPhysics website.

Problems

6–1 Work, Constant Force

1. (I) A 75.0-kg firefighter climbs a flight of stairs 28.0 m high. How much work does he do?

2. (I) The head of a hammer with a mass of 1.2 kg is allowed to fall onto a nail from a height of 0.50 m. What is the maximum amount of work it could do on the nail? Why do people not just "let it fall" but add their own force to the hammer as it falls?

3. (II) How much work did the movers do (horizontally) pushing a 46.0-kg crate 10.3 m across a rough floor without acceleration, if the effective coefficient of friction was 0.50?

4. (II) A 1200-N crate rests on the floor. How much work is required to move it at constant speed (*a*) 5.0 m along the floor against a friction force of 230 N, and (*b*) 5.0 m vertically?

5. (II) What is the minimum work needed to push a 950-kg car 710 m up along a 9.0° incline? Ignore friction.

6. (II) Estimate the work you do to mow a lawn 10 m by 20 m with a 50-cm-wide mower. Assume you push with a force of about 15 N.

7. (II) In a certain library the first shelf is 15.0 cm off the ground, and the remaining four shelves are each spaced 38.0 cm above the previous one. If the average book has a mass of 1.40 kg with a height of 22.0 cm, and an average shelf holds 28 books (standing vertically), how much work is required to fill all the shelves, assuming the books are all laying flat on the floor to start?

8. (II) A **lever** such as that shown in Fig. 6–35 can be used to lift objects we might not otherwise be able to lift. Show that the ratio of output force, F_O, to input force, F_I, is related to the lengths ℓ_I and ℓ_O from the pivot by $F_O/F_I = \ell_I/\ell_O$. Ignore friction and the mass of the lever, and assume the work output equals the work input.

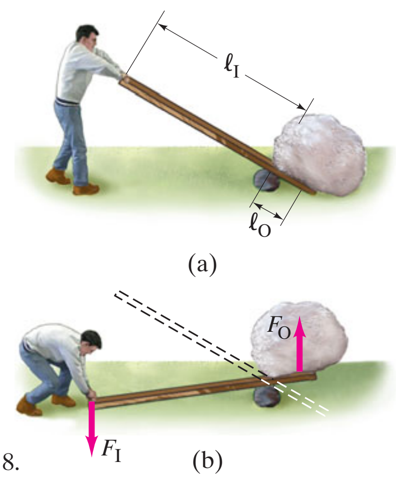


FIGURE 6–35 A lever. Problem 8.

9. (II) A box of mass 4.0 kg is accelerated from rest by a force across a floor at a rate of 2.0 m/s^2 for 7.0 s. Find the net work done on the box.

10. (II) A 380-kg piano slides 2.9 m down a 25° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (Fig. 6–36). Determine: (*a*) the force exerted by the man, (*b*) the work done on the piano by the man, (*c*) the work done on the piano by the force of gravity, and (*d*) the net work done on the piano. Ignore friction.

FIGURE 6–36 Problem 10.

11. (II) Recall from Chapter 4, Example 4–14, that you can use a pulley and ropes to decrease the force needed to raise a heavy load (see Fig. 6–37). But for every meter the load is raised, how much rope must be pulled up? Account for this, using energy concepts.

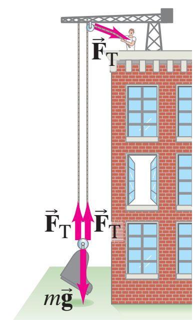


FIGURE 6–37 Problem 11.

12. (III) A grocery cart with mass of 16 kg is being pushed at constant speed up a 12° ramp by a force F_P which acts at an angle of 17° below the horizontal. Find the work done by each of the forces $(m\vec{g}, \vec{F}_N, \vec{F}_P)$ on the cart if the ramp is 7.5 m long.

*6–2 Work, Varying Force

***13.** (II) The force on a particle, acting along the *x* axis, varies as shown in Fig. 6–38. Determine the work done by this force to move the particle along the *x* axis: (*a*) from $x = 0.0$ to $x = 10.0$ m; (*b*) from $x = 0.0$ to $x = 15.0$ m.

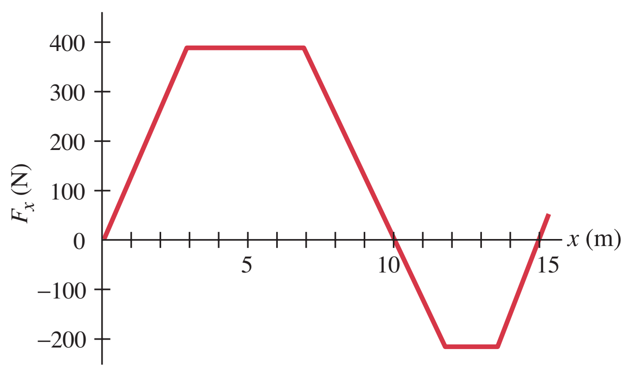
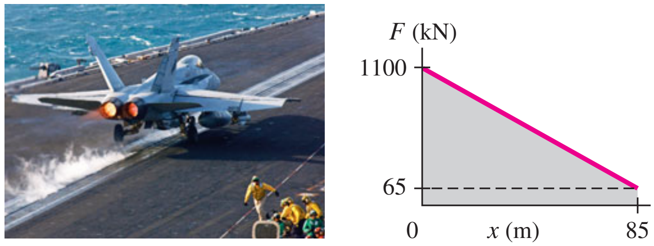


FIGURE 6–38 Problem 13.

*14. (III) A 17,000-kg jet takes off from an aircraft carrier via a catapult (Fig. 6–39a). The gases thrust out from the jet's engines exert a constant force of 130 kN on the jet; the force exerted on the jet by the catapult is plotted in Fig. 6–39b. Determine the work done on the jet: (*a*) by the gases expelled by its engines during launch of the jet; and (*b*) by the catapult during launch of the jet.



(a) (b)

FIGURE 6–39 Problem 14.

6–3 Kinetic Energy; Work-Energy Principle

15. (I) At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a kinetic energy of about 6.21×10^{-21} J. How fast is it moving?

16. (I) (*a*) If the kinetic energy of a particle is tripled, by what factor has its speed increased? (*b*) If the speed of a particle is halved, by what factor does its kinetic energy change?

17. (I) How much work is required to stop an electron $(m = 9.11 \times 10^{-31}\,\text{kg})$ which is moving with a speed of 1.10×10^{6} m/s?

18. (I) How much work must be done to stop a 925-kg car traveling at 95 km/h?

19. (II) Two bullets are fired at the same time with the same kinetic energy. If one bullet has twice the mass of the other, which has the greater speed and by what factor? Which can do the most work?

20. (II) A baseball $(m = 145\,\text{g})$ traveling 32 m/s moves a fielder's glove backward 25 cm when the ball is caught. What was the average force exerted by the ball on the glove?

21. (II) An 85-g arrow is fired from a bow whose string exerts an average force of 105 N on the arrow over a distance of 75 cm. What is the speed of the arrow as it leaves the bow?

22. (II) If the speed of a car is increased by 50%, by what factor will its minimum braking distance be increased, assuming all else is the same? Ignore the driver's reaction time.

23. (II) At an accident scene on a level road, investigators measure a car's skid mark to be 78 m long. It was a rainy day and the coefficient of friction was estimated to be 0.30. Use these data to determine the speed of the car when the driver slammed on (and locked) the brakes. (Why does the car's mass not matter?)

24. (III) One car has twice the mass of a second car, but only half as much kinetic energy. When both cars increase their speed by 8.0 m/s, they then have the same kinetic energy. What were the original speeds of the two cars?

25. (III) A 265-kg load is lifted 18.0 m vertically with an acceleration $a = 0.160\,g$ by a single cable. Determine (*a*) the tension in the cable; (*b*) the net work done on the load; (*c*) the work done by the cable on the load; (*d*) the work done by gravity on the load; (*e*) the final speed of the load assuming it started from rest.

6–4 and 6–5 Potential Energy

26. (I) By how much does the gravitational potential energy of a 54-kg pole vaulter change if her center of mass rises about 4.0 m during the jump?

27. (I) A spring has a spring constant k of 88.0 N/m. How much must this spring be compressed to store 45.0 J of potential energy?

28. (II) If it requires 6.0 J of work to stretch a particular spring by 2.0 cm from its equilibrium length, how much more work will be required to stretch it an additional 4.0 cm?

29. (II) A 66.5-kg hiker starts at an elevation of 1270 m and climbs to the top of a peak 2660 m high. (*a*) What is the hiker's change in potential energy? (*b*) What is the minimum work required of the hiker? (*c*) Can the actual work done be greater than this? Explain.

30. (II) A 1.60-m-tall person lifts a 1.65-kg book off the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to (*a*) the ground, and (*b*) the top of the person's head? (*c*) How is the work done by the person related to the answers in parts (*a*) and (*b*)?

6–6 and 6–7 Conservation of Mechanical Energy

31. (I) A novice skier, starting from rest, slides down an icy frictionless 8.0° incline whose vertical height is 105 m. How fast is she going when she reaches the bottom?

32. (I) Jane, looking for Tarzan, is running at top speed (5.0 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine affect your answer?

33. (II) A sled is initially given a shove up a frictionless 23.0° incline. It reaches a maximum vertical height 1.22 m higher than where it started at the bottom. What was its initial speed?

34. (II) In the *high jump*, the kinetic energy of an athlete is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must the athlete leave the ground in order to lift his center of mass 2.10 m and cross the bar with a speed of 0.50 m/s?

35. (II) A spring with $k = 83$ N/m hangs vertically next to a ruler. The end of the spring is next to the 15-cm mark on the ruler. If a 2.5-kg mass is now attached to the end of the spring, and the mass is allowed to fall, where will the end of the spring line up with the ruler marks when the mass is at its lowest position?

36. (II) A 0.48-kg ball is thrown with a speed of 8.8 m/s at an upward angle of 36°. (*a*) What is its speed at its highest point, and (*b*) how high does it go? (Use conservation of energy.)

37. (II) A 1200-kg car moving on a horizontal surface has speed $v = 85$ km/h when it strikes a horizontal coiled spring and is brought to rest in a distance of 2.2 m. What is the spring stiffness constant of the spring?

38. (II) A 62-kg trampoline artist jumps upward from the top of a platform with a vertical speed of 4.5 m/s. (*a*) How fast is he going as he lands on the trampoline, 2.0 m below (Fig. 6–40)? (*b*) If the trampoline behaves like a spring of spring constant 5.8×10^{4} N/m, how far down does he depress it?

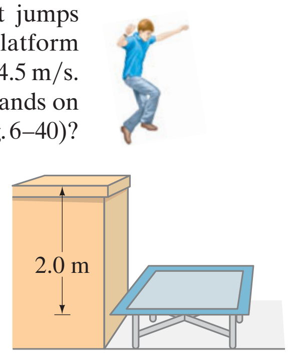


FIGURE 6–40 Problem 38.

39. (II) A vertical spring (ignore its mass), whose spring constant is 875 N/m, is attached to a table and is compressed down by 0.160 m. (*a*) What upward speed can it give to a 0.380-kg ball when released? (*b*) How high above its original position (spring compressed) will the ball fly?

40. (II) A roller-coaster car shown in Fig. 6–41 is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3, and 4.

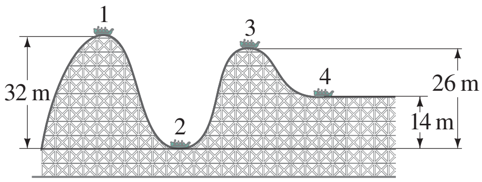


FIGURE 6–41 Problems 40 and 50.

41. (II) Chris jumps off a bridge with a bungee cord (a heavy stretchable cord) tied around his ankle, Fig. 6–42. He falls for 15 m before the bungee cord begins to stretch. Chris's mass is 75 kg and we assume the cord obeys Hooke's law, $F = -kx$, with $k = 55$ N/m. If we neglect air resistance, estimate what distance d below the bridge Chris's foot will be before coming to a stop. Ignore the mass of the cord (not realistic, however) and treat Chris as a particle.

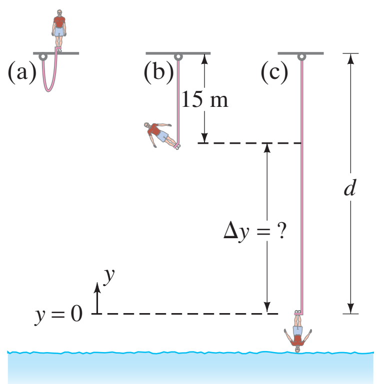


FIGURE 6–42 Problem 41. (a) Bungee jumper about to jump. (b) Bungee cord at its unstretched length. (c) Maximum stretch of cord.

42. (II) What should be the spring constant k of a spring designed to bring a 1200-kg car to rest from a speed of 95 km/h so that the occupants undergo a maximum acceleration of 4.0 g?

43. (III) An engineer is designing a spring to be placed at the bottom of an elevator shaft. If the elevator cable breaks when the elevator is at a height h above the top of the spring, calculate the value that the spring constant k should have so that passengers undergo an acceleration of no more than 5.0 g when brought to rest. Let M be the total mass of the elevator and passengers.

44. (III) A block of mass m is attached to the end of a spring (spring stiffness constant k), Fig. 6–43. The mass is given an initial displacement x_0 from equilibrium, and an initial speed v_0. Ignoring friction and the mass of the spring, use energy methods to find (*a*) its maximum speed, and (*b*) its maximum stretch from equilibrium, in terms of the given quantities.

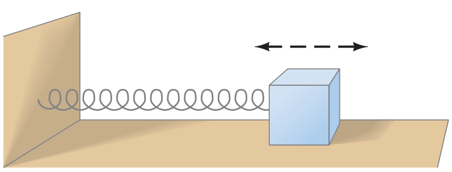


FIGURE 6–43 Problem 44.

45. (III) A cyclist intends to cycle up a 7.50° hill whose vertical height is 125 m. The pedals turn in a circle of diameter 36.0 cm. Assuming the mass of bicycle plus person is 75.0 kg, (*a*) calculate how much work must be done against gravity. (*b*) If each complete revolution of the pedals moves the bike 5.10 m along its path, calculate the average force that must be exerted on the pedals tangent to their circular path. Neglect work done by friction and other losses.

6–8 and 6–9 Law of Conservation of Energy

46. (I) Two railroad cars, each of mass 66,000 kg, are traveling 85 km/h toward each other. They collide head-on and come to rest. How much thermal energy is produced in this collision?

47. (I) A 16.0-kg child descends a slide 2.20 m high and, starting from rest, reaches the bottom with a speed of 1.25 m/s. How much thermal energy due to friction was generated in this process?

48. (II) A ski starts from rest and slides down a 28° incline 85 m long. (*a*) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (*b*) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.

49. (II) A 145-g baseball is dropped from a tree 12.0 m above the ground. (*a*) With what speed would it hit the ground if air resistance could be ignored? (*b*) If it actually hits the ground with a speed of 8.00 m/s, what is the average force of air resistance exerted on it?

50. (II) Suppose the roller-coaster car in Fig. 6–41 passes point 1 with a speed of 1.30 m/s. If the average force of friction is equal to 0.23 of its weight, with what speed will it reach point 2? The distance traveled is 45.0 m.

51. (II) A skier traveling 11.0 m/s reaches the foot of a steady upward 19° incline and glides 15 m up along this slope before coming to rest. What was the average coefficient of friction?

52. (II) You drop a ball from a height of 2.0 m, and it bounces back to a height of 1.6 m. (*a*) What fraction of its initial energy is lost during the bounce? (*b*) What is the ball's speed just before and just after the bounce? (*c*) Where did the energy go?

53. (II) A 66-kg skier starts from rest at the top of a 1200-m-long trail which drops a total of 230 m from top to bottom. At the bottom, the skier is moving 11.0 m/s. How much energy was dissipated by friction?

54. (II) A projectile is fired at an upward angle of 38.0° from the top of a 135-m-high cliff with a speed of 165 m/s. What will be its speed when it strikes the ground below? (Use conservation of energy.)

55. (II) The Lunar Module could make a safe landing if its vertical velocity at impact is 3.0 m/s or less. Suppose that you want to determine the greatest height h at which the pilot could shut off the engine if the velocity of the lander relative to the surface at that moment is (*a*) zero; (*b*) 2.0 m/s downward; (*c*) 2.0 m/s upward. Use conservation of energy to determine h in each case. The acceleration due to gravity at the surface of the Moon is 1.62 m/s^2.

56. (III) Early test flights for the space shuttle used a "glider" (mass of 980 kg including pilot). After a horizontal launch at 480 km/h at a height of 3500 m, the glider eventually landed at a speed of 210 km/h. (*a*) What would its landing speed have been in the absence of air resistance? (*b*) What was the average force of air resistance exerted on it if it came in at a constant glide angle of 12° to the Earth's surface?

6–10 Power

57. (I) How long will it take a 2750-W motor to lift a 385-kg piano to a sixth-story window 16.0 m above?

58. (I) (*a*) Show that one British horsepower (550 ft·lb/s) is equal to 746 W. (*b*) What is the horsepower rating of a 75-W lightbulb?

59. (I) An 85-kg football player traveling 5.0 m/s is stopped in 1.0 s by a tackler. (*a*) What is the original kinetic energy of the player? (*b*) What average power is required to stop him?

60. (II) If a car generates 18 hp when traveling at a steady 95 km/h, what must be the average force exerted on the car due to friction and air resistance?

61. (II) An outboard motor for a boat is rated at 35 hp. If it can move a particular boat at a steady speed of 35 km/h, what is the total force resisting the motion of the boat?

62. (II) A shot-putter accelerates a 7.3-kg shot from rest to 14 m/s in 1.5 s. What average power was developed?

63. (II) A driver notices that her 1080-kg car, when in neutral, slows down from 95 km/h to 65 km/h in about 7.0 s on a flat horizontal road. Approximately what power (watts and hp) is needed to keep the car traveling at a constant 80 km/h?

64. (II) How much work can a 2.0-hp motor do in 1.0 h?

65. (II) A 975-kg sports car accelerates from rest to 95 km/h in 6.4 s. What is the average power delivered by the engine?

66. (II) During a workout, football players ran up the stadium stairs in 75 s. The distance along the stairs is 83 m and they are inclined at a 33° angle. If a player has a mass of 82 kg, estimate his average power output on the way up. Ignore friction and air resistance.

67. (II) A pump lifts 27.0 kg of water per minute through a height of 3.50 m. What minimum output rating (watts) must the pump motor have?

68. (II) A ski area claims that its lifts can move 47,000 people per hour. If the average lift carries people about 200 m (vertically) higher, estimate the maximum total power needed.

69. (II) A 65-kg skier grips a moving rope that is powered by an engine and is pulled at constant speed to the top of a 23° hill. The skier is pulled a distance $x = 320$ m along the incline and it takes 2.0 min to reach the top of the hill. If the coefficient of kinetic friction between the snow and skis is $\mu_k = 0.10$, what horsepower engine is required if 30 such skiers (max) are on the rope at one time?

70. (II) What minimum horsepower must a motor have to be able to drag a 370-kg box along a level floor at a speed of 1.20 m/s if the coefficient of friction is 0.45?

71. (III) A bicyclist coasts down a 6.0° hill at a steady speed of 4.0 m/s. Assuming a total mass of 75 kg (bicycle plus rider), what must be the cyclist's power output to climb the same hill at the same speed?

General Problems

72. Spiderman uses his spider webs to save a runaway train moving about 60 km/h, Fig. 6–44. His web stretches a few city blocks (500 m) before the 10^4-kg train comes to a stop. Assuming the web acts like a spring, estimate the effective spring constant.

FIGURE 6–44 Problem 72.

73. A 36.0-kg crate, starting from rest, is pulled across a floor with a constant horizontal force of 225 N. For the first 11.0 m the floor is frictionless, and for the next 10.0 m the coefficient of friction is 0.20. What is the final speed of the crate after being pulled these 21.0 m?

74. How high will a 1.85-kg rock go from the point of release if thrown straight up by someone who does 80.0 J of work on it? Neglect air resistance.

75. A mass m is attached to a spring which is held stretched a distance x by a force F, Fig. 6–45, and then released. The spring pulls the mass to the left, towards its natural equilibrium length. Assuming there is no friction, determine the speed of the mass m when the spring returns: (*a*) to its normal length $(x = 0)$; (*b*) to half its original extension $(x/2)$.

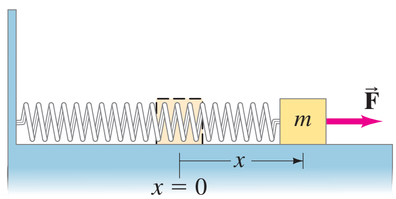


FIGURE 6–45 Problem 75.

76. An elevator cable breaks when a 925-kg elevator is 28.5 m above the top of a huge spring $(k = 8.00 \times 10^4$ N/m$)$ at the bottom of the shaft. Calculate (*a*) the work done by gravity on the elevator before it hits the spring; (*b*) the speed of the elevator just before striking the spring; (*c*) the amount the spring compresses (note that here work is done by both the spring and gravity).

77. (*a*) A 3.0-g locust reaches a speed of 3.0 m/s during its jump. What is its kinetic energy at this speed? (*b*) If the locust transforms energy with 35% efficiency, how much energy is required for the jump?

78. In a common test for cardiac function (the "stress test"), the patient walks on an inclined treadmill (Fig. 6–46). Estimate the power required from a 75-kg patient when the treadmill is sloping at an angle of 12° and the velocity is 3.1 km/h. (How does this power compare to the power rating of a lightbulb?)

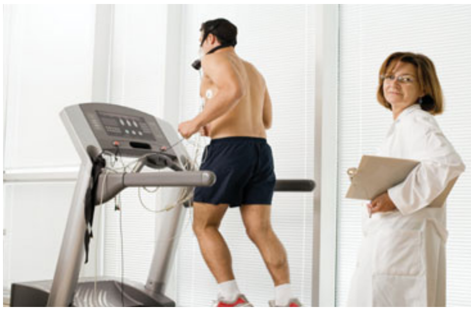


FIGURE 6–46 Problem 78.

79. An airplane pilot fell 370 m after jumping from an aircraft without his parachute opening. He landed in a snowbank, creating a crater 1.1 m deep, but survived with only minor injuries. Assuming the pilot's mass was 88 kg and his speed at impact was 45 m/s, estimate: (*a*) the work done by the snow in bringing him to rest; (*b*) the average force exerted on him by the snow to stop him; and (*c*) the work done on him by air resistance as he fell. Model him as a particle.

80. Many cars have "5 mi/h (8 km/h) bumpers" that are designed to compress and rebound elastically without any physical damage at speeds below 8 km/h. If the material of the bumpers permanently deforms after a compression of 1.5 cm, but remains like an elastic spring up to that point, what must be the effective spring constant of the bumper material, assuming the car has a mass of 1050 kg and is tested by ramming into a solid wall?

81. In climbing up a rope, a 62-kg athlete climbs a vertical distance of 5.0 m in 9.0 s. What minimum power output was used to accomplish this feat?

82. If a 1300-kg car can accelerate from 35 km/h to 65 km/h in 3.8 s, how long will it take to accelerate from 55 km/h to 95 km/h? Assume the power stays the same, and neglect frictional losses.

83. A cyclist starts from rest and coasts down a 4.0° hill. The mass of the cyclist plus bicycle is 85 kg. After the cyclist has traveled 180 m, (*a*) what was the net work done by gravity on the cyclist? (*b*) How fast is the cyclist going? Ignore air resistance and friction.

84. A film of Jesse Owens's famous long jump (Fig. 6–47) in the 1936 Olympics shows that his center of mass rose 1.1 m from launch point to the top of the arc. What minimum speed did he need at launch if he was traveling at 6.5 m/s at the top of the arc?

FIGURE 6–47 Problem 84.

85. Water flows over a dam at the rate of 680 kg/s and falls vertically 88 m before striking the turbine blades. Calculate (*a*) the speed of the water just before striking the turbine blades (neglect air resistance), and (*b*) the rate at which mechanical energy is transferred to the turbine blades, assuming 55% efficiency.

86. A 55-kg skier starts from rest at the top of a ski jump, point A in Fig. 6–48, and travels down the ramp. If friction and air resistance can be neglected, (*a*) determine her speed v_B when she reaches the horizontal end of the ramp at B. (*b*) Determine the distance *s* to where she strikes the ground at C.

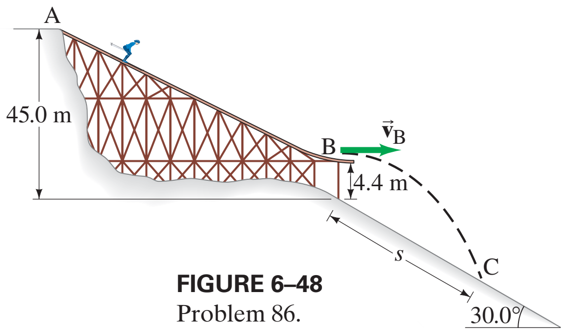


FIGURE 6–48 Problem 86.

87. Electric energy units are often expressed in "kilowatt-hours." (*a*) Show that one kilowatt-hour (kWh) is equal to 3.6×10^6 J. (*b*) If a typical family of four uses electric energy at an average rate of 580 W, how many kWh would their electric bill show for one month, and (*c*) how many joules would this be? (*d*) At a cost of $0.12 per kWh, what would their monthly bill be in dollars? Does the monthly bill depend on the *rate* at which they use the electric energy?

88. If you stand on a bathroom scale, the spring inside the scale compresses 0.60 mm, and it tells you your weight is 760 N. Now if you jump on the scale from a height of 1.0 m, what does the scale read at its peak?

89. A 65-kg hiker climbs to the top of a mountain 4200 m high. The climb is made in 4.6 h starting at an elevation of 2800 m. Calculate (*a*) the work done by the hiker against gravity, (*b*) the average power output in watts and in horsepower, and (*c*) assuming the body is 15% efficient, what rate of energy input was required.

90. A ball is attached to a horizontal cord of length ℓ whose other end is fixed, Fig. 6–49. (*a*) If the ball is released, what will be its speed at the lowest point of its path? (*b*) A peg is located a distance *h* directly below the point of attachment of the cord. If $h = 0.80\ell$, what will be the speed of the ball when it reaches the top of its circular path about the peg?

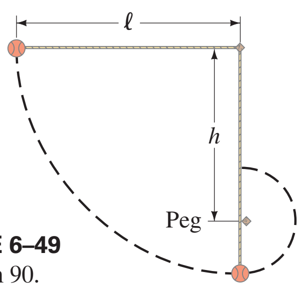


FIGURE 6–49 Problem 90.

91. An 18-kg sled starts up a 28° incline with a speed of 2.3 m/s. The coefficient of kinetic friction is $\mu_k = 0.25$. (*a*) How far up the incline does the sled travel? (*b*) What condition must you put on the coefficient of static friction if the sled is not to get stuck at the point determined in part (*a*)? (*c*) If the sled slides back down, what is its speed when it returns to its starting point?

92. A 56-kg student runs at 6.0 m/s, grabs a hanging 10.0-m-long rope, and swings out over a lake (Fig. 6–50). He releases the rope when his velocity is zero. (*a*) What is the angle θ when he releases the rope? (*b*) What is the tension in the rope just before he releases it? (*c*) What is the maximum tension in the rope during the swing?

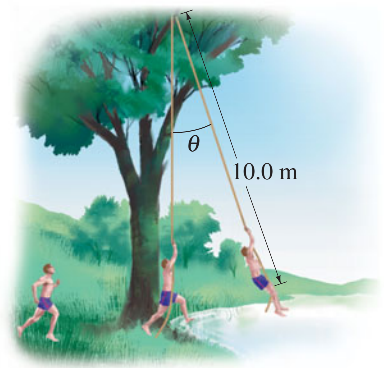


FIGURE 6–50 Problem 92.

93. Some electric power companies use water to store energy. Water is pumped from a low reservoir to a high reservoir. To store the energy produced in 1.0 hour by a 180-MW electric power plant, how many cubic meters of water will have to be pumped from the lower to the upper reservoir? Assume the upper reservoir is an average of 380 m above the lower one. Water has a mass of 1.00×10^3 kg for every $1.0\ m^3$.

94. A softball having a mass of 0.25 kg is pitched horizontally at 120 km/h. By the time it reaches the plate, it may have slowed by 10%. Neglecting gravity, estimate the average force of air resistance during a pitch. The distance between the plate and the pitcher is about 15 m.

Search and Learn

1. We studied forces earlier and used them to solve Problems. Now we are using energy to solve Problems, even some that could be solved with forces. (*a*) Give at least three advantages of using energy to solve a Problem. (*b*) When must you use energy to solve a Problem? (*c*) When must you use forces to solve a Problem? (*d*) What information is not available when solving Problems with energy? Look at the Examples in Chapters 6 and 4.

2. The brakes on a truck can overheat and catch on fire if the truck goes down a long steep hill without shifting into a lower gear. (*a*) Explain why this happens in terms of energy and power. (*b*) Would it matter if the same elevation change was made going down a steep hill or a gradual hill? Explain your reasoning. [*Hint:* Read Sections 6–4, 6–9, and 6–10 carefully.] (*c*) Why does shifting into a lower gear help? [*Hint:* Use your own experience, downshifting in a car.] (*d*) Calculate the thermal energy dissipated from the brakes in an 8000-kg truck that descends a 12° hill. The truck begins braking when its speed is 95 km/h and slows to a speed of 35 km/h in a distance of 0.36 km measured along the road.

3. (*a*) Only two conservative forces are discussed in this Chapter. What are they, and how are they accounted for when you are dealing with conservation of energy? (*b*) Not mentioned is the force of water on a swimmer. Is it conservative or nonconservative?

4. Give at least two examples of friction doing positive work. Reread parts of Chapters 4 and 6.

5. Show that on a roller coaster with a circular vertical loop (Fig. 6–51), the difference in your apparent weight at the top of the loop and the bottom of the loop is 6.0 times your weight. Ignore friction. Show also that as long as your speed is above the minimum needed (so the car holds the track), this answer doesn't depend on the size of the loop or how fast you go through it. [Reread Sections 6–6, 5–2, and 4–6.]

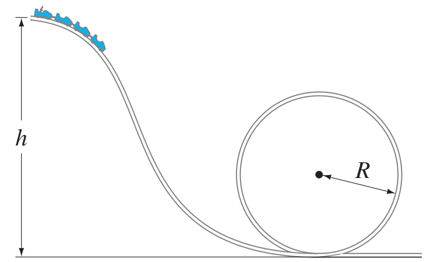


FIGURE 6–51 Search and Learn 5 and 6.

6. Suppose that the track in Fig. 6–51 is not frictionless and the values of h and R are given. (See Sections 6–9 and 6–1.) (*a*) If you measure the velocity of the roller coaster at the top of the hill (of height h) and at the top of the circle (of height $2R$), can you determine the work done by friction during the time the roller coaster moves between those two points? Why or why not? (*b*) Can you determine the average force of friction between those two points? Why or why not? If not, what additional information do you need?

ANSWERS TO EXERCISES

A: (*c*).
B: (*a*) Less, because $(20)^2 = 400 < (30)^2 - (20)^2 = 500$; (*b*) 2.0×10^5 J.
C: No, because the speed v would be the square root of a negative number, which is not real.
D: (*a*) $\sqrt{2}$; (*b*) 4.
E: Yes. It is nonconservative, because for a conservative force $W = 0$ in a round trip.
F: (*e*), (*e*); (*e*), (*c*).